TEKA Kom. Mot. Energ. Roln. – OL PAN, 2008, 8, 213–221

PARAMETRICAL OPTIMIZATION OF WORM GEARS BASED ON LOSSES IN GEARING

Valentin Shishov, Pavel Nosko, Pavel Fil, Alexandr Muhovatiy, Julia Sklyar

The Volodymir Dahl East-Ukrainian National University

Summary. A mathematical model of definition of losses in gearing of worm gears and their reductions by 10 … 15 % is determined based on a mathematical experiment concerning rational parameters of transfers.

Key words: mathematical model, worm gears, experiment, parameters.

INTRODUCTION

Gearing worm gears are commonly used in driving gear of modern cars because of evenness and quiet work and big transmissive number in one degree. However, they have big losses on friction in gearing of worm gears [5]. That is why it there is a need for solution of a problem of optimal design of machine-building constructions [9].

There are a lot of reports on research concerning the problem of definition of losses in gearing worm (e,g. 1, 2, 5,15). In these reports it has been proved proved that geometry of working surfaces of worm gearing has a great influence on the quantity of losses in engagement. Losses in engagement in the reported cases are determined by an experiment (5) or by liquid friction (1, 2) between working surfaces. The last one has not been confirmed experementally so far as it was installed in the report that there is a treatment of boundary friction in worm gearing.

The present report is devoted to the creation of mathematical model of definition of losses on friction in points of the field of gearing of worm gears in the treatment of boundary friction. A policy is being created of optimization parameters of worm pair for the purpose of minimization of overall losses on friction of working surfaces of worn turns and worn wheel teeth on this basis.

OBJECTS AND PROBLEMS

1. Mathematical model of definition of losses on friction in gearing

Look at worm gears equation of worn turn, which is connected with it in a coordinate system $x_1y_1z_1$ (the axis o_1z_1 is directed along the axis of worn)

$$x_1 = \lfloor f_{1(\lambda)} - R \rfloor \cos\mu, \; y_1 = \lfloor f_{1(\lambda)} - R \rfloor \sin\mu; \; z_1 = f_2(\lambda) + P\mu, \tag{1}$$

here, μ, λ – parameter; f_1, f_2 – optional functions, describing a profile of worm turns in axial section (in further λ in the definition of function f_1, f_2 will be missed); R – radius of worm dividing cylinder; P – turn parameter.

Values of function f_1 and f_2 you can define using the references of the report [10] depending on geometry of abrasive wheel used in the last worm processing.

Capacity of frictional force in gearing worm with worm wheel equals:

$$\Delta P^* = q_n f v^{12} d\ell, \tag{2}$$

where q_n is a loading on the unit of length of contact lines in gearing, directed along a perpendicular of surface (1);

f – a coefficient of friction between working surfaces, determined with using references [4, 13];

v^{12} – slip velocity in gearing worm gears [6, 14];

$d\ell$ – length of contact line of working surfaces [14].

We have observed that there is a linear contact of working surfaces, including possible reduction of burn-in worm pair even in that case when the first contact was localized in a boundary gearing field. We also assume that as a result of running-in wear of working surfaces of contact, voltages by Gerts take on a fixed value on the whole line of a momentary contact line. It is possible if constant load is in gearing.

Then, using Gerts formula we will determine contact voltages and have:

$$q_n = \frac{T_1}{X_{np} \int \frac{\{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} d\ell}{X_{np}}}, \tag{3}$$

where: T_1 – rotational moment of worm barrel;

X_{np} – reduced curve of contact surfaces of worn turn and worn wheel [14];

$[\bar{r}_1 \times \bar{e}_1]_{Z1}$ – moment projection of perpendicular crosscut (1) on axis of worm;

$[\bar{r}_1 \times \bar{\tau}]_{Z1}$ – moment projection of perpendicular crosscut of surface (1), directed along relative velocity vector on worm axis;

$\bar{r}_1$ – vector with coordinates (1);

$\bar{e}_1, \bar{\tau}$ – vectors of perpendicular crosscut and tangent, in the direction of the vector of slip velocity to surface (1), look [6, 14].

Capacity on worm barrel, made by force q_n from (3) equals:

$$P_1^* = q_n \{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} \omega_1 d\ell. \tag{4}$$

It is known that losses on friction in gearing are characterized by a coefficient of losses. It equals thickness ratio of friction force to available capacity [5]. Consequently, momentary coefficient of losses on friction for a point of contact line equals (reference ΔP^* to P_1^*):

$$\psi_{мгн} = \frac{f v^{12}}{\{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} \omega_1}. \tag{5}$$

It follows that a coefficient of losses on friction depends on geometrical parameters of a contact point and on the whole on geometry of surface of turn worm. For momentary contact line capacity of force friction using (2) equals:

$$\Delta P = \int q_n f v^{12} d\ell. \tag{6}$$

Available capacity on a worm barrel for this contact line with using (4) will have the value:

$$P_1 = \int q_n \{ [[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1} \int \omega_1 d\ell. \} \tag{7}$$

A coefficient of losses on friction for momentary contact line has the value (reference ΔP to P_1):

$$\psi_{\kappa\pi} = \frac{\int q_n f v^{12} d\ell}{\int q_n \{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} \omega_1 d\ell}. \tag{8}$$

Consequently, in a boundary field of gearing worm gears a coefficient of losses on friction changes when it transitions from one contact line to another. It follows, there is an important conclusion that in any process in gearing worm pair with a permanent whirling moment, on the shaft of worm wheel the whirling moment in a boundary step in gearing on worm barrel is of variable quantity. It makes the load on the motional gear variable. This condition is valid for all kinds of gears. We will notice that the boundary of integration for every contact line in (8) is determined depending on its position in the gearing field.

Now we will determine the overall coefficient of losses on friction in gearing worm gears. Work of friction forces in the gearing field with using (6) equals:

$$A_m = \frac{1}{\omega} \sum_{i=1}^{\kappa_1} \int q_n v^{12} d\ell \Delta\varphi, \tag{9}$$

where: $\Delta\varphi = \frac{\varphi_{1\kappa} - \varphi_{1н}}{\kappa_1}$;

$\varphi_{1\kappa}, \varphi_{1н}$ – angles of worm twist, corresponding to the gearing field. They are determined according to references [2, 6];

κ_1 – number of momentary lines in the gearing field. The current meaning of angle of turning is determined according to the formula:

$$\varphi_{1u} = \varphi_{1н} + i\Delta\varphi \quad (i = 1,2...k_1), \tag{10}$$

ω_1 – angular velocity of worm.

The meaning of payload with using (7) equals:

$$A_n = \sum_{i=1}^{\kappa_1} \int q_n \{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} d\ell \Delta\varphi. \tag{11}$$

Common coefficient of losses on friction in worm gears will equal:

$$\psi_з = \frac{A_m}{A_n} = \frac{\sum_{i=1}^{\kappa} \int q_n f v^{12} d\ell \Delta\varphi}{\omega_1 \sum_{i=1}^{k} \int q_n \{[\bar{r}_1 \times \bar{e}_1]_{Z1} + f[\bar{r}_1 \times \bar{\tau}]_{Z1}\} d\ell \Delta\varphi}. \tag{12}$$

The higher meaning “k_1”, the higher coefficient of losses is:

– The order of calculation of coefficient of losses is the following: with using [6, 14]; we determine $\varphi_{1н}$ *u* $\varphi_{1\kappa}$, boundaries of fields of gearing according to the reference [4]:

– Put the value “k_1”;

– according to the formula (10) we can determine φ_{1i}:

– for each φ_{1i} we can determine a boundary of integration in (12) (we notice, that the line φ_{1i} = *const* can consist of two filiations, as far as there is a recurrent contact of working surface of worm);

– by a formula (12) we determine the coefficient of losses on friction in gearing of worm gear.

2. Mathematical model of optimization of geometrical parameters of worm gears

By choosing a geometry of profile of worm turn we can improve the conditions of lubrication of working surfaces and reduce losses of worm gear in gearing of worm pair. So, for example, using ZT – worms with concave profile of turns reduces losses in gearing by 14%, compared to the gears which have involute worm [12]. You can notice the same if it is the gear with convex worm in case of choosing an acceptable coefficient of deposition. For example, if we use deposition with a coefficient of $X < -1$ for worm gears with involute worm (if number of turns $z_1^1 > 1$), we can receive the most favorable arrangement of contact lines relative to vector of sliding [11].

Loading ability and quality of worm gears are determined by Nimann, Nimann-Davis and Block criterions, except for the losses on friction in gearing [2]. Contact soundness of teeth of worm wheel and heat generation in a contact zone of working surfaces are characterized by these criterions. Consequently, optimization of parameters of worm gears should be made by providing acceptable values of specified criterions and quantity of losses on friction in gearing means, that it needs to use multi-objective optimization. However, we can reduce the number of criterions of optimization. It needs the planning of an experiment where we consider a row of functions. These functions determine specified criterions and quantity of losses on friction in gearing. Then we can use the method of correlation analysis. So, it is shown in work [8] that the main criterion of optimization of parameters of worm gears is quantity of losses on friction in gearing. The function (12) should be put in a row, using the plan of mathematical experiment. For example, with matrix of central composition orotrabasic uniform of planning (CCOUP) the second order [7]. This row looks like:

$$\psi_3 = b_0 + \sum_{u=1}^{k} b_u Z_u + \sum_{u=1}^{k}\sum_{l=1}^{k} b_{ul} Z_u Z_l + \sum_{u=1}^{k} b_{uu} Z_{uu}^2, \tag{13}$$

where b_0, b_u, b_{ul}, b_l, coefficients determined by definition on the basis of plan of mathematical experiment;

k – number of plan factors;

Z_u, Z_l – normalized values of factors.

We can accept four factors, which are independent parameters: X – a coefficient of deposition; q – a number of modules in dividing diameters of worms; U – a reduction rate; α – an angle of turn profile on dividing diameters of worms.

Normalized factor will be denoted Z_u and we will determine it according to the formula:

$$Z_u = \frac{X_u - X_{u0}}{\Delta X_u} \quad (u = 1,2,3,4), \tag{14}$$

where $X_{u0} = \frac{X_{u\max} + X_{u\min}}{2}$ – an average value of factor;

$\Delta X = \frac{X_{u\max} - X_{u\min}}{2}$ – an interval of factor variation;

X_{umax}, X_{umin} – levels of factor values.

From here follows the meaning of coeficient X_u through its determined value Z_u

$$X_u = \Delta X_u Z_u + X_{u0}. \tag{15}$$

If $X_1 = X$, $X_2 = q$, $X_3 = u$, $X_4 = \alpha$ and, for example, we can see warm gears with axle base mm with involute worm and worm ZT with concave and convex turn profile, it is possible to determine a coefficient of losses on friction in gearing for parameters of gears. We will determine the coefficient of the row (13) according to references of work [7] for a chosen plan of mathematic experiment.

When we determine optimal geometrical parameters of worm gears we should pay attention to these circumstances:

– an overlap coefficient of worm pair should be bigger than one unit;
– there is no tagging and undercutting of teeth in worm wheel;
– the worm flexure should be in acceptable boundaries.

When we choose the geometrical parameters,m the first circumstance is always fulfilled. The second circumstance is provided by corresponding choice of worm deposition value and using worm gears with unequal axle step, if it is necessary. When we choose the value q you should pay attention to worm flexure and soundness.

An analysis of the quantity of worm flexure with using [5] shows that acceptable worm flexure for worm gears with axle base a_w = 80 mm is provided with $d_{f1} \cong$ (22...24) mm (d_{f1} – a diameter of worn turn cavity). The firmness of worms is provided for spreading materials and thermal treatment used when they were produced [3].

3. Examples of optimization of parameters of worm gears with minimization of losses on friction in gearing

3.1. Worm gears with involute worms

In this chapter we are going to pay attention to worm gears with gear-ratio U = 31. The axle base equals $a_w = 80 мм$. We will determine geometrical dimensions according to references [5], boundaries of field engagement and contact lines with constant angular displacement will be determined according to references [2, 6].

The levels of factors, planned by mathematical experiment (for the whole factor experiment PFE), equal:

$$-0{,}75 \le X_1 = X \le 0{,}75;\ 7 \le X_2 = q \le 11;\ 29 \le X_3 = U \le 33;\ 16^\circ \le X_4 = \alpha \le 24^\circ.$$

As a result of data procesing of the mathematical experiment used in the above-presented models we have got the following coefficients of regression equation (13):

$b_0 = 0{,}1710$; $b_1 = -0{,}0203$; $b_2 = 0{,}0264$; $b_3 = 0{,}0026$; $b_4 = -0{,}0221$; $b_{12} = +0{,}0009$; $b_{13} = -0{,}0009$; $b_{14} = 0{,}0087$; $b_{23} = 0{,}0004$; $b_{24} = -0{,}0070$; $b_{34} = -0{,}0019$; $b_{11} = 0{,}0075$; $b_{22} = -0{,}0005$; $b_{33} = 0$; $b_{44} = +0{,}0054$.

These coefficients were determined by rotating velocity of worm n_1 = 1500 t/m, T_1 = 8 Hm, HB_2 = 100 (firmness of teeth in worm wheel), R_{a1} = 0,32 (surface roughness of turn worm), ν = 50 cCt (oil body), $E_{пр}$ = 1,1· 10^5 MPa (coerced coefficient of elasticity), material of worm wheel Бр010Ф1.

The most preferred worms with losses on friction in gearing are worm gears with parameters, indicated in Table 1.

Table 1. Optimal parameters of worm gears

Gears with involute worms							
Number of gears	z_1'	U	q	X	m, mm	$\alpha°$	$\psi_з$
1	1	33	7	0,75	3,86	24	0,1315
2	1	30	7	0,75	4,16	24	0,1283
3	1	31	5	4,44	0	20	0,1188
4	1	30	8	4	1	20	0,1450
Ост	1	31	9	4	0	20	0,1707
Gears with ZT-type worm with concave profile of turns							
Number of gears	z_1'	U	q	X	m, mm	$\alpha°$	$\psi_з$
1	1	30	9	0,9	3,92	19	0,1194
2	1	30	7	0,9	4,12	19	0,1101
3	1	31	8	1,0	3,90	21	0,1197
4	1	31	6	0,8	4,15	21	0,1038
5	1	31	8	0,8	3,94	21	0,1201
Gears with ZT-type worm with convex profile of turns							
Number of gears	z_1'	U	q	X	m, mm	$\alpha°$	$\psi_з$
1	1	33	8	-1,25	4,16	23,5	0,1172
2	1	33	8	-1,25	4,16	18,5	0,1195
3	1	31	10	-2,00	4,32	21	0,0888
4	1	31	6	-1,50	4,71	21	0,0812
5	1	33	8	-1,75	4,27	23,5	0,0960

You can see the changing diagrams of coefficient of losses in gearing in Fig. 1. It depends on q, X, α. Gear-ratio is u = 31 Quantity of modules in the dividing diameter of worm gearing has the main influence on friction in gearing in the dividing diameter of worm. You can notice it from an analysis of the diagram. When the quantity of modules changes in boundaries $5 \leq q \leq 13$, losses on friction change by 45….100%, the least values of losses with minimal value q have different values α and X.

According to geometrical calculation (look [5]) the gear-ratio of worm pair is u = 30 with d_{f1} = 23,2, m = 4 and q = 8. For a given value q and number of teeth in wheel Z_2 = 30 we have X = 1. This gear has losses in gearing $\psi_з$ = 0,145, α = 20° and $\psi_з$ = 0,14, α = 24° . According to OCT –2H21 – 4 – 84 the worm of this gear is made with q = 9, m = 4 mm and coefficient of displacement X = 1. Losses in gearing of this worm gear are: $\psi_з$ = 0,14 (look at Fig. 1). It means that it has 17% more of losses in gearing. So, using the above-mentioned regression equation we can provide the choice of parameters of worm gear according to OCT – 2H21 – 4 – 84. Parameters of worm gear q = 8, X = 8, α = 20...24° of u = 30 are called optimal. They provide a reduction of losses in gearing with preservation of worm firmness and inflexibility.

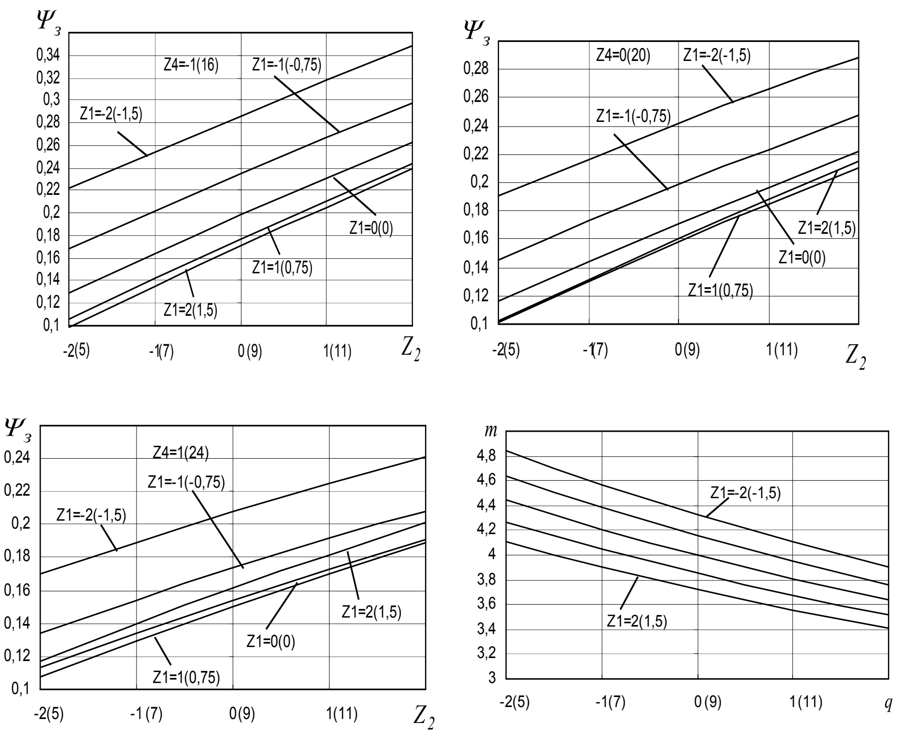

Fig. 1. Meanings of coefficient of losses on friction in gearing (U=31, involute worm, the unfixed values of factors are given in the brackets, m-module of engagement in mm)

3.2. Worm gears with ZT-type worm with concave profile of turns

We will look at worm gears with average gear-ratio u = 31 and axel base a_w = 80 mm. Number of worm turns is z_1 = 1, the diameter of abrasive disc is D_{kp} = 80 m, the radius of profile arc of abrasive disc is r_{kp} = 5 m (m is a module of gear), a coefficient of altitude of turn capping is h^*_{a1} = 1, an angle of incidence of abrasive disc on its dividing cylinder equals a hill climbing ability when a worm is ground. According to geometrical calculation and definition of field boundaries in gearing the references [2, 6] were used.

The values of factor levels were accepted (for PFE):

$$0{,}7 \le X_1 = X \le 0{,}9\,;\; 7 \le X_2 = q \le 9\,;\; 30 \le X_3 = U \le 32\,;\; 23^\circ \le X_4 = \alpha \le 19^\circ. \qquad .$$

Coefficients of regression equation (13) for the definition of coefficient of losses on friction equal:

$$b_0 = 0{,}1201\,;\quad b_1 = -0{,}0126\,;\quad b_2 = 0{,}0138\,;\quad b_3 = 0{,}0058\,;\quad b_4 = -0{,}0039\,;\quad b_{12} = -0{,}0048\,;$$
$$b_{13} = 0{,}0030\,;\; b_{14} = 0{,}0078\,;\; b_{23} = 0{,}0051\,;\; b_{24} = -0{,}0029\,;\; b_{34} = -0{,}0027\,;\; b_{11} = 0{,}0103\,;\; b_{22} = 0{,}0040\,;$$
$$b_{33} = 0{,}0008\,;\; b_{44} = 0{,}0061\,.$$

According to the diagram and regression equation it was determined that the most preferable gears on losses on friction in gearing are worm gears with parameters in Table 1.

Looking through data analysis (Table 1) you can notice that the area of optimal value of worm gears with ZT- worms with concave profile of turns when is determined by values:

$$6 \le q \le 9\,;\ 0{,}8 \le X \le 1\,;\ 19^\circ \le \alpha \le 21^\circ.$$

Losses in gearing of these gears are lower by 1,42…1,64 times than losses in gearing of worm gear with involute worm OCT 2H21-4-84. From condition of worm firmness and inflexibility the most optimal ones are the gears 3 and 5. These gears have coefficients of losses in gearing. They have 83% of coefficients of losses in gear with involute worm with the same parameters (Table 1, gear 4, which coincides with information in work [12]. According to this work, losses in gearing of worm gear with ZT- worm consist of 86% of losses in gearing with involute worm).

It follows, that when we project worm gears with TZ-type with concave profile of turns, it is necessary to pay attention to these points:

– minimal, from condition of worm firmness and inflexibility, value q;
– positive value of coefficient of deposition in boundary $0{,}8 \le X \le 1$;
– the value of profile angle of instrument $19° \le \alpha \le 21°$.

3.3. Worm gears with ZT- worm with convex profile of turns

We will consider the worm gears with average gear-ratio $u = 31$ and axel base $a_w = 80$ mm. The number of worm turns is $z_1' = 1$, the diameter of abrasive disc is $D_{kp} = 80\ m$, the radius of profile arc of abrasive disc is $r_{kp} = 3{,}75\ m$, a coefficient of altitude of turn capping is $h^*_{a1} = 1$, geometrical sizes of worm and worm wheel will be determined according to reference for gears with involute worm [5]. The axis of grinding coincides with axis of gearing of worm pair [6].

The values of factor levels with mathematical experiment equal (for PFE):

$$-1{,}75 \le X_1 = X \le -1{,}25\,;\ 8 \le X_2 = q \le 12\,;\ 29 \le X_3 = U \le 33\,;\ 18{,}5^\circ \le X_4 = \alpha \le 23{,}5^\circ.$$

The values of regression equation (13) for definition of losses on friction in gearing equal:

$$b_0 = 0{,}1172\,;\ b_1 = 0{,}0143\,;\ b_2 = 0{,}0121\,;\ b_3 = 0{,}0016\,;\ b_4 = -0{,}0026\,;\ b_{12} = 0{,}0043\,;\ b_{13} = -0{,}0009\,;$$
$$b_{14} = -0{,}0010\,;\ b_{23} = 0{,}0002\,;\ b_{24} = 0{,}0002\,;\ b_{34} = 0{,}0012\,;\ b_{11} = 0{,}0011\,;\ b_{22} = -0{,}0006\,;\ b_{33} = -0{,}0008\,;$$
$$b_{44} = 0{,}0006\,.$$

According to the regression equation, it was determined that the most preferable gears on losses on friction in gearing are gear parameters, which you can see in Table 1.

Looking through data analysis (Table 1) you can notice that the area of optimal value of worm gear are determined by values: $6 \le q \le 10$; $-2{,}00 \le X \le -1{,}25$; $18° \le \alpha \le 21°$. Losses in gearing of these gears are lower by 1,42-2,09 times than losses in gearing of worm gear with involute worm OCT 2H21-4-84.

CONCLUSIONS

1. A mathematical model of definition of losses on friction in worm gears was developed. This model considers the values in different points of contact patch.

2. A mathematical model of function expansion was developed. It determines losses on friction in gearing of worm gears according to a mathematical experiment.

3. Optimal parameters of worm gears were determined, when u = 31 with involute worms. Quantity of losses on friction in gearing is lower by 1.17 times than gears according to OCT.

4. Optimal parameters of worm gears were determined when u = 31 with ZT- worms. Quantity of losses on friction in gearing are lower by 1,42-2,09 times than gears according to OCT.

5. The findings can be used when projection of worm gears with optimal geometrical parameters with minimization of losses on friction in gearing is made.

REFERENCES

Baturin N.U., 1988. Analys and synthesis of worm gears with the improved quality indicators//Diss. … Cand.Tech. Sciences. – Novocherkassk: 210.

Bernatskij I.N., Vjushkin N.I., Gerasimov V. K., 1974. Rational a choice of parametres of gearing of worm cylindrical transfers//In book Gear and worm gears. – L: Mechanical engineering: 193-210.

Boyko L.S., Vysotsky A.Z., Galichenko E.N., etc., 1984. Reducers and motors-reducers the general machine-building applications. The directory. – M: Mechanical engineering: 247.

Kudryavtsev V. N., Derzhavets J.A., Gluharev E.G., 1971. Design and calculation of gear reducers. The directory. – L: Mechanical engineering: 328.

Levitan J.V., Obmornov V. P, Vasilev V. I., 1985. Worm reducers. The directory. – L: Mechanical engineering: 168.

Litvin F.L., 1968. Theory of gear gearings. – M: the Science: 584.

Nalimov V.V., Chernov N.A., 1965. Static methods of planning of extreme experiments. – M: the Science: 328.

Nosko P. L, Muhovatiy A.A., Shishova N.V., 2003. Optimisation of parametres of worm gears with cylindrical involute worm.//Vesnik to the VNU name V.Dalja. Lugansk. № 12 (70).: 34-40.

Nosko P. L., 1999. Optimum designing of machine-building designs. – Lugansk: Publishing house East-Ukrainian. State University: 392.

Nosko P. L, Malkov V. N, Muhovatiy A.A. 2003. Geometry of a worm processed by disk grinding circle//Technology and processings by pressure of materials in mechanical engineering. The collection of scientific works in 2 p 1. – Lugansk: publishing house VNU of a name of V.Dalja: 126-131.

Parubets V. I., 1985. Peculiarites of contact in worm gears with involute worm.//the mechanical engineering Bulletin. № 6: 17-21.

Predki Wolfgang., 1989-91. Tragfahigneitsvegltich von Schnenen.//Techn., № 13: 80-81.

Kragelsky I.V., Alisin V.V., 1979. Friction, wear process and greasing. The directory. In 2 books. Book. 2. – M: mechanical engineering: 358.

Shishov V.P., 1994. Theory, mathematic security and synthesis realisation highly loaded transfers by gearing for industrial transport./Diss. … Doct. Techn. Sciences. – Lugansk: 525.

Schultz V.V., Tiunov V.V., Levitan Ju.V., 1974. Loss on a friction in worm gears with various geometry //In work Gear and worm gears. – L: Mechanical engineering: 323-330.