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IMITATING MODEL OF THE AGRICULTURAL TRACTOR UNIT FOR RESEARCH OF DYNAMIC LOADINGS IN TRANSMISSION AND SMOOTHNESS OF A MOTION

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Summary. The article presents the developed mathematical imitation model for research of dynamic loadings in transmission and smoothness of a motion of a machine-tractor unit on the basis of chassis with all driving wheels as well as the hinge-jointed agricultural tractor with the wheel formula 6K6.

Key words: the hinge-jointed agricultural tractor chassis, mathematical imitation model, loadings in transmissions, smoothness of a motion

INTRODUCTION

One of directions of development of pull-power supply sources in machine-tractor units (MTU) will consist in a creation, on the basis of serial tractors, of the universal hinge-jointed chassis with the wheel formula 6K6 agricultural assignment with a technological platform on back half-chassis on which can be aggregated, for example, a self-unloading platform, capacities for transportation and scatterings of liquid fertilizers and toxic chemical protection of plants with pipes and many other kinds of the equipment. Differing in the increased passableness, high shunting qualities and the best fitness to work in heavy road conditions, such MTU allow to realize new principles concerning the chassis in structure of multioperational technological agricultural units, that essentially raises efficiency of their application.

However, these data do not suffice for designing such chassis, in particular, the loadings given about dynamics on carrying system and in the ramified transmission in real conditions of operation MTU. Natural tests are dear and long. Therefore, it represents a scientific and practical interest to create a mathematical imitation model MTU of the given kind and carrying out of modelling on the COMPUTER of work of units of various assignment in the whole spectrum of operational operating modes.

According to needs of practice, mathematical models are developed for research dynamic loads transmissions, half-chassis and their hinge-joint connections in characteristic operating conditions, and also smoothness of a course of machines. Therefore it is necessary to consider dynamic systems of machines in which interactions of their movement of subsystems determining dynamics among themselves and with roughnesses of a basic surface are reflected. By virtue of it the mathematical model should be universal and intended for both modelling and an estimation of several dynamic properties of researched machines. Despite of complexity of its subsequent machine realization, similar mathematical models allow to take into account with greater reliability features of object of modelling and a condition of its work [Bogdan and Tajanowskij 2000, Krasowski (red.) 2005]. At the same time more simple models for research of separate properties of a machine, for example smoothness of a course, due to comprehensible adequacy and smaller complexity, allow to achieve faster and with smaller expenditures of labour the estimations of smoothness of a course of a machine. That is why they are also widely used in design researches [Tajanowskij and Tanas 2004, 2006].

Separate scientific positions of the methodical approach to imitating modelling tractor chassis of agricultural assignment are developed by authors in earlier works [Bogdan, Tajanowskij 2000, Tajanowskij and Tanas 2004, 2006] and also are used in the given model.

THE DIFFERENTIAL EQUATIONS OF MOVEMENT THE TRACTOR UNIT ON A BASIC SURFACE

Settlement dynamic systems of the tractor unit are developed at assumptions, characteristic for the decision of similar problems [Bogdan and Tajanowskij 2000, Krasowski (red.) 2005, Tajanowskij and Tanas 2006] the subsystem engine-transmission-wheels installations represents discrete dynamic rotating system with non-mass resilient extinguishing connections; movement of the machine – rectilinear on a casual microstructure of horizontal road; the running system is considered as flat symmetric concerning the longitudinal axis; the skeleton of the machine represents a firm body with a longitudinal axis of symmetry; statistical parameters of road influences on the right and left wheels are equal; the seat of the driver is located in a longitudinal – vertical plane of symmetry and makes vertical fluctuations; weights of a seat and the driver are concentrated in one point; the trunk is considered elastic in a radial direction, and properties of horizontal interactions with a deformable basic surface are taken into account by the so-called curve of slipping of a wheel. Wheels make continuous movement; the racetrack of the trunk is considered as a non-mass hoop with the radius equal to radius rotation, contact of a wheel to road takes into account smoothing ability of trunks; rigidity of trunks, transmissions, suspension brackets are constant, extinguishing resistance are linear concerning speed of deformation; high-frequency fluctuations are not considered; fluctuations of weights of dynamic systems of the tractor chassis are small.

In Fig. 1 the settlement circuit of the tractor chassis in a transport operating mode is submitted, which includes dynamic system of transmission (Fig. 1a) and oscillatory system of the machine at movement on roughnesses of a basic surface.

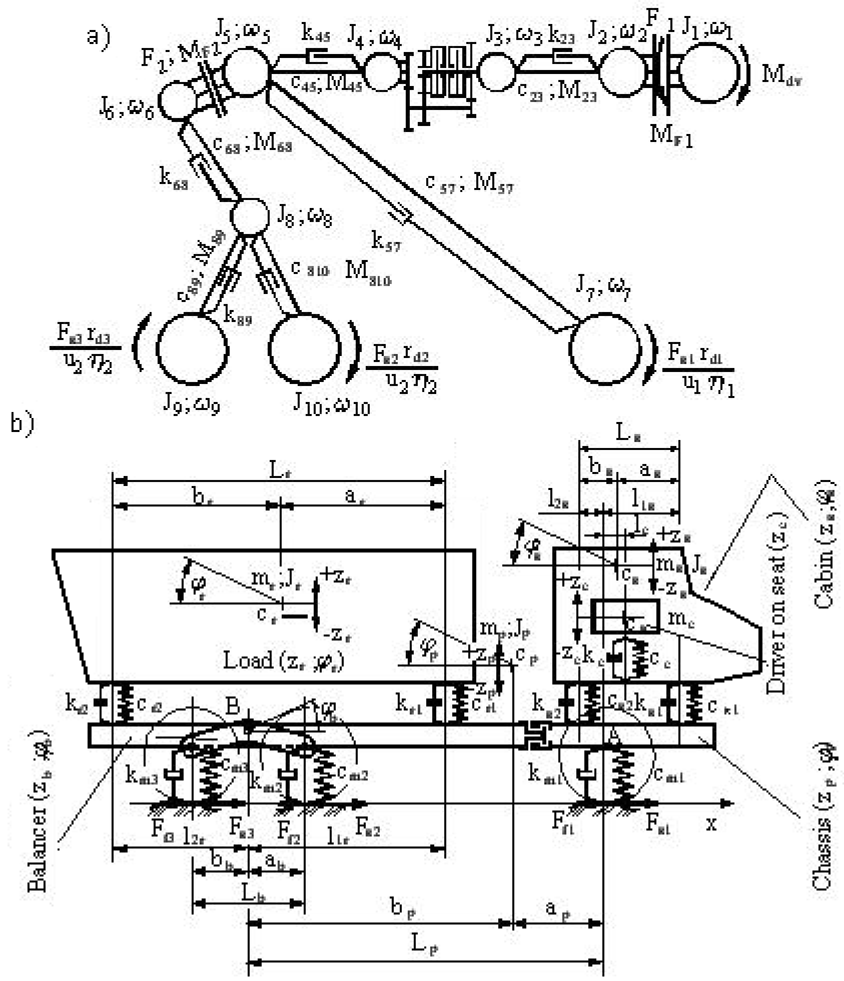

Fig. 1. Settlement dynamic model of the tractor unit

In dynamic system of transmission the following designations are accepted: I_1, ω_1 – the moment of inertia of rotating details of the engine and angular speed of rotation of its cranked shaft; I_2, ω_2 – the moment of inertia of conducted details of coupling and its angular speed; I_3, I_4, ω_3, ω_4 – the moments of inertia and angular speeds, accordingly, details on conducting and conducted shaft of a box of transfers and their angular speeds; I_5, ω_5 – the moment of inertia of details of a distributing box and its angular speed; I_6, ω_6 – the moment of inertia of a conducted part unit pattern managements of back conducting bridges and its angular speed; I_7, ω_7 – the moment of inertia of forward driving wheels and their angular speed; I_8, ω_8 – the moment of inertia of details of the conducting back bridge, including a cogwheel of onboard transfer from which the twisting moment is distributed on the first and second back wheels, and its angular speed; I_9, I_{10}, ω_9, ω_{10} – the moments of inertia of back wheels and their angular speeds; c_{ij}, k_{ij} – rotated rigidity and extinguishion in *ij-X* sites of transmission (i = 2, 4, 5, 6, 8; j = 3, 5, 7, 8, 9, 10); *Mij* – the dynamic moments in *ij-X* sites of transmission; F_1, F_2 – accordingly, coupling and unit pattern inclusions – deenergizings of the back bridge; *Mdv* – the twisting moment of the engine; *Fkiri/(Uiηi)* – the twisting moments on driving wheels *(i* = 7, 9, 10*)*.

All elements which are included in the dynamic system of transmission of the machine, are resulted in a cranked shaft of the engine.

The system of the differential equations describing work of transmission of the tractor chassis, equipped with a self – unloading platform with the device of loading for transportation of agricultural cargoes, for example sugar beet from heaps in a field to a place of processing, in a transport mode, has the following appearance:

$$
\begin{array}{ll}
\text{At } \omega_1 \neq \omega_2 & \text{At } \omega_1 = \omega_2 \\
I_1\dot{\omega}_1 = M_{dv} - M_{F1}; & (I_1 + I_2)\dot{\omega}_1 = M_{dv} - M_{23}; \\
I_2\dot{\omega}_2 = M_{F1} - M_{23}; & (I_1 + I_2)\dot{\omega}_2 = M_{dv} - M_{23};
\end{array}
$$

$$
\begin{gathered}
(I_3 + I_4)\dot{\omega}_3 = M_{23} + K_{23}(\omega_2 - \omega_3) - [M_{45} + K_{45}(\omega_3 - \omega_5)]; \\
(I_3 + I_4)\dot{\omega}_4 = M_{23} + K_{23}(\omega_2 - \omega_3) - [M_{45} + K_{45}(\omega_4 - \omega_5)]; \\
(I_5 + I_6)\dot{\omega}_5 = M_{45} + K_{45}(\omega_4 - \omega_5) - [M_{68} + K_{68}(\omega_5 - \omega_8) + M_{57} + K_{57}(\omega_5 - \omega_7)]; \\
(I_5 + I_6)\dot{\omega}_6 = M_{45} + K_{45}(\omega_4 - \omega_5) - [M_{68} + K_{68}(\omega_6 - \omega_8) + M_{57} + K_{57}(\omega_5 - \omega_7)]; \\
I_7\dot{\omega}_7 = M_{57} + K_{57}(\omega_5 - \omega_7) - \frac{F_{k1} r_{d1}}{U_1 \eta_1}; \\
I_8\dot{\omega}_8 = M_{68} + K_{68}(\omega_5 - \omega_8) - [M_{89} + M_{810} + K_{89}(\omega_8 + \omega_9) + K_{810}(\omega_8 - \omega_{10})]; \qquad (1) \\
I_9\dot{\omega}_9 = M_{89} + K_{89}(\omega_8 - \omega_9) - \frac{F_{k3} r_{d3}}{U_2 \eta_2}; \\
I_{10}\dot{\omega}_{10} = M_{810} + K_{810}(\omega_8 - \omega_{10}) - \frac{F_{k2} r_{d2}}{U_2 \eta_2}; \\
\dot{M}_{23} = C_{23}(\omega_2 - \omega_3); \\
\dot{M}_{45} = C_{45}(\omega_4 - \omega_5); \\
\dot{M}_{57} = C_{57}(\omega_5 - \omega_7); \\
\dot{M}_{68} = C_{68}(\omega_6 - \omega_8); \\
\dot{M}_{89} = C_{893}(\omega_8 - \omega_9); \\
\dot{M}_{810} = C_{810}(\omega_8 - \omega_{10});
\end{gathered}
$$

where:

F_{ki} – the tangent force of draft developed by wheels *i* of the bridge;
r_{di} – dynamic radius of wheels *i* the bridge;
$U_i = U_{кп} U_{рк} UM$ – transfer number from the engine to wheels *i* the bridge, here $U_{кп}$, $U_{рк}$,
UM – transfer numbers, accordingly, boxes of transfers, the distributing box, the conducting bridge;
η_i – efficiency of a drive to wheels;
i – the bridge.

The system of the equations (1) has zero entry conditions, except for $\omega_1 = \omega_{10}$, i.e. during the initial moment of time shaft the engine has some angular speed ω_{10}.

For definition of a tangent of force of draft of the bridge wheels the formula is used:

$$F_{ki} = \varphi \cdot G_i \cdot \left(1 - e^{-k\delta_i}\right),$$

where:

φ – factor of coupling of wheels with a basic surface;

G_i – the vertical loading falling;

i – the bridge;

k – the factor dependent on properties of a basic surface;

δ_i – slipping of wheels.

Slipping of wheels of conducting bridges are based on expression:

$$\delta_i = \frac{\vartheta_{Ti} - \vartheta}{\vartheta_{Ti}} = 1 - \frac{\vartheta}{\vartheta_{Ti}} = 1 - \frac{\dot{x}}{\omega_i r_i}, \tag{2}$$

where:

ϑ_{Ti} – theoretical speed of the centers of wheels; *i* the bridge;

$\vartheta = \dot{x}$ – the valid speed of the machine;

ω_i – angular speed of wheels, determined from the decision of the system of the equations (1).

Mathematical descriptions of work of the engine, unit couplings at beginning of movement and dispersal of the machine, and also work units switchings of transfers during switching transfers in the synchronized box of transfers and at switching without break of a stream of capacity, in case of a hydromechanical box of transfers with hydrolics frictional units of booster type, are typical and widely used at research of dynamic loadings in transmission of tractors on mathematical models on the COMPUTER and in the given article are not resulted.

Solving system of the equations (1), it is possible to define the dynamic moments in the basic parts of transmission of the machine at its movement in various conditions of operation, slipping of wheels of conducting bridges and a tangent developed by their force of draft, speed of movement of the machine, etc.

Let us consider oscillatory system of the machine in Fig. 1. In Figure designations are accepted: m_p, L_p; m_k, L_k; m_r, L_r; m_b, L_b – weights and the moments of inertia accordingly with frames of the machine with the equipment (the engine, transmission, a platform, grasping the manipulator, etc.), cabins, a cargo, the balance weight; m_c – weight of a seat with the driver; c_P, c_K, c_R – the centers of weights accordingly with frames, cabins, a cargo; a_P, b_P; a_κ, b_κ; a_R, b_R; a_b, b_b – distances from a forward and back support up to the center of weights accordingly to frames, cabins, a cargo, the balance weight; c_{mi}, k_{mi}; c_{ki}, k_{ki}; c_{Ri}, k_{Ri} - rigidity and extinguishing accordingly with trunks, a suspension bracket of a cabin, a suspension bracket of a cargo; cc, кс – rigidity and extinguishion suspension brackets of a seat of the driver.

From Fig. 1, it can be seen, that the oscillatory system of the machine includes the following oscillatory subsystems; a frame with the equipment, the balance weight, a cabin, a cargo and a seat with the driver. First four oscillatory subsystems can move in a vertical direction $\left(\pm Z_p, \pm Z_b, \pm Z_k, \pm Z_r\right)$ which is counted from position of static balance, and can turn on small corners $\pm\varphi_p, \pm\varphi_b, \pm\varphi_к, \pm\varphi_r$. Thus it is possible to accept $tg\,\varphi \approx \varphi$ $\quad u \quad$ $sin\,\varphi \approx \varphi$ [Bogdan and Tajanowskij 2000, Krasowskiego (red.) 2005]. The last subsystem – a seat with the driver – can make only the vertical movings $\pm Z_c$ also counted from position of static balance of sitting.

Hence, the oscillatory system of the machine has nine degrees of freedom and fluctuation of the machine will be described by nine differential equations of the second order.

Each subsystem receives indignation from the various sources influencing points of a support of subsystems. So, the balance weight receives indignations from roughnesses of a basic surface which are transferred through elastic trunks. And at equality of base of balance weight L_b and lengths of roughness $l_н$, the point of fastening of the balance weight In will describe roughness, that is moving of a point I_n and axes of wheels of the balance weight will be identical. Otherwise, the point I_n will make the movings which are distinct from movings of axes of wheels, connected with the balance weight. The frame with the equipment receives indignation from roughnesses of a basic surface which are transferred to a point and through elastic forward trunks and the indignations received by a point In from the balance weight. The cabin receives indignations from a frame in points of fastening of a suspension bracket of a cabin. Indignations are transferred to a cabin through its suspension bracket. Similarly, it receives indignation from a frame and a cargo. The weight of sitting with the driver receives indignation from a floor of a cabin in a point of fastening of sitting through its suspension bracket. Hence, the first four oscillatory subsystems represent the weights leaning in two points on elastic elements in which a part of energy is lost. Fluctuation of such subsystems is described by the following system of the differential equations:

$$\begin{aligned} M_1\ddot{Z}_1 + M_3\ddot{Z}_2 + 2K_{p1}\left(\dot{Z}_1 - \dot{\xi}_1\right) + 2c_{p1}\left(Z_1 - \xi_1\right) = 0; \\ M_3\ddot{Z}_1 + M_2\ddot{Z}_2 + 2K_{p2}\left(\dot{Z}_2 - \dot{\xi}_2\right) + 2c_{p2}\left(Z_2 - \xi_2\right) = 0; \end{aligned} \tag{3}$$

where:

M_1, M_2, M_3 – factors;

Z_1, Z_2 and their derivatives – movings, speed and acceleration of points above suspension brackets according to forward and back;

ξ_1, ξ_2 and their derivatives – movings, speed and acceleration of points of fastening of a suspension bracket (for the balance weight – heights of roughnesses under wheels balanced carriages);

c_{pi}, c_{pi} – rigidity and extinguishion in a suspension bracket.

$$M_1 = m_i \frac{a_i^2 + \rho_i^2}{L_i^2}; \quad M_2 = \frac{b_i^2 + \rho_i^2}{L_i^2}; \quad M_3 = \frac{a_i b_i - \rho_i^2}{L_i^2},$$

where:

m_i – weight i subsystems;

a_i, b_i – coordinates of the center of weights i subsystems;

ρ_i - radius of inertia i subsystems;

L_i – distances between elastic elements i subsystems.

The system of the equations (3) has zero entry conditions, that is at $t = 0, \quad Z_1 = \dot{Z}_1 = Z_2 = \dot{Z}_2 = 0.$

So, fluctuation of the machine at movement on roughnesses of a basic surface will be described by the following system of the differential equations:

– The balance weight

$$M_{1b}\ddot{Z}_{1b} + M_{3b}\ddot{Z}_{2b} + 2K_{m2}\left(\dot{Z}_{1b} - \dot{q}_2\right) + 2c_{m2}\left(Z_{1b} - q_2\right) = 0;$$
$$M_{3b}\ddot{Z}_{1b} + M_{2b}\ddot{Z}_{2b} + 2K_{m3}\left(\dot{Z}_{2b} - \dot{q}_3\right) + 2c_{m3}\left(Z_{2b} - q_3\right) = 0.$$

Frame with the equipment

$$M_{1p}\ddot{Z}_{1p} + M_{3p}\ddot{Z}_{2p} + 2K_{m1}\left(\dot{Z}_{1p} - \dot{q}_1\right) + 2c_{m1}\left(Z_{1p} - q_1\right) = 0;$$
$$M_{3p}\ddot{Z}_{1p} + M_{2p}\ddot{Z}_{2p} = 0; \tag{4}$$

a cabin

$$M_{1k}\ddot{Z}_{1k} + M_{3k}\ddot{Z}_{2k} + 2K_{k1}\left(\dot{Z}_{1k} - \dot{\xi}_{1k}\right) + 2c_{k1}\left(Z_{1k} - \xi_{1k}\right) = 0;$$
$$M_{3k}\ddot{Z}_{1k} + M_{2k}\ddot{Z}_{2k} + 2K_{k2}\left(\dot{Z}_{2k} - \dot{\xi}_{2k}\right) + 2c_{k2}\left(Z_{2k} - \xi_{2k}\right) = 0;$$

a platform with a cargo and the mechanism of loading

$$M_{1\Gamma}\ddot{Z}_{1\Gamma} + M_{3r}\ddot{Z}_{2r} + 2K_{r1}\left(\dot{Z}_{1r} - \dot{\xi}_{1r}\right) + 2c_{r1}\left(Z_{1r} - \xi_{1r}\right) = 0;$$
$$M_{3r}\ddot{Z}_{1r} + M_{2r}\ddot{Z}_{2r} + 2K_{r2}\left(\dot{Z}_{2r} - \dot{\xi}_{2r}\right) + 2c_{r2}\left(Z_{2r} - \xi_{2r}\right) = 0;$$

a seat with the driver

$$m_c\ddot{Z}_c + K_c\left(\dot{Z}_c - \dot{\xi}_c\right) + c_c\left(Z_c - \xi_c\right) = 0.$$

The system of the equations (4) has zero entry conditions. Indignation for oscillatory system of the machine are smoothed on length of stain – contact of the trunk and roughness of a basic surface under lobbies *q1*, average *q2* and back wheels *q3*.

For definition of a way, speed and acceleration of progress of the machine it is necessary to work out one more equation:

$$m\ddot{x} = F_{k1} + F_{k2} + F_{k3} - F_f; \tag{5}$$

where:

m – weight of the machine;

F_f – force of resistance to movement;

x and its derivatives – a way, speed and acceleration of the machine.

By force of resistance of air it is neglected, as it is small in comparison with force of resistance to movement.

The equation (5) has zero entry conditions, that is at $t = 0 \quad x = \dot{x} = 0.$

The equations of communication are still necessary for the decision of initial system of the equations. We shall write down these equations:
the balance weight – a frame

$$Z_b = Z_{2p} = \frac{a_b Z_{1b} + в_b Z_{2b}}{L_b},$$

a frame – a cabin

$$\xi_{1к} = \left(Z_{1p} - Z_{2p}\right)\frac{a_p + l_{1k}}{L_p} + Z_{2p};$$

$$\xi_{2к} = \left(Z_{1p} - Z_{2p}\right)\frac{a_p - l_{2k}}{L_p} + Z_{2p};$$

a frame – a cargo

$$\xi_{1r} = \left(Z_{1p} - Z_{2p}\right)\frac{b_p - l_{1r}}{L_r} + Z_{2p};$$

$$\xi_{2r} = \left(Z_{1p} - Z_{2p}\right)\frac{b_p + l_{1r}}{L_P} + Z_{2p};$$

a cabin – sitting of the driver

$$\xi_c = \left(Z_{1p} - Z_{2p}\right)\frac{a_p + l_c}{L_p} + Z_{2p}.$$

The derivatives $\dot{Z}_{2p}, \dot{\xi}_{1k}, \dot{\xi}_{2k}, \dot{\xi}_{1r}, \dot{\xi}_{2r}, \dot{\xi}_c$ which are included in the system of equations (4), are by differentiation of the equations of communication (connection) on time.

Thus, deciding(solving) in common systems of the equations (1), (4) and the equation (5), we shall obtain all data on dynamic loading in transmission, progress of the machine and fluctuations of its subsystems. At the decision of systems of the equations speed of movement of the machine, received as a result of the decision of the equation (5) is substituted in the formula (2) for calculation of slipping of driving wheels, and tangents of force of draft of driving wheels – in the system of equations (1) and the equation (5).

Dynamic force factors, in places of fastening of wheels, balance weights, cabins and a cargo, working on half-chassis machines that is necessary for calculation of the strength of the last, and also hinges with vertical and longitudinal horizontal axes of relative turn half-chassis, are defined as the sum of corresponding elastic forces and forces of not elastic resistance through determined at the decision of the equations of movement of the machine relative deformations of elastic elements and relative speeds in places resilient extinguishing connections of dynamic model.

Acceleration of any point of a frame of the machine and the mentioned forces in trunks and suspension brackets are under the following formulas:

– acceleration i points through known acceleration k points, moving off from each other on distance L_i:

$$\ddot{z}_i = \ddot{z}_k + \frac{\partial^2 \varphi}{\partial t^2} \cdot L_i,$$

– vertical effort in trunks of wheels of the bridge

$$Đ_{mi} = c_{mi} \cdot f_{mi} + K_{mi} \cdot \dot{f}_{mi}, (i = 1...3);$$

– vertical effort in resilient extinguishing of an element of a suspension bracket

$$Đ_i = c_i \cdot f_i + K_i \cdot \dot{f}_i + F_i \cdot sign(\dot{f}_i),$$
$$K_i = \begin{cases} K_{i1}, \dot{f}_i \le 0 \\ K_{i2}, \dot{f}_i > 0, (i = 1, c), \end{cases} \quad (6)$$

where:

c – rigidity of an elastic element;

K_i – factor extinguishion (with an index 1 – on a course of a release, 2 – on a course of compression);

F_i–absolute value of force of dry friction;

$f, \dot{f}$ – deformation on coordinate z and speed of change of deformation elastic - extinguishing parts of dynamic system.

Deformations are as difference of absolute movings of corresponding points of system which are defined under the formula:

$$z_i = z_j \pm \varphi_j L_i$$

Using system of the equations (4), it is possible to define the lowest Ωn and the maximum (supreme) Ωb own frequencies of each subsystem. We shall show that it is possible to do this by the example of system of the equations (3) to which can describe fluctuations of subsystems a frame with the balance weight, a cabin, a cargo.

In system of the equations (3) we shall neglect extinguishion in system of a suspension bracket ($кр_1 = кр_2 = 0$) as the account extinguishion in system changes sizes of own frequencies on 3 … 5%. Besides, as we shall consider own fluctuations, on the springing system indignations do not operate ($\xi_1 = \xi_2 = 0$). We shall divide the first equation on M_1, and the second on M_2. Hence, the system of the equations (3) can be presented in the following way:

$$\ddot{Z}_1 + \frac{M_3}{M_1}\ddot{Z}_2 + \frac{2c_{p1}}{M_1} Z_1 = 0;$$

$$\ddot{Z}_2 + \frac{M_3}{M_1}\ddot{Z}_1 + \frac{2c_{p2}}{M_2}Z_2 = 0.$$

Let's enter designations:

$$\frac{M_3}{M_1} = \eta_1; \quad \frac{M_3}{M_2} = \eta_2; \quad \frac{2c_{p1}}{M_1} = \omega_1^2; \quad \frac{2c_{p2}}{M_2} = \omega_2^2. \tag{7}$$

Factors ω_1 and ω_2 name part frequencies - frequencies of fluctuations of complex oscillatory system on one of degrees of freedom (Z_1 or Z_2) if moving on the second degree of freedom is impossible.

Let's copy system of the equations in the accepted designations:

$$\begin{aligned} &\ddot{Z}_1 + \eta_1\ddot{Z}_2 + \omega_1^2 Z_1 = 0; \\ &\ddot{Z}_2 + \eta_2\ddot{Z}_1 + \omega_2^2 Z_2 = 0. \end{aligned} \tag{8}$$

Let's reduce system of the equations (8) to two equations of the fourth order: is relative Z_1

$$(1 - \eta_1\eta_2)Z_1^{IV} - (\omega_1^2 + \omega_2^2)\ddot{Z}_1 + \omega_1^2\omega_2^2 Z_1 = 0; \tag{9}$$

is relative Z_2

$$(1 - \eta_1\eta_2)Z_2^{IV} - (\omega_1^2 + \omega_2^2)\ddot{Z}_2 + \omega_1^2\omega_2^2 Z_2 = 0. \tag{10}$$

Own frequencies we shall find as roots of the characteristic equation which will be identical to the differential equations (9) and (10):

$$(1 - \eta_1\eta_2)\Omega^4 - (\omega_1^2 + \omega_2^2)\Omega^2 + \omega_1^2\omega_2^2 = 0.$$

Solving the received equation of the fourth degree and taking into account only positive roots, as frequencies cannot be negative, we shall receive two own frequencies of fluctuations of the considered oscillatory system:

The lowest frequency

$$\Omega_{ni} = \sqrt{\frac{1}{2(1 - \eta_1\eta_2)}\left[(\omega_1^2 + \omega_2^2) - \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\eta_1\eta_2\omega_1^2\omega_2^2}\right]} \tag{11}$$

The maximum frequency

$$\Omega_{bi} = \sqrt{\frac{1}{2(1 - \eta_1\eta_2)}\left[(\omega_1^2 + \omega_2^2) + \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\eta_1\eta_2\omega_1^2\omega_2^2}\right]} \tag{12}$$

Own frequency of fluctuations of the driver on a seat pays off under the formula

$$\omega_{в} = \sqrt{\frac{c_c}{m_c}}. \tag{13}$$

Thus, having calculated on expressions (7) factors η_i and ω_i for each subsystem, under formulas (11) and (12) it is possible to define(determine) the lowest and maximum frequency of each subsystem, and on expression (13) the own frequency of fluctuations of the driver on a seat.

Thus, solving the common developed systems of equations we shall receive the full information on movement of MTA in the set conditions of operation, fluctuations of inertial weights and the dynamic moments in transmission, and also about parameters of casual fluctuations of subsystems of the unit, at movement on roughnesses of a basic surface and dynamic force factors, in places of fastening of wheels, balance weights, cabins and a cargo, working on half-chassis machines, which is necessary for the precise calculation of the last and for an estimation of smoothness of a course.

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