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CIRCULAR-SCREW GEARS WITH ASYMMETRIC FUNCTION OF TRANSMISSION RATIO

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Summary. A mathematical model of synthesis of circular-screw gears with asymmetric function of transmission ratio based on the offered main and additional conditions of synthesis is stated in the article. The advantage of application of transmissions by noncircular gears for struggling against resonance oscillations that allows extending the possibility of their application is shown in this work.

Key words: noncircular gears, variable transmission ratio, struggling against resonance oscillations.

INTRODUCTION
PURPOSE AND RESEARCH PROBLEMS

Creation of reliable and durable transmission gears is an important scientific and practical problem of modern machine-building industry which can be solved on the basis of gear transmissions improvement by toothing synthesis. One of the ways of gear transmission development by toothing synthesis, extending their functional capabilities, is a design of gears with a variable transmission ratio (transmissions by noncircular gears).

Experience of such kind of gears implementation created on the basis of involute mesh showed the advantage of their use in chain mechanisms and drives of machines for equalization of chain link speeds and elimination of their internal dynamic loads.

In full measure, it is reasonable to apply this method for an improvement of the anti-resonance stiffness of circular-screw gears, which have a high load-carrying capacity and are common in the reduction gearboxes of heavy engineering industry. Practice shows that 3 – 5 % of reduction gears failures are concerned with some type of vibrations and resonance phenomenon.

However, the development of gearing by synthesis of efficient toothing geometrics with variable transmission ratio providing the assigned transformation law of motion demands the decision of a number of questions such as selection of transfer function ratio, determination of main and additional conditions of transmission by noncircular gears, elaboration of mathematical model of synthesis of efficient geometrics of circular-screw toothing and estimation of their influence upon gearing working capacity etc.

At present time the circular-screw gears which offer a high load-carrying capacity have become prevalent for use in reduction gearboxes of heavy engineering industry; the development of these gears can be achieved by means of synthesis of efficient toothing geometrics.

ANALYSIS OF PUBLICATIONS, MATERIALS

The scientific works of M.L. Novikov, R.V. Fedyakin, V.A. Chesnokov, A.F.Kirichenko, A.V. Pavlenko, V.A. Krasnoshekov, V.N. Sevruk, V.M. Gribanov, V.P. Shishov and others deal with the problems of synthesis of circular gears with circular-screw toothing.

N.I. Mercalov, M.A. Skuridin, O.A. Pyj, N.A. Gaevskiy, N.I. Kolchin, F.L. Litvin, R.S. Varsimashvili, N.L. Ututov, D. Gunter, B. Raingard, M. Kanchiti, I. Kisuko and others have made a considerable contribution in research of gearing with variable transmission ratio. They have laid the foundation for the development of gearing with noncircular gears and considered the examples of their practical use.

The scientific works of B.M. Abramov, E.L. Airapetov, M.D. Genkin, A.I. Petrusevich A.P. Fillipov, V.K. Grinkevich, S.S. Gutyrya, T. Toshima, K. Masan, D. Wallas, A. Seireg, G. Opits and others are dedicated to study of the problems of vibroactivity in reduction gearings with circular gears. These works have shown that the existing various methods of resonance vibration control of reduction gearboxes (designation of supercritical and subcritical shaft rotational speeds; rise of manufacturing accuracy of gearing production and assembling; modification of construction of gears, housings and shafts; application of special covering of reduction gearbox parts; using of dynamic dampeners and so on) lead to a rise in the price of construction, increase of mass and size, and in many cases these methods are ineffective and unreliable [1, 3, 4].

THE MAIN CHAPTER

It is determined by investigations that the solvation of anti-resonance stiffness problems of circular-screw gears is possible by using the variable transfer function which allows extending of noncircular gear application including the resonance vibration control of gears. In this case the function of transmission ratio has to have the asymmetric law of variation.

One of the types of asymmetric function of transmission ratio, which provides assigned transformation law of motion, can be obtained as follows:

$$i(\varphi_1)=\frac{r\cdot[\xi+\cos(j_1\varphi_1)]+B\cdot\sin(j_1\varphi_1)}{u\cdot r\cdot[\xi+\cos(j_1\varphi_1)]-B\cdot\sin(j_1\varphi_1)}, \tag{1}$$

which has three main indexes of asymmetric function of transmission ratio: ζ, j_1 and B characterize the degree of asymmetry, frequency and magnitude of transmission ratio changing, respectively.

In function (1) i and u are the transmission ratio and transmission number of noncircular gear; r is the mean radius of driving gear centrode; φ_1 is the turning angle of driving noncircular gear; j_1 is the coefficient of asymmetric function of transmission ratio which equals the quantity of maximum values of centrode radius of driving noncircular gear.

Fig. 1 represents the transmission ratio i – turning angle of driving gear φ_1 diagram.

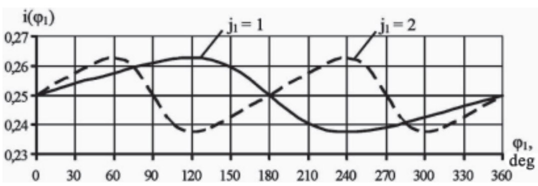

Fig. 1. Charts of asymmetric function of transmission ratio

Mathematical analysis $i(\varphi_1)$ (see Fig. 1) shows the following: function (1) is asymmetrical in regard to its one-half period under $\xi > 1$ and recommended value ξ should be 2; j_1 is a whole number. It is recommended to take the quantity of maximum values of centrode radius $j_1 \geq 2$ in order to avoid the mass imbalance.

Doing a mathematical analysis of index B, which characterizes the variation value of transmission ratio, it is possible to determine the dependence of index B from the transmission number of gearing u, the center-to-center spacing aw, and the coefficient of nonuniformity of mechanism motion δ:

$$B = \frac{a_w u \sqrt{3}}{\delta \cdot (u+1)} \cdot \left(\sqrt{u^2 + 2u + \delta^2 + 1} - u - 1 \right). \qquad (2)$$

Numerous investigations show that for the existing dimension-type reduction gearboxes the values B lie in the range $0 \leq B \leq 13{,}74$ mm.

Thus, under the assigned parameters u and aw the relation (2) allows to select the efficient value B subject to the required coefficient δ from additional synthesis criterion $B \leq B_\delta$ where B_δ is the index of asymmetric function for the required δ.

Fig. 2 shows the gearing centrodes whose radii are described by the equations:

for driving gear
$$r_1 = r + \frac{B \sin(j_1 \varphi_1)}{2 + \cos(j_1 \varphi_1)}; \qquad (3)$$

for driven gear
$$r_2 = u \cdot r - \frac{B \sin(j_1 \varphi_1)}{2 + \cos(j_1 \varphi_1)}. \qquad (4)$$

In order to estimate the strength factor of gearing we determine the sizes of contact area by the tooth length:

$$C = \frac{\{r[2 + \cos(j_1 \varphi_1)] + B \sin(j_1 \varphi_1)\}(\varphi_1^* - \varphi_2^*)}{[2 + \cos(j_1 \varphi_1)] \sin \beta}, \qquad (5)$$

where φ_1^* and φ_2^* are the angles which count out from straight lines normal to lines which connect the gear's centers.

The analysis of relation (5) shows that contact patch moves around the tooth on the constant distance along its height and the maximum change of contact patch sizes in toothing does not exceed 4,2% from the value of other circular gears.

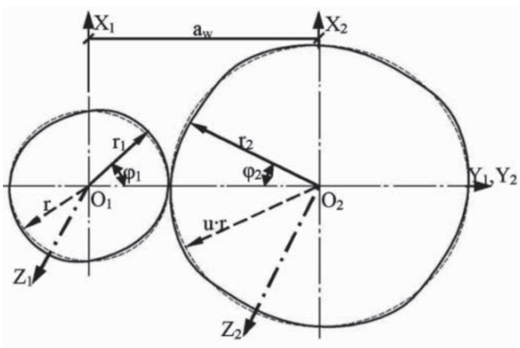

Fig. 2. Gearing centrodes in a fixed coordinate system $X_1Y_1Z_1$ and $X_2Y_2Z_2$ under $j_1 = 2$: r and u·r are the mean radii of centrodes of driving and driven gears

Using the geometrical-kinematic criterions and forced factors it is possible to realize the theoretical estimate of working capacity of synthesized circular-screw gearings with asymmetric function of transmission ratio by means of their comparison with other types of gearings which have a constant transmission ratio.

Dependance of absolute value of motion relative speed of mesh point from turning angle of driving gear is described by the equation:

$$V_C = \frac{\sqrt{K_{VC}}}{r^2\left[2+\cos(j_1\varphi_1)\right]^2(u+1)^2\left\{u\cdot r\cdot\left[2+\cos(j_1\varphi_1)\right]-B\sin(j_1\varphi_1)\right\}^2}, \tag{6}$$

where K_{VC} is the coefficient of absolute value of motion relative speed of mesh point.

Mathematical analysis shows that traverse speed of mesh point along contact line is the variable quantity and depends on the turning angle of gears and value B; changing of value V_C does not exceed 5,7% relative to values for circular gears.

In order to evaluate the wear we generate relations for determination of teeth slip coefficients ϑ_1 and ϑ_2 :

for driving gear

$$\vartheta_1 = \frac{\sqrt{K_{VCX}^2 + K_{VCY}^2}}{\left\{u\cdot r\cdot\left[2+\cos(j_1\varphi_1)\right]-B\sin(j_1\varphi_1)\right\}\sqrt{K_{VK1}}}; \tag{7}$$

for driven gear

$$\vartheta_2 = \frac{\sqrt{K_{VCX}^2 + K_{VCY}^2}}{\sqrt{K_{VK1}}}, \tag{8}$$

where K_{VK1} is the coefficient characterizing the absolute value of motion relative speed of mesh point of teeth along the contact line of driving gear; K_{VCX}, K_{VCY} are the coefficients of absolute value of motion relative speed of mesh point, relatively to the axes of coordinates.

Changing of slip coefficients ϑ on the driving and driven gears is equally and directly proportional to value B, and does not exceed 4%, relatively to values ϑ for circular gears.

Taking into consideration that rotational speed of driving gear ω_1 and moments of inertia of gear's reduced mass $I_{red.1}$ and $I_{red.2}$ are constant, the equation of motion of machine with noncircular gears [5] becomes as following:

$$T_{mot.} = \left[T_{u.r} + T_{add.}\right]\cdot i(\varphi_1), \tag{9}$$

where $T_{mot.}$ is the moment of motive force on shaft of driving noncircular gear; $T_{u.r.}$ is the moment from forces of useful resistances on shaft of driven noncircular gear; $T_{add.}$ is the additional moment caused by variability of transmission ratio and determined by relation:

$$T_{add.} = I_{red.2}\varepsilon_1\frac{r\cdot\left[2+\cos(j_1\varphi_1)\right]+B\sin(j_1\varphi_1)}{u\cdot r\cdot\left[2+\cos(j_1\varphi_1)\right]-B\sin(j_1\varphi_1)} + \frac{I_{red.2}\omega_1^2 B j_1 r(1+u)(1+2\cos(j_1\varphi_1))}{\left[u\cdot r\cdot(2+\cos(j_1\varphi_1))-B\sin(j_1\varphi_1)\right]^2} +, \tag{10}$$
$$+\frac{1}{2}\frac{dI_{red.2}}{d\varphi_1}\left[\frac{\omega_1\left\{r\cdot\left[2+\cos(j_1\varphi_1)\right]+B\sin(j_1\varphi_1)\right\}}{u\cdot r\cdot\left[2+\cos(j_1\varphi_1)\right]-B\sin(j_1\varphi_1)}\right]^2,$$

where ε_1, ε_2 are the angular accelerations of rotation of driving and driven gears.

Analysis of results represented in Fig. 3 shows that the change of additional moment $T_{add.}$ per one rotation of driving gear does not exceed 5,8% from the value of external loading moment.

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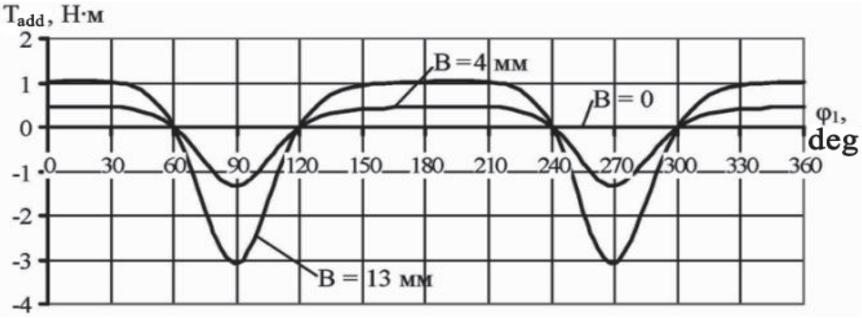

Fig. 3. Additional moment $T_{add.}$ – turning angle φ_1 relation

Equation for normal force determination in toothing for driving and driven circular-screw gears looks as the following:

$$F_{N1(N2)} = \frac{T_{Z1\ (Z2\ )}\left[2+\cos\left(j_1\varphi_1\right)\right]\sqrt{K_{1N(2N)}}}{p\left\{r\left[2+\cos\left(j_1\varphi_1\right)\right]+B\sin\left(j_1\varphi_1\right)\right\}\sqrt{A_1}\,\sin\lambda_{1(2)}}, \tag{11}$$

where T_{Z11} and T_{Z22} are the total moments acting upon driving and driven gears respectively; K_{1N} and K_{2N} are the coefficient of normal vector scalar for teeth surface, λ_1 and λ_2 are the turning angles of tool tips under cutting of driving and driven gears; p is the helix parameter.

The mathematical analysis shows that in gearing with asymmetric function of transmission ratio the normal forces in gears mesh under $T_{mot.}$ = const and $T_{u.r.}$ = const have variable values; at the same time a change of F_N in gears mesh per one revolution of driving gear becomes no more than 3,7% from the magnitude for circular gears.

According to the results of the conducted comparative analysis for synthesized gearing with circular gear transmissions we conclude that there is a possibility to use the circular-screw gearing with asymmetric function of transmission ratio for application in reduction gearboxes of heavy engineering industry.

Field experience [1, 3, 4] confirms that impulse excitation (teeth concussion at the time of input and output out of mesh) is the main reason for vibration onset provoked in gearing. In reduction gearboxes the first pass of gearing is the most vibro-active zone. Under the coincidence or multiplicity of frequency of natural and forced vibrations the resonance occurs.

Analytical dependence for finding a frequency of natural vibrations of transmission by noncircular gears has the following type:

$$f_c = 3{,}15\cdot 10^5\cdot\frac{(1+u)\cdot\sqrt{1+u^2}}{2a_w u}. \tag{12}$$

Equation for determination of tooth mesh frequency of forced vibrations of transmissions with asymmetric function of transmission ratio under the impulse excitation is obtained as follows:

$$f_z = \frac{\omega_1 a_w\left\{r\cdot\left[2+\cos\left(j_1\varphi_1\right)\right]+B\sin\left(j_1\varphi_1\right)\right\}}{286{,}5\cdot m\cdot(u+1)\cdot r\cdot\left[2+\cos\left(j_1\varphi_1\right)\right]}, \tag{13}$$

where ω_1 is the rotational speed of driving gear; m is the toothing module.

Taking into account (12) and (13), the resonance (critical) frequency of noncircular gear expressed by the relation:

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$$\omega_{1crit.} = 8{,}8 \cdot 10^4 \cdot \frac{m \cdot r \cdot [2 + \cos(j_1\varphi_1)] \cdot (1+u)^2 \sqrt{1+u^2}}{a_w^2 u \cdot p \cdot \{r \cdot [2 + \cos(j_1\varphi_1)] + B \sin(j_1\varphi_1)\}}. \tag{14}$$

The graph of $\omega_{1crit.} - \varphi_1$ curve is represented in Fig. 4.

The analysis of equations (12) – (14) shows that critical rotational speed of shaft ω_{1res} with noncircular gear changes for its one revolution, B is the variation value of critical rotational speed (see Fig. 4). It is determined that in transmissions by means of noncircular gears the tooth mesh frequency f_z of forced vibrations is a variable value, and does not coincide or multiply its natural frequency f_{nat} of vibrations. The conducted theoretical investigations of resonance vibrations of transmissions by means of noncircular gears came to conclusion that asymmetric law of changing of transmission ratio function prevents from resonance occurrence.

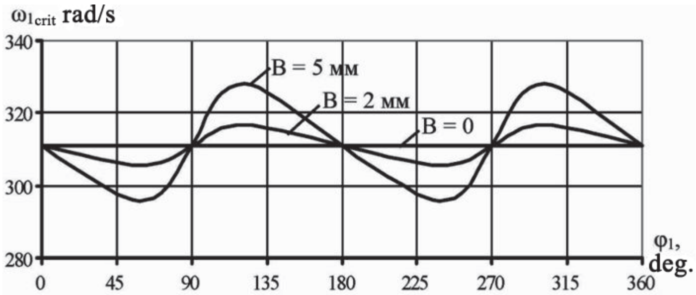

Fig. 4. Resonance rotational speed of driving gear from impulse excitation

In order to decrease the resonance [3] risk of rotational speed ω_1 of driving noncircular gear it is necessary to follow the criterion:

$$\begin{cases} \omega_1 \geq (1+K) \cdot \omega_{1crit}^{\hat{\imath}} \\ \omega_1 \leq (1-K) \cdot \omega_{1crit}^{\hat{\imath}} \end{cases}, \tag{15}$$

where K is the coefficient which defines margin of resonance origin zone, $\omega_{1crit.}^{\hat{\imath}}$ is the resonance rotational speed of driving gear of transmissions by means of noncircular gears, which is determined by the relation:

$$\omega_{1crit}^{o} = 8{,}8 \cdot 10^4 \cdot \frac{m \cdot (1+u)^2 \sqrt{1+u^2}}{a_w^2 u \cdot p}. \tag{16}$$

Considering (14) – (16) and using mathematical transformations, the equation for $B_{crit.}$ definition will have the type:

$$B_{crit} = 0{,}82 \cdot K \cdot \frac{r \cdot [2 + \cos(j_1\varphi_1)]}{\sin(j_1\varphi_1)}. \tag{17}$$

Therefore, value B has to satisfy the conditions of additional synthesis matter: $B \geq B_{crit}$.

Thus, value B of function $i(\varphi_1)$, which characterises variation value of transmission ratio, is chosen from condition $B_{crit} \leq B \leq B_\delta$.

Experimental investigations for the determination of coefficient K and comparative tests of gearing by means of noncircular gears with asymmetric function of transmission ratio and transmissions by means of circular gears were conducted. Experimental investigations provided with the purpose of practical approbation of results and conclusions were derived under theoretical study of toothing and contain the following: control of transmission ratio, comparative assessment of vibration resonance of transmissions by noncircular gears with asymmetric function of transmission ratio (under the different meanings of coefficient K) and transmissions by circular gears. For these:

– device for gear-milling machine 5K32 which makes the teeth cutting of noncircular gear was engineered and manufactured;

– according to results of theoretical calculations, the experimental noncircular gears with circular-screw toothing for double-reduction gearbox were synthesized and produced;

– measurement complex including the stand for inspection of centrode production accuracy and transmission ratio accuracy was prepared;

– complex for vibration measurement of gearbox under rotational speed of driving shaft up to 356 rad/sec was prepared;

– procedure of experimental investigations of resonance vibrations of gearing was developed;

– bench tests of experimental gearings and circular gearings, as well as their comparison characteristics were presented.

Experimental gearings made by steel 40X GOST (State Standard) 4543-71. Heat treatment: pinion gear – refining up to HB 269…302, gearwheel – refining up to HB 235…262. A characteristic of transmissions is represented in Table. 1.

Table 1. Characteristic of experimental transmissions

<table>
<tr><td rowspan="2">Designation of parameter</td><td colspan="4">1st pass</td><td colspan="4">2nd pass</td></tr>
<tr><td colspan="3">Non circular</td><td>Circular</td><td colspan="3">Non circular</td><td>Circular</td></tr>
<tr><td>Normal module m_n, mm</td><td colspan="3">3,0</td><td>3,0</td><td colspan="3">3,0</td><td>3,0</td></tr>
<tr><td>Transmission ratio u</td><td colspan="3">2,0</td><td>2,0</td><td colspan="3">2,0</td><td>2,0</td></tr>
<tr><td>Number of teeth:
–pinion gear z_1
– gearwheel z_2</td><td colspan="3">
21
42</td><td>
21
42</td><td colspan="3">
32
64</td><td>
32
64</td></tr>
<tr><td>Axle base a_w, mm</td><td colspan="3">100</td><td>100</td><td colspan="3">150</td><td>150</td></tr>
<tr><td>Coefficient K</td><td>0,06</td><td>0,08</td><td>0,15</td><td>0</td><td>0,06</td><td>0,08</td><td>0,15</td><td>0</td></tr>
</table>

In Fig. 5 the transmission with noncircular gears was obtained.

Estimation of transmission ratio of experimental gearings and gearbox as a whole was conducted under the testing (Fig. 6).

Fig. 5. Double-reduction gearbox with noncircular gears

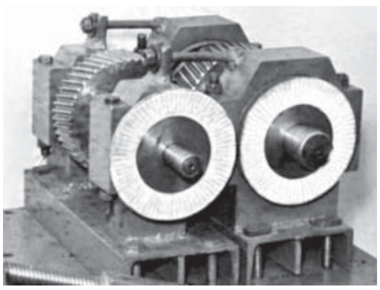

Fig. 6. Stand for transmission ratio inspection

As as result of the conducted experiment the following conclusion was made: change of transmission ratio strictly corresponds to relative change of centrode radius of noncircular gears. At the same time, the maximum discrepancy between theoretical and experimental results of transmission ratio values of gearbox amounts to 6%.

In order to define the vibration level in gearbox with noncircular gears, the stand with closed-loop power was engineered; vibration-measuring apparatus ВИ6-6ТН together with self-recording instrument Н 327-3 was used for experiments (Fig. 7). On the housing of research gearbox the following objects were set up: vibration detector ДВ-1СГ for detection of vibration in horizontal plane and ДВ-1-СВ – for vertical plane.

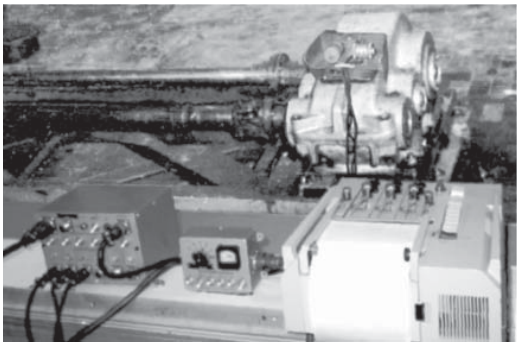

Fig. 7. Stand for measurement of gearing vibrations

Testings were carried out in regime of smooth variations of rotational speed of driving shaft ω_1 from 0 to 356 rad/sec, and vibrations recorded by using self-recording instrument Н 327-3.

According to thec results of the conducted experiment the following things were determined: with the same character of vibrations, the vibration amplitude in vertical plane is much greater than vibration amplitude in a horizontal plane; in gearbox with circular gears under ω_1 = 298 rad/sec, a sharp increase of vibration amplitude Y (Fig. 8, a) was stated; in gearbox with noncircular gear under K = 0,15, on the full scale of rotational speed of driving gear, no increase of vibration amplitude was observed (Fig. 8, d); under K = 0,08 the maximum magnitude of vibration amplitude, compared to the magnitude under K = 0,15, increased 1,5 times in speed range ω_1 from 272 up to 323 rad/sec (Fig. 8, c); under K = 0,06 the maximum magnitude of vibration amplitude compared to the magnitude under K = 0,15, increased 3,5 times in speed range ω_1 from 281 up to 315 rad/sec, and at the same time the maximum values of amplitudes approached to amplitude values for circular gears (Fig. 8, b).

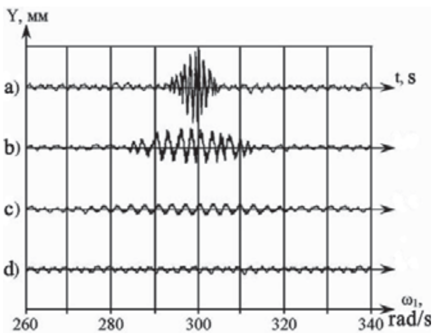

Fig. 8. Oscillogram of vibrations in gearboxes:
a) with circular gears; b) with noncircular gears under K = 0,06;
c) with noncircular gears under K = 0,08;
d) with noncircular gears under K = 0,15

Graphs analysis (Fig. 8) showed that the border of resonant zone appearance observed under K = 0,08. It allows giving the recommendations by definition of index B in asymmetric function of transmission ratio.

The conducted research of two-stage gearbox with noncircular gears and common constant transmission ratio for recommended value K = 0,08 under B from 2 up to 7 mm showed the following: at a full range of rotational speed of driving shaft there was no increase of vibration amplitudes to be observed, which conforms the theoretical background.

CONCLUSIONS

The obtained results of the conducted theoretical and experimental researches let us to come to the following conclusions:

1. A mathematical model of synthesis of circular-screw gears with asymmetric function of transmission ratio was developed. The efficient geometrical parameters of toothing (upon proposed supplementary conditions of synthesis) which secure the operating regime of gearing with no resonance effect ($B \geq B_{crit}$) and required coefficient of nonuniformity of motion δ ($B \leq B_{\delta}$)were determined on the basis of synthesis task solution.

2. Theoretical analysis of working capacity of the synthesised gearings by means of comparing with gearings with constant transmission ratio was carried out.

3. Estimation of resonance vibrations of transmissions with noncircular gears with asymmetric function of transmission ratio from impulse excitation was carried out. Dependancies for definition of border of resonant vibrations zone appearance were detected.

4. Experiment-calculated works for the purpose of transmission ratio control and estimation of resonance vibrations of gearings by noncircular gears with circular-screw mesh were carried out. As a result of testing it was determined that in gearbox with noncircular gears with symmetric function of transmission ratio in the whole range of ω_1 for the recommended value K, no increase of vibration level was observed.

5. One of the ways of circular-screw gears development, by synthesis of rational mesh geometrics with asymmetric function of transmission ratio, which guarantees the specified low degree of energy conversion and expands the functional capabilities of application of transmissions with noncircular gears for resonance vibrations struggling, is proposed.

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