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OPTIMIZATION OF WORM GEARS PARAMETERS BY THE NON-LINEAR PROGRAMMING METHODS

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Summary. The purpose of the given research: to determine the optimum combination of the WG parameters, which could provide the minimum value of the target function, depending on the given criteria of working capacity.

Key words: WG, parameters, non-linear programming.

INTRODUCTION

The existing standards determining the parameters of the warm gearing for the cylindrical worms ZA, Z1, ZN, in fact are invariant with respect to the conditions of operation: loading conditions, required service life, etc. It means that the choice of the main parameters for a worm gear (W.G.) – [z_1,q,x] - is done according to the common recommendations. At the same time, a difference in the exploitation period – long or short, loading conditions – T_2=*const* or T_2=var *ia*, required sybarites, thermal regime etc, under the same sets [z_1,q,x] will lead to different results as for the working capacity of WG. In this way the problems of optimization of W.G. parameters in numerous investigations, for example in [1,2,3], were not considered. In work [4] the choice of the rational values [z_1,q,x] is done in the first approach, then the target function is represented by the sum of the normalized functions, determining the main standards of WG working capacity.

STATEMENT OF THE TASK

The purpose of the given research was to determine the optimum combination of the WG parameters [z_1,q,x], which could provide the minimum value of the target function $S = S$ [z_1,q,x], depending on the given criteria of working capacity.

1. Criterion function for optimization of WG

As the criteria of WG working capacity loading ability (I), relative efficiency of gearing (II), heat resistance (III), wear resistance (IV), jamming resistance (V), were under consideration.

It is possible to unify the procedure of optimization of W.G. parameters if it is necessary to minimize all the criteria functions I, II, III, IV and V for it. In criterion III, IV and V it is going on in a natural way as the minimum values of these criteria correspond to the best induces of WG working. As for criteria I and II we deal with them in a different way. In criterion I instead of the loading factor T_2 the interaxial a_w will be considered. Under the fixed T_2 minimizing a_w will correspond to the top loading ability of WG. As for criterion II instead of gearing efficiency (η) the reverse function $\eta_0 = 1/\eta$, whose minimum corresponding to η = max, will be under consideration. So minimization of criteria functions I,II,III,IV,V (previously normalized) will direct the value of the target function S to S = max.

The target function $S = S[z_1,q,x]$ for the real WG with the limits for the modifying parameters $[z_1,q,x]$ in the mathematical sense does not have even local minimum if the optimization is done through more then two criteria. That is why by $S = S[z_1,q,x]$ = min we will agree to take the least of all the possible values S under the given initial conditions.

We will produce each of the five criteria functions analytically or as algorithm.

1.1. Criterion I.

Initial dependence for a_w is widely used in the project calculation of WG and expresses the condition of the contact durability of the worm wheel teeth:

$$a_w = c_a\left(1+\frac{u\cdot z_1}{q+2x}\right)\cdot\left(\frac{q+2x}{u\cdot z_1}\right)^{2/3},$$

where $c_a = \left(5300/[\sigma_н]\right)^{2/3}\cdot\left(K_н\cdot T_2\right)^{1/3}$ - is the constant for the given initial conditions. For the optimization procedure dimensionless function $\varphi_1 = a_w / c_a$, proportional to a_w will be used:

$$\varphi_1 = \varphi_1(z_1,q,x) = \left(1+\frac{u\cdot z_1}{q+2x}\right)\cdot\left(\frac{q+2x}{u\cdot z_1}\right)^{2/3}. \quad (1)$$

Minimization of function (1) under the fixed $[\sigma]$ and T_2 corresponds to reaching WG of the maximum loading ability.

1.2. Criterion II.

The index of the energy losses of WG – efficiency of gearing is defined according to method [5]:

$$\eta = 1-\left(\psi'_c+\psi'_k\right). \quad (2)$$

The given coefficients of losses from slippage ($\psi'_ń$) and rolling (ψ'_k) finally are the functions from the optimized parameters of gearing z_1,q,x

$$\psi'_c = \left(B_c - C_c\cdot\ln V_s\right)\cdot\left[\frac{q}{z_1}+tg\left(\gamma_w+\varphi'\right)\right];$$

$$\psi'_k = \left(B_k - C_k\ln V_\Sigma\right)\cdot 2{,}6\frac{1+1{,}2|x|}{u\cdot z_1},$$

where: B_c, C_c, B_k, C_k are the constants depending on the material of the wheel teeth and the sort of the lubrication, [5, c.15, table. 5];

$V_s = \dfrac{m\cdot n_1\cdot(q+2x)}{19100\cdot\cos\gamma_w}$ - the speed of sliding of the worm coils, m/s,

$V_\Sigma = 2V_2\cdot\sin\alpha_{tw} = \dfrac{m\cdot z_1 n_1}{955}\cdot\sin\alpha_{tw}$ - the speed of rolling of the WG operating surface, m/s.

After normalizing parameters V_s and V_Σ will be defined by the following relations:

$$V_s = V_s(z_1, q, x) = \sqrt{(q+2x)^2 + z_1^2} \,/\, (q + 2x + z_1);$$
$$V_\Sigma = V_\Sigma(z_1, q, x) = 2tg\alpha_{tw} \cdot z_1 \cdot V_s \,/\, \sqrt{(q+2x)^2 \cdot (1 + tg^2\alpha_{tw}) + (z_1 \cdot tg\alpha_{tw})^2}.$$

As a result the unknown criterion function of the efficiency will be defined

$$\varphi_2 = \varphi_2(z_1, q, x) = 1/\eta = \frac{1}{1 - (\psi_c' + \psi_л')} \tag{3}$$

Minimizing $\varphi_2 = \varphi_2(z_1,q,x)$ the value $\eta = \max$ is automatically reached under the given initial conditions (T_2,$[\sigma_i]$,$ï_1$...).

1.3. Criterion III.

The thermal criterion is presented on the basis of common for engineer practice condition of the thermal balance for worm gears in the natural cooling regime:

$$\Delta t^0 = t_м^0 - t_в^0 = \frac{P_1(1-\eta)}{K_e \cdot A \cdot (1+\psi)}. \tag{4}$$

After rate setting and corresponding transformations heat resistance criterion function looks like this

$$\varphi_3 = \varphi_3(z_1, q, x) = \frac{(q+2x) \cdot [f_c \cdot (q+2x) + z_1] \cdot (1-\eta)}{u \cdot z_1 \cdot [(q+2x) - f_c \cdot z_1]}, \tag{5}$$

where $f_c = B_c - C_c \cdot \ln V_s$, defined in criterion II.

1.4. Criterion IV.

Despite a big number of researches on wear resistance as a whole and as for WG in particular, [6, 7, 8] and others, this criterion of working capacity has not got the universal and rather reliable method of calculation. In this work criterion IV is evaluated quantitatively not by its absolute value but in comparative aspect under different parameter combination [z_1,q,x]. For this reason the linear dependence of wearing intensity (J) on specific pressure (q_n) in the point of operating areas and speed of their sliding (V_s) is adopted:

$$J = q_n \cdot V_s \cdot a. \tag{6}$$

In conformity with the given research expression (1.6) in the fixed aspect will give unknown criterion function of wear resistance φ_4:

$$\varphi_4 = \varphi_4(z_1, q, x) = \frac{(q + 2x + z_1)\sqrt{(q+2x)^2 + z_1^2}}{u \cdot z_1 (q+2x)}. \tag{7}$$

1.5. Criterion V.

The calculation criterion for the second stage of seizing (distraction of the oily film and friction of the clean juvenile surfaces) after Block [9], is presented as dimensionless function φ_5:

$$\varphi_5 = \varphi_5(z_1,q,x)\ T_2 / T_{2a}, \tag{8}$$

where: T_2 - is the rolling moment on the wheel given under the initial conditions of the WG calculation; $T_{2a} = \left(\theta^0_{кр} - t^0_в\right) / \left(A + B\right)$ - is the rolling moment on the wheel under the initial condition of no searing.

All geometry – kinematics characteristics included in to T_{2a} [9] given through the optimized parameters $[z_1,q,x]$. That is why the unknown function φ_5, under the given $T_2 = const$, will be minimized under T_{2a} = max. It corresponds to the maximum stableness of WG according to the criterion under the given T_2.

Boundary conditions.

The limits of the optimized gearing parameters $[z_1,q,x]$ are defined by the existing standards on projecting power WG with the worms of the types *ZA*, *Z*1, *ZN*, *ZK*:

z_1 = 1; 2; 4.

q = 6,3; 7,1; 8; 9; 10; 11,2; 12,5; 14; 16; 18; 20; 22,4; 25.

x = [–1,0... + 1,0], taking any current value in these limits.

1. The task formalization through Lagrange multiplier.

As it was mentioned about the task multicriterion is conditioned by the essence of the scientific and technical problems which are being solved here. In the case when it is necessary to solve a multicriterion task punctually and in time it is necessary to use cybernetics principles both in formalizing and algorithmization of the task and in its solving. We would remind that the core of the algorithms of automatic taking decisions in descriptive modeling is the tasks of the non-linear mathematical programming (NMP). In the time of formalizing for determined and quasidetermined processes we will use the task NMP which following [10] has the form:

$$\min_x F(x), \tag{9}$$

$$g_j(x) \geq 0, j = 1,2,3,..., m, \tag{10}$$

$$a_i \leq x_i \leq b_i, i = 1,..., n, x \in S \subset R^n. \tag{11}$$

The components of vector x are sought for parameters of the initial task. In our case $m \leq 7$, $n = 3$, and according to the designations introduced for the components of vector x are: $x_1 = z1$; $x_2 = q$; $x_3 = x$ Taking into consideration the importance of limits in temperature as the main target function for the initial problem we will introduce criterion *III*, including Furje thermal equation [11]. The other criteria and limits (10) are introduced into the target function as items with positively defined values realizing one of the types of the error criterion [10] – in this way the initial task of the conditional minimization is converted into the sucession of tasks of the unconditional minimization. It is caused by the fact that there is a reliable number methods of solving only for solving of absolute optimization. Modern information technology makes high demands of the integral error in all the stages of the project realization. That is why in the initial stage of estimations in the calculation experiment we used a number of methods in which the heuristic aspect is dominating that is the engineering approach in technology of calculating was under consideration. To ground the descriptive modeling the calculations in the simplified scheme were made keeping to the tactics given above.

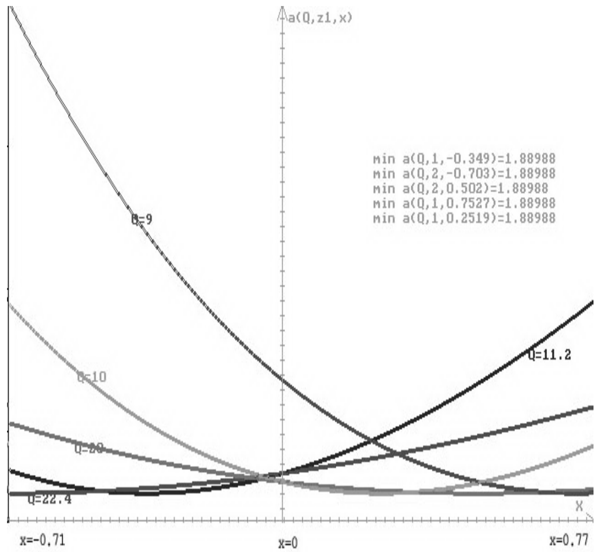

Fig.1 Graphical representation of the task solving for the extremum search according the first criterion with interval (-0.71, 0.77)

For this the task NMP without introducing functional limitations (other criteria), but considering evident restrictions for parameters (boundary conditions) was represented in the same (1) aspect with preserving notations:

$$\min_{z1,q,x} aw(z1,q,x) = \left(\left(1+u\cdot z1/(q-2x)\right)\cdot\left((q-2x)(u\cdot z1)\right)^{2/3}\right). \tag{12}$$

were *aw* as before is the target function according to the criterion *I*, and $z1$, q, x - are the sought parameters under the given u. The task was solved by Monte-Carlo method through the previously given type sizes of parameters $z1$, q, and variable parameter x on which the limitation of aspect (2.3) was put on. The least values *aw* were memorized in the lines of bimeasured massif 10×4 to extract data for their graphical specification. In Fig.1 there were shown diagrams of functions (1) for five pairs of typesizes ($z1$, q) which deliver similar in its meaning target function (1.1), minimum in parameter x. There will be the same result if the necessary condition of extremum is used. For this we will write down (1) as $aw(\zeta) = \left(1+\left(1/(\zeta)\right)\cdot(\zeta)^{2/3}\right)$, where $\zeta = \left((q-2x)/(u\cdot z1)\right)$ - is a newly introduced variable as the function of the same parameters. For the search of the solution in this case instead of task (12) we solve the equations $aw(\zeta) = 0$ for the same five pairs type sizes ($z1$, q). It is easy to see that at this stage we imitated the traditional sorting out of variants (method of «Tests and Errors»). Now the change is being grounded (12) for the limitation like (10), that is inequality $aw(z1, q, x) \geq 1.88988$ for using it directly in the descriptive model. For many years experience of research has shown that criteria *II – V* also represented by the limitations of (11) type would be calculation of formulas not linearized to that degree.

During the practical realization in the process of the task transformation there were used the quadratic error criterion and the error criterion set by the reverse function. The choice algorithm of the parameter error criterion R is based on the usage of increasing degree of number 10 or 2. In case when under the definition of global extremum expenditures on the derivatative calculations are great, we will use the Monte-Carlo method. On the first stage the given obvious limitation on parameters (11, afforded to realize the following effective and simplest procedure:

1. We calculate the value of the target function for the point of approaching to coordinates $(a_i + b_i)/2$.
2. We generate n random numbers belonging to intervals (a_i,b_i) and the value of the target function is calculated.
3. If the value of the target function is less than initial, the points are memorized in the line of not big depth (otherwise it is rejected).
4. Steps 2 and 3 are repeated the set number of time or the procedure is interrupted according to the criterion of precision.

Further, the formed records of the results are used, first for forming vector-direction of the local minimum search, second as the initial approach in Newton multiparametrical method. For the convenience of formalization in the choice of better algorithm (the method of inside or outside point, the type of etc.), we will introduce two numerical indexes of productivity of programmed modulus: productivity (the volume of information processed by the system within a unit of time) and reactivity (the time between the introduction of incoming data to the system and appearance of corresponding outcoming daft). Through Monte-Carlo method, if reactivity is ignored, the result can be received with any in advance set precision. For this it is enough to change limitations (11), using the method of enclosed segments, this is what we have done. Once again we will apply the graphic interpretation of the solution of the set task in extremum. The minimum of the target function received through Monte-Carlo method is represented by the optimum three parameters (the others were introduced: n1=955, u=21, T2=1148, also efficiency=90%, aw=1.88988):

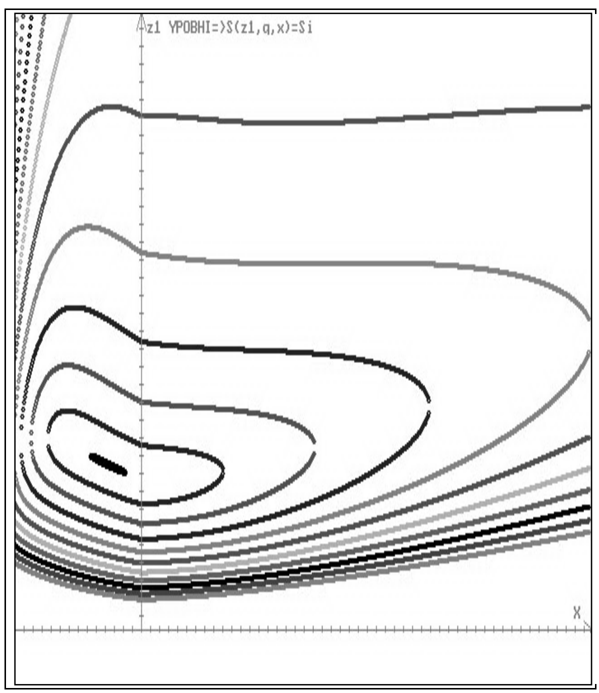

Fig.2 Graphical representation of the lines of the target function level without limitations put down as the task NMP (in interval (-7.0,25))

I, $S_{\min}$ = 0.00000156, *z1* = 1.306, *q* = 15.343, *x* = –0.816,
II, $S_{\min}$ = 0.0740, *z1* = 3.98, *q* = 6.307, *x* = –0.8861,
I – II, $S_{\min}$ = 0.0056, *z1* = 1.235, *q* = 16.073, *x* = –0.8565,
I – III, $S_{\min}$ = 4.471, *z1* = 1.599, *q* = 6.33, *x* = –0.88577,
I – IV, $S_{\min}$ = 0.684, *z1* = 2.00, *q* = 6.41, *x* = –0.9954,
I – V, $S_{\min}$ = 0.685, *z1* = 2.05, *q* = 6.371, *x* = –0.9815.

Fig. 2 presents ten lines of the constant value of the target function constructed according to the first two criteria and constants S_i, calculated according to formula $S_i = S_{min} + iH$, where $H = (S_{min} – S_{min})/n$, $i = 1,...,n$, $n = 10$. In the diagram you can see the defects of the set task. Firstly, in the zero there is a rupture of the surface (the broken smoothness in criterion *II*, as multiplier $(1 + 1{,}2\ |x|)$ is introduced, look the estimation of efficiency), secondly, the line of the level look like ellipse with the great variation of semi axes as the target function is reached. Here there are difficulties as for calculating: special number methods of solving tasks in extremum and algorithms of approximation of derivatives are required.

CONCLUSIONS.

Thus, the salvation with great integral error is received. In perspective, with introduction of the virtual model of worm gear (WG) for conducting calculation experiments, the descriptive model can be improved. Then it will be possible to check the WG working capacity in dynamics, conducting virtual experiments and imitating WG work within any time interval.

REFERENCES

Dusev I.I., Baturina N.Y.: Multicriterion synthesis of worm gears with optimum parameters /Novocherk. polytech. in-te. – Novocherkassk, 1987. № 344.
Schulz V.V., Tihomirov V.V.: Geometrical optimization on the cylindrical worm gear wearing. // Engineering news. – 1985. - № 9. – p. 27…44.
Bernatskyi I.P., Vjushking N.I., Gerasimov V.K., Komkov V.N.: Rational choice of parameters of gearing of worm cylindrical gears.//In book: Toothed and worm gears. L.: Engineering, 1974. – p. 193…210.
Shevchenko S.V.: Optimization of the parameters of the worm gears. – Voroshilovgrad, 1986. – 16 p.
Worm cylindrical reductors of general engineering application. The calculation of the coefficient. – M., 1985. – 41 p.
Kragelskyi I.V., Mihin N.M.: Friction units of the machine. Reference book. – M.: Engineering, 1984. – 280 p.
Friction, wearability and lubrication. Reference book Book 2. /Editor I.V. Kragelskyi and V.V. Alisin. – M.: Engineering, 1979. – 358 p.
Kodnir D.S.: Contakt hydrodynamics of lubrication of machine units. – M.: Engineering, 1976. – 303 p.
Worm cylindrical transmissions. Calculations of strength and seizing. – M.: Scientific-research institute of Engineering, 1982. – 84 p.
Recleytis G., Reyvindrfn A., Regedel K.: Optimization in engineering: In 2 nd books. M.: Mir, 1986. v.1. 349 p.

Vishnevskyi A.V.: Stochastic contact model of two rough bodies and calculation of temperature distribution in each of them. //News of East-Ukrainian National University. – 2003. – №8(66) . – 30 p.

OPTYMALIZACJA PARAMETRÓW WG PRZY UŻYCIU METOD NIELINEARNEGO PROGRAMOWANIA

Streszczenie. Celem niniejszych badań było ustalenie optymalnej kombinacji parametrów WG, które zapewniłyby minimalną wartość funkcji docelowej w zależności od danych kryteriów zdolności roboczej.

Słowa kluczowe: WG, parametry, programowanie nielinearne.