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DESIGN OF PROCESS OF FRICTION OF ROLLING WITH SLIDING

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Summary. In the article the question of design of processes of friction is considered at rolling with sliding. A mathematical model of the interaction between wheel pairs and rails for different friction conditions and arbitrarily assigned position of wheel pairs relative to rails was made. The proposed model is the basis for determining tractive and braking properties for railway vehicles on design stages and gives input characteristics for the development of control strategies for a microprocessor control system.

Key words: the rolling with sliding, external friction, rolling contact

INTRODUCTION

A sliding friction and friction of rolling with sliding are the most widespread types of friction in engineering. So, for example a friction of rolling with sliding is in toothed and toothed-spiral transmissions or between wheels and rails.

An experimental investigation on the process of friction interaction between the wheels of a railway vehicle and rails was carried out by using many methods for different locomotives and conditions. The obtained results allow for the possibility to put forth the following common laws for the adhesion process:

- The basis of adhesion is the force of external friction,
- Realization of adhesion force is impossible without a slip of the wheel relative to the rail,
- Decrease of the adhesion coefficient, when the velocity of a locomotive has increased, depends on the dynamic characteristics of wheel–rail interaction,
- Critical slip, the wheel slip corresponding to the maximum adhesion coefficient, depends on friction conditions of the contact,
- If friction condition of the contact begins to be worse, then the maximum value (peak value) of adhesion coefficient on dependence of adhesion coefficient from slip is decreasing and shifted to the zone of large value of the slip,
- When the load from the wheel to the rail has increased, then the adhesion coefficient decreases.

The calculation of wheel–rail contact forces is possible by means of various mathematical models. At the present time all of these models can be divided into several groups. However, most of these models use the exact theory of Kalker [1] or the simplified theory [2]. The main disadvantage of these models is the long calculation time.

Other models use the fast methods for calculations (for example, simple saturation functions, pre-calculated values, etc.) [3–5]. The main advantage of these methods is the short time needed for calculations of wheel–rail contact forces. Almost all of the models do not take into account the real process in wheel–rail contact and friction conditions.

TASK AND MODEL FORMULATION

This paper is focused on the development of an adhesion model, which includes real contact conditions and contains a fast method for calculation of adhesion force depending on wheel load, slip velocity and real characteristics in the contact zone.

The contact interaction between a wheel pair and railway track can be formulated by finding the adhesion force between the wheel and the rail with defined friction conditions of the contact, wheel load, a vector of slip, profiles and elastic properties of contact bodies.

Therefore, for undertaking this task we should account for friction conditions in wheel–rail contact, because almost all existing models use an ideal contact model. However, for a real process of the contact interaction the third body is presented in the contact zone.

One can see from [6] that the third body has a wide range of thickness from a few micrometers to several tens of micrometers. The third body makes contact interaction analysis more complex. Under these conditions, the third body has a major influence on friction and adhesion coefficient in the contact zone.

To develop a more realistic mathematical model, it should take into account the influence of temperature in the friction zone, movement velocity and slip, initial friction condition and its variability. It is necessary to investigate the interdependence between wear of contact bodies and temperature in the contact zone, and also the interdependence between energy liberation and temperature, which is dependent on parameters of the contact, such as its dimension, contact area, normal and tangential stresses in the contact area, value of friction coefficient, slip velocity and motion velocity of a railway vehicle.

MATHEMATICAL MODEL

To obtain contact stresses, the classic approach according to Hertz was applied, based on the use of singular integral equations. However, the theory by Hertz is sometimes impossible to use for obtaining a solution of a contact interaction between the wheel and the rail [10]. In the past few years, in order to reach this goal, many numerical algorithms were developed [11, 12].

The adhesion model can be, in general, divided into two main problems — normal and tangential. The normal contact problem was solved by presenting contact bodies as elastic bodies with the application of numerical methods. As a result of the solution of the normal contact problem, the coordinates of starting points of the contact area (area E), the form, dimensions of the contact spot and normal stress distribution $\sigma(i, j)$ for this spot were obtained (Fig. 1).

These parameters are the initial data for the computational solution of the tangential contact problem of the adhesion model.

The goal of the solution of the tangential contact problem is to determine the adhesion force between a wheel (wheel pair) of a railway vehicle and a rail (rail track). In the wheel rolling process relative to the rail, the wheel has the normal load from the wheel on the rail and movement velocity V. Under the action of wheel load and due to properties of the wheel, the contact area E is formed. The form and dimensions of the contact zone E, distribution of normal stresses, wheel load are known from the solution of the normal contact problem of the adhesion model.

Besides this, the friction law, elastic and thermophysical properties are considered as given. The vector of relative slip can be found from the solution of the dynamic model of vehicle movement.

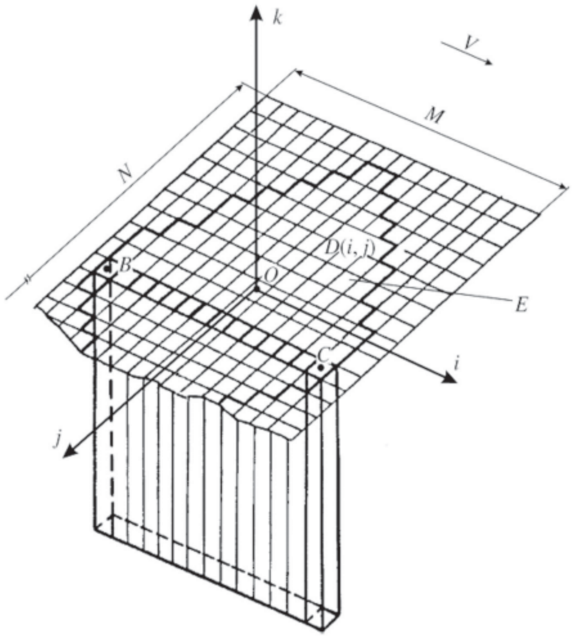

Fig. 1. Three-dimensional rolling contact in the proposed model: calculation scheme and coordinates system

To solve the tangential contact problem of the model, a rectangular mesh with MxN dimension with $\Delta i, \Delta j$ sides is employed, and this mesh covers the contact zone between the wheel and the rail.

The rolling process with a slip is considered a stationary process. The assumption is that the tangential contact problem of the adhesion model should be taken into consideration as quasistatic task.

Based on the above, it is possible to formulate the tangential contact problem by finding the vector-function of tangential stresses, with determinant functions of distribution of normal stresses $\sigma(i, j)$, slip $\vec{v}\ (i, j)$ and the dependence $f\ (\sigma,\theta)$ of the slip-friction coefficient in the rolling mode with a slip on the pressure and temperature in the contact area E, by means of the following equation:

$$\vec{\tau} = \sigma(i, j) f(\sigma\theta)\vec{v} \mid \vec{v} \mid, \tag{1}$$

where: $f\ (\sigma,\theta)$ is obtained with the experiment described in [7]. In this case, the adhesion force $\vec{F}_a$ equals:

$$\vec{F}_a = \int_E \vec{\tau} dE. \tag{2}$$

For computing it is necessary to determine the temperature field in the contact area.

In this case, the temperature field should be calculated first. For this purpose, we would consider the point *D(i, j)*, which belongs to the rail and is situated in the contact zone (Fig. 1). In contact area *E* the heat transfer rate is:

$$g(\theta) = \alpha\sigma(i,j) f(\sigma,\theta) \,|\, \bar{v}(i,j) \,|, \tag{3}$$

where: α is the distribution coefficient of heat (in our case we used $\alpha = 0.5$). The heat transfer rate depends on the pressure, temperature in the contact area *E* and the value of the local vector of the slip.

The temperature *T* at point *D* at the time moment t is:

$$t = s/V, \tag{4}$$

where: *s* is the dimension from the leading edge to point *D* along *i*-axle; *V* is the movement velocity of the railway vehicle.

For the solution of a non-stationary thermal conductivity problem, in the case of independence from thermophysical properties of materials, it is possible to use the following equation:

$$\beta \left(\frac{\partial^2 T}{\partial i^2} + \frac{\partial^2 T}{\partial j^2} + \frac{\partial^2 T}{\partial k^2} \right) \pm q = c_\rho \theta \frac{\partial^2 T}{\partial t}, \tag{5}$$

where: β is the heat conduction coefficient; *c* is the volumetric thermal capacity; *q* is the rate of energy generation per unit volume; *T* is temperature; *t* is time. An analytical solution for equations (3) and (5) does not exist, so it is necessary to use a numerical method.

The numerical method requires making the following assumptions:

• The points of the wheel and rail, when they are in the contact zone, move in parallel with the *i*-axle. This assumption is admissible because of the short time of the single contact (it is about 10^{-3}–10^{-5} seconds). Taking this into account, the movement in the direction of *j* -axle can be excluded.

• The heat, which is generated due to the relative slip of contact surfaces, distributes only along the normal to the contact. This assumption is correct, if the Peclet number *Pe* is:

$$Pe = vl / a > 20, \tag{6}$$

where: *V* is the movement velocity of the railway vehicle; *l* is the contact length; *a* is the thermal diffusivity for the wheel and rail materials.

Equation (6) is valid for real contact conditions and movement velocities of locomotive. Under these assumptions, it is possible to model contact interaction by means of the set of bars with heat-insulated sidewalls. Thus, one can write equation (5) for each bar in the following form:

$$\beta \frac{\partial^2 T}{\partial i^2} \pm q = c\rho \frac{\partial T}{\partial t}. \tag{7}$$

The functions $\sigma(i, j)$, $\vec{v}(i, j)$, and $\vec{\tau}(i, j)$ are accounted as piecewise constant functions, and in the same cell with a center *(m, n)*, one has:

$$\sigma(i,j) = \sigma(i_m, j_n); \vec{\tau}(i,j) = \vec{\tau}(i_m, j_n); \vec{v}(i,j) = \vec{v}(i_m, j_n). \tag{8}$$

The contact spot is considered as the sum of N strips, which are in parallel to the i-axle (Fig. 1). First the adhesion force is calculated for each strip, and then for the whole contact. This is possible by means of the assumptions described above.

The temperature can be calculated with the following equation from [13].

$$T = \frac{2q\sqrt{t}}{\sqrt{\lambda c \rho}} \left[\frac{1}{\sqrt{\pi}} e^{\left(k^2/4at\right)} - \frac{-k}{2\sqrt{at}} Erfc\left(\frac{-k}{2at}\right) \right]. \tag{9}$$

Under the condition that one need only the value of temperature on the contact surface, and in this case $k = 0$, one can write:

$$T = \frac{2q\sqrt{t}}{\sqrt{\lambda c \rho}}. \tag{10}$$

where: λ is the thermal conductivity, and ρ is the material density.

It is necessary to take up the set of bars again; each of these bars has a number from 1 to N, and also has the cross section $\Delta i x \Delta j$ (Fig. 1). For example, the upper ends of bars create a strip BC with the number n $(1 \leq n \leq N)$. This strip is parallel to the i-axle and is situated in the plane of the contact. Each bar in the strip n has its own number $(1 \leq m \leq M)$.

Now one can use equation (7) to calculate the rate of energy generation per unit volume $q1$ at the moment of time when the wheel moves and the first bar is situated in the contact zone at point C (Fig. 1). The time t equals:

$$t = \Delta i / v, \tag{11}$$

where: Δi is the step of the mesh along the i-axle.

One can make the following assumptions that the initial distribution of temperature along the bar length is equal to zero and the rate of energy generation per unit volume at the moment of time t is constant.

Since the source power depends on temperature in the contact zone, equation (7) can be solved with a successive refinement of $q(\theta)$ by means of the simple iteration method. After a good accuracy of determination of $q(\theta)$ is attained, the next bar should be chosen. On the second bar, the heat source q_2 is active and at the same time the heat source q_1 becomes inactive. In this case, one has to calculate the temperature change in the bar from q_2 and to add the temperature change, which is what happens after the source q_1 becomes inactive in time t . It can be calculated as the difference between the temperature from the heat source q_1 during time $2t$ and the temperature from the heat source q_2 with the same power during time t . Then, it is necessary to define more exactly the value of q_2. For the third bar, the temperature change from the source q_3 is added with the temperature change in the bar, which happens after both q_1 and q_2 become inactive. The temperature change is calculated as the sum of the differences between temperatures for the source q_1 during time $3t$ and the source q_1 during time $2t$, and the differences between the source q_2 during time $2t$ and the source q_2 during time t. After that the value of q_3 needs to be defined more exactly. The calculation procedure should be performed M times, and then it is necessary to consider the next strip of the contact. The initial conditions for the next strip are the same as written above for the first one.

The example of calculation of temperature values under wheel load of 115 kN and slip velocity 5 m/s was made. The scheme of temperature distribution with the proposed method is shown on Fig. 2.

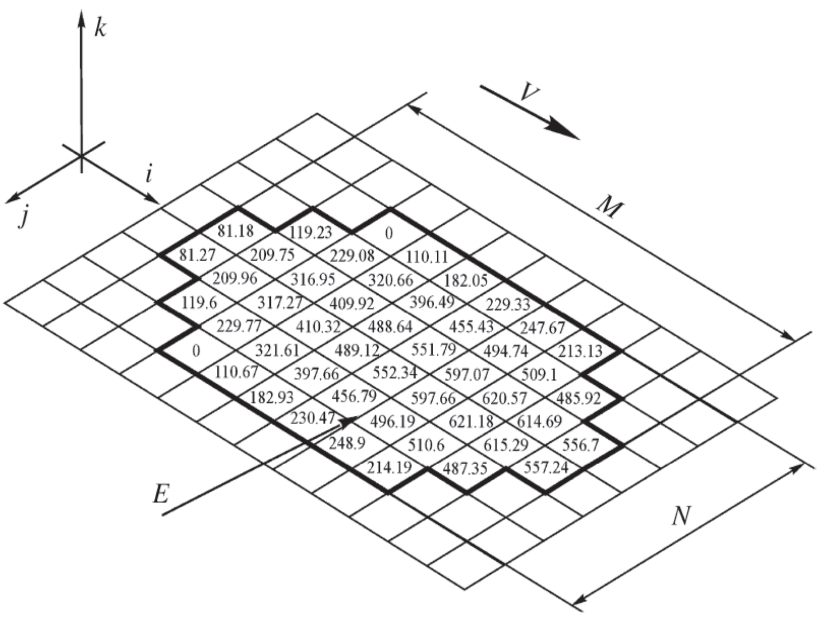

Fig. 2. Temperature distribution (degree Celsius) in the contact

Then one can determine the value of the adhesion force, *F*a, as:

$$F_a = \sum_{m=1}^{M} \sum_{n=1}^{N} \sqrt{\tau_i^{\ 2}}(i_m, j_n)\Delta i \Delta j. \tag{12}$$

The proposed solution describes the adhesion force computation method for a single point between a wheel and a rail. However, during the same time, the wheel and the rail have a two-point contact, especially in the case when movement takes place on a curved track. The two-point contact has a major influence on tractive and dynamic characteristics of a railway vehicle.

In this situation, two tasks should be carried out. The first one is to detect the force distributions for two points; the second is to calculate the form, dimensions of contact and stress distribution.

When the calculation process of the dynamic model of a railway vehicle goes, it is possible to find a solution for the normal contact problem of the adhesion model, in other words, to find the form, values of dimensions for wheel–rail contact and stress distribution. The solution of the adhesion model for the two points is more complicated, because interference for two points of contact should be taken into account.

From the results of calculations, one can see that the elastic deformation for a wheel (wheel load is 100 kN) is decreased to minimal value at 3–4 millimeters from the boundaries of contact. The same process is for a rail. One can use the proposed model to calculate the adhesion force for a two-point contact. For this purpose, each contact should be considered separately.

The examples of calculation with the proposed model are shown in Figs 3 and 4. The results do agree with experimental data obtained [14, 15].

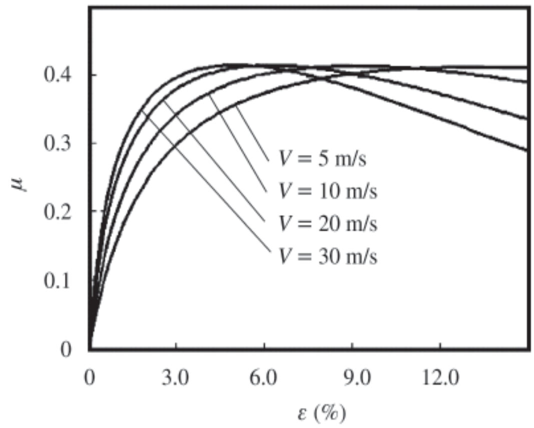

Fig. 3. Adhesion coefficient μ against relative slip ε for different values of movement velocity of the railway vehicle

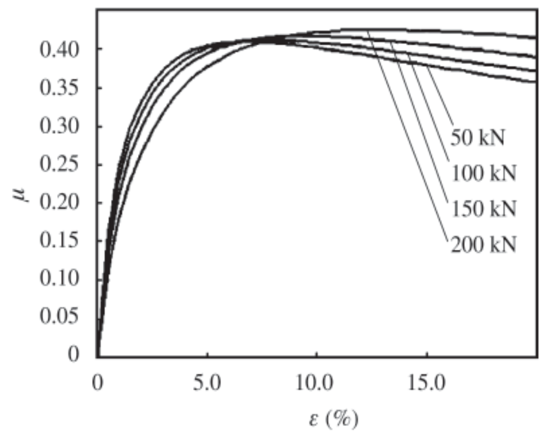

Fig. 4. Adhesion coefficient μ against relative slip ε for different values of wheel load

CONCLUSIONS

A mathematical model of the interaction between wheel pairs and rails for different friction conditions and arbitrarily assigned position of wheel pairs relative to rails was made. A pattern for the adhesion force depending on relative slip of the wheel relative to the rail for different adhesion conditions, temperature in the contact zone, and stresses was developed.

A method for determining the adhesion force between a wheel (wheel pair) of a railway vehicle and rail (rail track) was obtained. This relation was obtained as a result of experimental and analytical researches. The proposed dependence can also be used for the equations of movement of a railway vehicle for modeling adhesion processes between wheels and rails The proposed model is the basis for determining tractive and braking properties for railway vehicles on design stages and gives input characteristics for the development of control strategies for a microprocessor control system.

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PLANOWANIE PROCESU TARCIA PODCZAS TOCZENIA ŚLIZGOWEGO

Streszczenie. W artykule przedstawiono problem planowania procesów tarcia podczas toczenia ślizgowego. Opracowano matematyczny model interakcji pomiędzy parami kół a szynami dla różnych warunków tarcia i dowolnie przydzielonych ustawień par kół w stosunku do szyn. Zaproponowany model jest podstawą dla określenia właściwości pociągowych oraz hamowania dla pojazdów szynowych na etapie planowania i zapewnia wstępne dane dla rozwoju strategii kontrolnych w systemie kontroli mikroprocesorów.

Słowa kluczowe: toczenie ślizgowe, tarcie zewnętrzne, styk toczenia.