# WORKING OUT THE EQUATION OF THE "GRAVIMET" TRACTOR MOVEMENT BY THE METHOD OF OSTROGRADSKY – JACOBY

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**Summary.** The paper presents a method that allows to improve the locomotion resources of the wheeled tractor by an addition of gravitational components of wheels that are loading by additional mass. The equation of shifting tractor with that equipment is also presented.

Key words: dynamics, tractor, theorem, system, method, equation, parabola.

## INTRODUCTION

The modernization problem of the wheeled tractor constructions, increase of locomotion resources, increase of the wheel mover efficiency and reduction of slipping has always been settled without any theoretical argumentation.



Fig. 1. The geometrical model of the leveling of the tractor "Gravimet" running: 1 – running device; 2 – weight m with mass t; 3,4 – wheel movers; 5 – dumping connection Hoever, one is needed for making new types of wheel movers and modernization of the old ones, for increasing such quality indices [4, 5, 6, 7, 8, 10] as the alteration of the theoretic approaches to the creation of the perspective (long – term) wheel moves and the practical measures as for gearing and accumulating of power in the movers, the dynamics of the interaction of additional loadings with the gravitational loading.

We offered a method of increasing the locomotion resources of the wheeled tractor 1 (figure 1) when at the moment of its run the gravitational component of the wheel movers 3, 4 loading from the additional mass 2 adds the pushing effort.

#### THE METHODOLOGY OF INVESTIGATION

Let's examine the work of power of weight m in the field of tractive effort.

Tractive effort, the work of which does not depend on the trajectory of its adjusting point, is the power that has a potential.

Let's find the leveling of running of traction device with the help of Ostrogradsky -Jacoby method. Let's connect axes Z and Y with a running device. Let's chose the dekart coordinates as generalized coordinates.

Then the kinetic energy of the traction device is the following:

$$T = \frac{1}{2}m(y^2 + z^2).$$
 (1)

The potential energy of the weight m we will calculate with formula:

$$\Pi = mgh. \tag{2}$$

For such view in construction plan of locomotion means Lagrange function looks like the following:

$$L = T + \Pi = \frac{1}{2m} (m^2 + \dot{z}^2) + mgz.$$
 (3)

We find the expressions for general impulses:

$$P_{y} = \frac{dL}{dy} = m\dot{y}; \ P_{z} = \frac{dL}{dz} = mz, \tag{4}$$

hence

$$\dot{y} = \frac{P_y}{m}; \ \dot{z} = \frac{P_z}{m}.$$

Then Hamiltonian function for this locomotion means is:

$$H = T + \Pi = \frac{1}{2m} \left( P_{y}^{2} + P_{z}^{2} \right) + mgz = h.$$
(5)

To define the characteristic function W we replace in the equation (5) general impulse by partial derivatives from characteristic function according to appropriate coordinates and we receive Ostrogradsky-Jacoby equality:

$$\left(\frac{d\omega}{dy}\right)^2 + \left(\frac{d\omega}{dz}\right)^2 + 2m^2gz = 2mh.$$
(6)

As coordinate Y does not enter explicity in the expression of the function H, this coordinate is cyclic.

Let's assume:

$$\omega = \beta y + f(z).$$

Calculate partial derivatives from W and substitute them in the equation.

$$\beta^2 + \left(\frac{df}{dz}\right)^2 + 2m^2 gz = 2mh.$$
<sup>(7)</sup>

Then from the equation (7) we receive:

$$\frac{df}{dz} = \pm \sqrt{2mh - \beta^2 - 2m^2 gz}$$

Let's integrate this equation within an additive constant:

$$f = (z, \beta, h) = \mp \frac{1}{3m^2g} \left(\sqrt{2mh - \beta^2 - 2m^2gz}\right)^3.$$
(8)

Finally, the characteristic function will be as follows:

$$W = \beta y + f(z, \beta, h) + c.$$
(9)

As per Ostrogradsky-Jacoby theorem the full independent integrals system of the canonical motion equations of the traction device will be as follows:

$$\frac{d\omega}{d\beta} = y + \frac{df}{d\beta} = b,$$
(10)
$$\frac{d\omega}{dh} = y + \frac{df}{dh} = t - t_{0}.$$

Apparently the equations will be as follows:

$$y + \frac{\beta}{m^2 g} F(z,\beta,h) = b$$

$$-\frac{1}{mg} F(z,\beta,h) = t - t_0$$
(11)

(12)

Where:

$$(y-b)^{2} = \frac{\beta^{2}}{m^{4}g^{2}}(2mh-\beta^{2}-2m^{2}gz).$$
(13)

The obtained parabola equation located in the Oyz, plain along with axis parallel to the Z – axis.

The second system – defined equation determines the m-mass motion equation as follows:

 $F(z,\beta,h) = \pm \sqrt{2mh - \beta^2 - 2m^2gz}.$ 

$$\frac{2mh - \beta^2 - 2m^2 gz}{m^2 g^2} = (t - t_0).$$
(14)

From equation we find:

$$z = \frac{2mh - \beta^2}{2m^2g} - \frac{g}{2}(t - t_0)^2.$$

#### CONCLUSION

1. Using Ostrogradsky-Jacoby method allows to estimate the dynamics of the process of moving of the locomotion hauling means.

2. There have been found the integrals of the canonical system of the hauling mean which simplify the integration of these equation system.

3 According to the method of Ostrogradsky-Jacoby the equation of the parabolic law of moving describes the hauling means.

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## ROZWIĄZANIE RÓWNANIA OPISUJĄCEGO RUCH CIĄGNIKA "GRAVIMET" METODĄ OSTROGRADSKY-JACOBY

**Streszczenie.** W artykule zaprezentowano metodę pozwalającą na usprawnienie napędu ciągnika kołowego poprzez dodanie elementów zamachowych do kół, które są obciążane przez dodatkową masę. Zaprezentowano również równie ruchu ciągnika wyposażonego w dodatkowe elementy.

Słowa kluczowe: dynamika, ciągnik, twierdzenie, system, metoda, równanie