TEKA Kom. Mot. Energ. Roln. – OL PAN, 2008, 8, 135–139

GROUND STRESS MODELING

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Summary. The effect of punctual deformation on soil tension was analysed. The condition of deformation destruction at optional durability e. g. share chisel was determined.

Key words: load, normal tension, tangential tension, soil.

INTRODUCTION

The problem of static tensions quantitative estimation in case of applying concentrated load to soil body was first formulated and solved by Z.Bussinek.

Specifically, he has obtained the equation establishing dependence of radial stresses σ_R on the value of applied force F, distance R to the load application point and angle β of deflection from the force direction.

$$\sigma_R = \frac{3}{2} \cdot \frac{F}{\pi \cdot R^2} \cos\beta . \tag{1}$$

Equation (1) shows that radial stresses are inversely proportional to the squared distance to the load F application point and directly proportional to the cosine of the angle of deflection from the direction of this force. Curve of radial stresses according to equation (1) is shown in Fig. 1. As it can be seen in the figure the curve is crescent, changing from 0 at Y-axis to maximum (amplitude) at X-axis.

Works by V.P. Goryachkin, V.A. Zheligovsky, M.E. Matsepuro, A.T. Vagin, Y.V. Chigarev, A. Vilde, A. Rucins, W. Tanaś and others include rich scientific, theoretical and practical material in this area.

MATERIALS AND METHODS

Let us inscribe circle of radius R/2 in the semicircle R (fig. 1) and choose two arbitrary directions l_1 and l_2 at angles β_1 and β_2 to X-axi, accordingly. Rays l_1 and l_2 cross semicircle R in points M and N and the inscribed circle in points M_1 and N_1. We will draw semicircles of radius R_1 and R_2 from the centre O through the obtained points M_1 и N_1.

Let us define radial stresses on the obtained scheme in the direction l_1. According to Bussinek's formula the radial stress σ_{RM} in the point M will be $\sigma_{RM} = 3F\cos\beta_1/(2\pi R^2)$ and in the point M_1 $\sigma_{RM1} = 3F\cos\beta_1/(2\pi R_1^2)$.

Radiuses R$_1$ and R are bound with simple proportion from ΔOM_1A: $R_1 / R = \cos\beta_1$ or $R_1 = R\cos\beta_1$. Then:

$$\sigma_{RM_1} = 3F/(2\pi R^2\cos\beta_1), \tag{2}$$

and because of equality of corresponding angles (see Fig. 1) the projection σ_{M1} of this stress on the normal to the inscribed circle is

$$\sigma_{M1} = 3F/(2\pi R^2) = \sigma_{RA}, \tag{3}$$

where σ_{RA} is radial stress in the point A (with $\cos\beta = 1$).

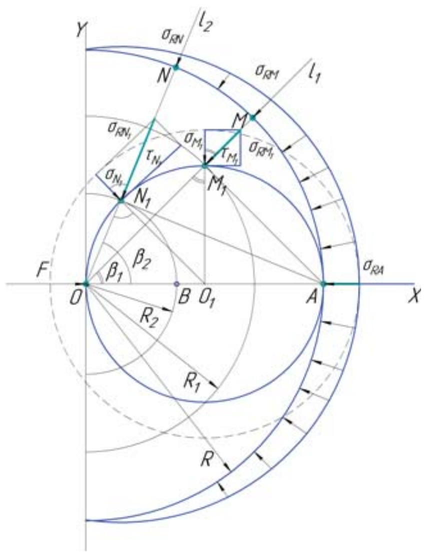

Fig. 1. Scheme to the analysis of stress field of point source

In the direction l_2, similarly to the discussed above: $\sigma_{RN1} = 3F\cos\beta_2/(2\pi R_2^2)$,
and as $R_2 = R\cos\beta_2$, then:

$$\sigma_{RN1} = 3F/(2\pi R^2\cos\beta_2), \tag{4}$$

and the concerning projection of this stress on the normal to inscribed circle

$$\sigma_{N1} = 3F/(2\pi R^2) = \sigma_{RA}. \tag{5}$$

As we can see, at any direction l normal components of radial stress to the inscribed circle are equal inter se and equal to radial stress at X-axis in the point A, i.e. curve of normal stresses σ for the inscribed circle shapes into a circle (see Fig. 1).

Thus, a circle is being formed (in the space it is spherical domain with a diameter $D=R$) in the soil body in front of the point of force F application that is compressed by normal stresses, whose value is directly proportional to force F and inversely proportional to squared diameter or sphere D surface area:

$$\sigma = 3F/(2\pi D^2). \tag{6}$$

Circle diameter decreases with an increase in force F and stress at the point O. From the point of view of static this area is a self-balanced system that is theoretically impossible to destruct: at any value of the force F the sphere collapses but remains in balance.

Let us estimate values of shearing stresses on the circle D in the point M_1. Considering equality of corresponding angles $\tau_{M1} = \sigma_{RM1} \cdot sin\beta_1$ or, putting value σ_{RM1} from expression (2) we will obtain:

$$\tau_{M1} = 3Ftg\beta_1/(2\pi R^2). \tag{7}$$

In the point N_1 shearing stresses will be

$$\tau_{N1} = 3Ftg\beta_2/(2\pi R^2), \tag{8}$$

and in general with consideration of expression (6), the change of shearing stresses on the circle D will be defined by the expression:

$$\tau = \sigma \cdot tg\beta. \tag{9}$$

Therefore, shearing stresses along the circle change from zero on the force direction (X-axis) to infinity on the direction that is perpendicular to force F (Y-axis). Hence, nearby point O, because of considerable shearing stresses, the conditions are created for destruction of deformer of any hardness. In practice sharp-ground ploughshare or chisel is growing blunt actively just in the initial period of work until the blade becomes so blunt that the effect of the above- mentioned shearing stresses shifts in the zone of soil that sticks on it, i.e. in area D itself. At this moment blunting rate decreases and for some time the working element parameters maintain relatively stable values (period of normal service).

When analysing stressed area we note one more property of stresses that are radial to point O: their projection on X-axis is constant and is equal to normal stresses $\sigma=\sigma_{RA}$. In fact, considering expressions (2), (4):

$$\sigma_x = \sigma_{RM1} \cdot cos\beta_1 = \sigma_{RN1} \cdot cos\beta_2 = \sigma = 3F/(2\pi D^2). \tag{10}$$

Circle D is displaced in relation to zero point for amount of $2a = D$. Equation of this circle takes form:

$$x^2 + y^2 = 2ax, \tag{11}$$

from which:

$$y = \sqrt{2ax - x^2} \text{ or } f(x) = \sqrt{2ax - x^2}. \tag{12}$$

In the polar coordinates for $x = \rho\cos\beta$ we obtain:

$$f(x) = \pm\sqrt{2a\rho\cos\beta - \rho^2\cos\beta}. \tag{13}$$

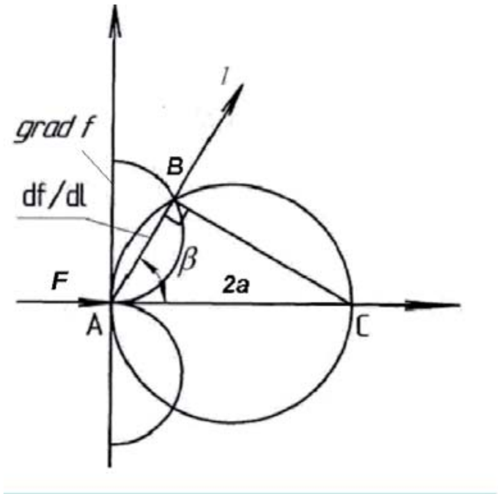

Fig. 2. Directional derivative and gradient of a function of shearing stresses

From ΔABC (fig. 2) $2a = \dfrac{\rho}{\cos\beta} \Rightarrow \rho = 2a\cos\beta$. After inserting value ρ in the formula (13) we will obtain:

$$f(x) = \pm\sqrt{\rho^2 - \rho^2\cos^2\beta} = \pm\rho\cdot\sin\beta. \tag{14}$$

It is known that derivative $\frac{df}{dl}$ of function *f* (*X*) in point *A* in the direction *l* is a limit in point *A* of ratio of the function increment on *l* to the distance ρ (*A, X*):

$$\frac{df}{dl} = \lim\frac{f(X) - f(A)}{\rho(A,X)}. \tag{15}$$

In our case the directional derivative takes form:

$$\frac{df}{dl} = \lim\frac{f(\rho\cos\beta, \rho\sin\beta)}{\rho} = \lim(\pm\frac{\rho\sin\beta}{\rho}) = \pm\sin\beta. \tag{16}$$

The $\pm$ sign in the expression (16) defines the position of sine curve branches in 1 and 4 quadrants *XOY* (Fig. 2).

Physical meaning of directional derivative is that it shows the rate of change in values of stress function in the direction of said vector *l* and the function *f* gradient is a vector showing the direction in which this rate of change is maximum.

Function *f* gradient in point *A* is defined as a vector of the projections which on coordinate axes are equal to the corresponding derivatives of function *f* (*X*) in point *A*. It is also known that directional derivative $\frac{df}{dl}$ is a scalar product of gradient and unit vector of direction *l*. Therefore, derivative in the direction *l* is equal to projection of gradient on this direction:

$$\frac{df}{dl} = gradf \cdot l = |gradf| \cdot \cos(90 - \beta) = |gradf| \cdot \sin \beta . \qquad (17)$$

Considering formula (16) we obtain:

$$|gradf| \cdot \sin \beta = \pm \sin \beta , \qquad (18)$$

from which:

$$gradf = \pm 1. \qquad (19)$$

CONCLUSIONS

As we can see, the directional derivative is a harmonic continuous function. It defines running values of parameters participating in the energy-transfer process. And the gradient is a constant value not depending on parameters of stress circle.

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