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SCALE-UP OF THERMOPLASTIC STARCH EXTRUSION

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Summary. The scale-up of processing equipment is an important and not very easy question that must be given careful consideration during designing the extrusion technique and its tools. As the size of equipment increases, the various parameters and dimensions change at different rates, which can have a profound impact on the process. There must be a clear understanding how the critical dimensions of the equipment change as well as what the critical parameters of the process are. Equipment differences may have more effect on some processes than on others. In the paper major engineering, kinematic, thermal and energy balance aspects in scale-up of thermoplastic starch extrusion equipment are discussed. The main rules presented were worked out and proved practically during the realization of the BIOPACK project – CT.

Key words: scale-up rules, extruders, thermoplastic starch, geometrical and kinematic similarity, energy balance

INTRODUCTION

Scale-up rules provide the possibility to transfer knowledge obtained on small scale laboratory equipment to large scale production units. The principle of scale-up is that equations describing the behaviour of process equipment can be written in a dimensionless form. If the resulting dimensionless groups are kept equal in the small scale and in the large scale equipment, the solutions of the various equations remain constant in a dimensionless form.

Scale up of thermoplastic starch extrusion suffers from the same general problems that are encountered in many other processes in the process industry:

– on scaling up, the surface to volume ratio decreases and therefore the possibilities for heat transfer are limited in large scale equipment,

– at equal temperature differences between the temperature gradients, and therefore the heat fluxes, are smaller in large scale equipment,

– at equal shear fields in large scale and small scale equipment diffusion limitations connected to distributive mixing can be more predominant in large extruders.

Various theories on the scale-up of single screw extruders exist. Due to the high viscosities a considerable amount of the process energy is transferred into heat by viscous dissipation. Therefore the thermal considerations will dominate the scale up rules and an important aspect is the extent to which the process is adiabatic or not. If the process can be considered to occur adiabatically a sufficient condition for scale-up will be that the energy input per unit throughput is constant and the average temperature of the end product will be the same in the small scale and the large scale equipment. If this is not the case, similar temperature profiles in both types of equipment, called complete thermal similarity, are required.

In order to obtain complete thermal similarity, the screw rotation rate has to decrease drastically with the increasing screw diameter, as compared to the adiabatic case. As a result, the scale factor for the throughput is only 1.5 for Newtonian fluids (and decreases even further for fluids with pseudo plastic behaviour). This scale-up factor (q) for the throughput is defined from:

$$\left[\frac{Q}{Q_0}\right] = \left[\frac{D}{D_0}\right]^q \tag{1}$$

where:

Q – denotes the throughput,
D – the screw diameter and the subscript
0 – indicates the small extruder.

In case of adiabatic scale-up a scale up factor up to 3 can be achieved.

Several types of similarities can play a role in the scale-up of an extruder:

Geometric similarity exists if the ratio between any two length parameters in the large scale equipment is the same as the ratio between the corresponding lengths in the small scale model. This is not necessarily the case, as will be seen later

For hydrodynamic similarity two requirements should be fulfilled: The dimensionless flow profiles should be equal and for twin screw extruders, both extruders should have the same (dimensionless) filled length. Equal dimensionless flow profiles lead to equal shear rates in corresponding locations, but not to equal velocities.

Similarity in residence times means equal residence times in the small scale and large scale equipment. This is a requirement that is often not fulfilled in extrusion processes, and in thermoplastic starch extrusion this can only be realized if the scale-up is adiabatic.

Absolute **thermal similarity** is difficult to achieve as stated before. This similarity indicates equal temperatures in all corresponding locations. A distinction has to be made between processes with small heat effects and those with high heat effects. For adiabatic processes where the heat generation is far more important than heat removal to the wall, similarity based on over-all energy balances is generally used. Although, strictly speaking, this does not lead to thermal similarity, equal average end temperatures of the product lead to far more favourable scale-up rules.

BASIC ANALYSIS

To derive rules for scale-up, all parameters are assumed to be related to the diameter ratio by a power-relation. For this purpose, in this report all basic parameters will be written in capitals, whereas the scale-up factors will be written in small print. This implies that all relevant parameters can be related to the screw diameter according to:

$$\left[\frac{N}{N_0}\right]=\left[\frac{D}{D_0}\right]^n;\left[\frac{P}{P_0}\right]=\left[\frac{D}{D_0}\right]^p;\left[\frac{\mu}{\mu_0}\right]=\left[\frac{D}{D_0}\right]^\upsilon;$$

$$\left[\frac{H}{H_0}\right]=\left[\frac{D}{D_0}\right]^h;\left[\frac{L}{L_0}\right]=\left[\frac{D}{D_0}\right]^l;\left[\frac{\tau}{\tau_0}\right]=\left[\frac{D}{D_0}\right]^t;\left[\frac{R}{R_0}\right]=\left[\frac{D}{D_0}\right]^r \tag{2}$$

where: τ

N – the rotation rate of the screws,

P – the die pressure and μ – the viscosity,

Q_b– the back flow is an important parameter for the working of a twin screw extruder.

H – denotes the cannel depth,

L – the screw length,

τ – the residence time in the extruder,

R – the pumping efficiency.

For thermal scale up rules two more parameters have to be defined, the Greaz number (Gz) and the Brinkmann number (Br) that will be defined later. The scale up notation for these dimensionless groups reads:

$$\left[\frac{Gz}{Gz_0}\right]=\left[\frac{D}{D_0}\right]^{gz} \; and \; \left[\frac{Br}{Br_0}\right]=\left[\frac{D}{D_0}\right]^{br} \tag{3}$$

SUMMARY OF EQUATIONS USED

Scale-up rules are necessarily rather mathematical in nature. In this paragraph the extruder equations used are summarized.

The throughput of a single screw extruder can be written as:

$$Q=\frac{1}{2}\pi^2 ND^2 H(1-a)\sin\theta\cos\theta \tag{4}$$

The width of the flight is neglected and it can be shown that this simplification has no influence on the final outcome. θ is an angle of flight. a is the throttle coefficient and can be written as:

$$a = \frac{H^2 \Delta P \tan\theta}{6\mu(\pi ND)L} \tag{5}$$

The equation for the motor power in the pump zone can be written as:

$$E = \frac{(\pi ND)^2 WL}{H\sin\theta}(\cos^2\theta + 4\sin^2\theta + 3a\cos^2\theta) \tag{6}$$

where:

W – a channel width.

For the use in scaling rules this equation can be simplified for screws with the same flight angle to:

$$E = \text{const}\,\frac{\mu D^3 N^2 L}{H} \tag{7}$$

The pumping efficiency of the extruder is the ratio of energy used for pumping the material and the total energy input into the extruder.

$$R = \frac{QP}{E} \tag{8}$$

Thermal similarity yields from the energy balances:

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + q \tag{9}$$

In this equation q is the heat produced by viscous dissipation:

$$q = 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \tag{10}$$

If the equations above are made dimensionless there remain two important dimensionless numbers that govern the heat balances in the extruder, the Graez number and the Brinkmann number.

$$Gz = \frac{UH^2}{aL} \quad \text{and} \quad Br = \frac{\mu U^2}{\lambda \Delta T} \tag{11}$$

where:

λ – thermal conductivity,

T – temperature,

$U = \pi ND$.

The Graez number accounts for the development of the temperature profile, while the Brinkmann number signifies the ratio between viscous dissipation and heat conduction to the wall.

KINEMATIC SIMILARITY

Kinematic similarity means equal shear levels in the small and large extruder. Its importance is coupled to the requirements for:

- equal mixing in small and large machines,
- equal distribution of viscous dissipation,
- equal influence of non-Newtonian rheological effects.

For the throughput of the small laboratory extruder we can write:

$$Q_0 = \frac{1}{2}\pi^2 N_0 D_0^2 H_0 (1-a_0)\sin\theta_0 \cos\theta_0 \tag{12}$$

and for the throughput of the production machine:

$$Q = \frac{1}{2}\pi^2 N D^2 H (1-a)\sin\theta \ \cos\theta \tag{13}$$

If the screws of the small and large machine have the same screw angle, which is the same as the same dimensionless pitch we may write:

$$\frac{Q}{Q_0} = \frac{ND^2 H}{N_0 D_0^2 H_0}\frac{(1-a)}{(1-a_0)} \tag{14}$$

and if we process both machines with the same throttle coefficient:

$$\frac{Q}{Q_0} = \frac{N}{N_0}\left(\frac{D}{D_0}\right)^2 \frac{H}{H_0} \tag{15}$$

Introducing the diameter ratios as defined before:

$$\left(\frac{D}{D_0}\right)^q = \left(\frac{D}{D_0}\right)^n \left(\frac{D}{D_0}\right)^2 \left(\frac{D}{D_0}\right)^h = \left(\frac{D}{D_0}\right)^{n+2+h} \tag{16}$$

gives the exponent equation:

$$q = n + 2 + h \tag{17}$$

Because both machines operate with the same throttle coefficient:

$$a=a_0 \rightarrow \frac{H^2 \Delta P}{6\mu(\pi N D)L} \tan\theta = \frac{H_0^2 \Delta P_0}{6\mu_0(\pi N_0 D_0)L_0} \tan\theta_0 \tag{18}$$

and equal throttle coefficients leads to:

$$2h + p - \upsilon - 1 - n - \ell = 0 \tag{19}$$

For equal velocity gradients an extra equation is necessary:

$$\frac{\pi N D}{H} = \text{constant}$$

and therefore:

$$h = n + 1 \tag{20}$$

for kinematic similarity both (18) and (19) must be valid:

$$p = \ell - h + \upsilon \tag{21}$$

These results have to be combined with geometrical considerations of with thermal scaling rules.

GEOMETRICAL SIMILARITY

Geometrical similarity is often used for its simplicity but it is not a strong requirement. Especially in processing thermoplastic starches, where temperature and temperature homogeneity are very important the principle of geometric similarity of small and large scale equipment can not always be retained. Geometric similarity means that all dimensions scale in the same way, or:

$$l = 1 \text{ and } h = 1 \tag{22}$$

Geometric and kinematic similarity follows from a combination of this equation with equations 1, 3 and 4 resulting in:

$$n = 0;\ q = 3 \text{ and } p = \upsilon \tag{23}$$

This means for our process that:

- rotation speed must remain the same
- throughput proportional to D^3
- the die should be designed such that the pressure ratio equals the ratio between the end viscosities.

MOTOR POWER AND TORQUE

The motor power in the extruder can be approximated to:

$$E = \text{const}\,\frac{\mu D^3 N^2 L}{H} \tag{24}$$

It should be realised that this equation does not comprises the power needed to transport the solid bed, however this last one is not important for the thermal considerations in the next paragraphs.

The scale factor of the motor power can be defined as:

$$\frac{E}{E_0} = \left(\frac{D}{D_0}\right)^e \tag{25}$$

and we find:

$$e = 3 + 2n + \ell + \upsilon - h \tag{26}$$

and for the torque:

$$m = 3 + n + \ell + \upsilon - h \tag{27}$$

EQUAL AVERAGE END TEMPERATURE

Two types of thermal similarities can be used: equal average end temperatures and similar temperature profiles. The concept of equal average end temperatures can be applied if the extruder operates adiabatically or if Br >> 1. In this case scaling up has to proceed according to equal motor power per unit throughput:

$$\frac{E}{Q} = \text{const} \quad \text{or} \quad e - q = 0 \tag{28}$$

With equations (1) and (5) this leads for equal viscosities ($\upsilon = 0$) to

$$2h = 1 + n + l \tag{29}$$

In this case still various degrees of freedom are retained.

SIMILAR TEMPERATURE PROFILES

From the dimensionless energy equation it follows that thermal similarity can be attained if the dimensionless numbers of Graez and Brinkmann are the same for both sizes of machines. Because:

$$Br = \frac{\mu(\pi ND)^2}{\lambda \Delta T} \tag{30}$$

we find for materials with the same heat conductivity (λ) that thermal similarity is attained if:

$$\upsilon + 2n + 2 = 0 \tag{31}$$

This means for materials with the same viscosity: $n = -1$.
From:

$$Gz = \frac{\pi NDH^2}{aL} \tag{32}$$

follows at equal heat diffusivity a:

$$1 + n + 2h - \ell = 0 \tag{33}$$

leading to thermal similarity (equal Br and Gz numbers) if:

$$2h = \ell \tag{34}$$

For extruders with equal length to diameter ratios ($\ell = 1$) yields that the channel depth must decrease according to h=½ which gives together with equation 17:

$$q = 2 + n + h = 1.5 \tag{35}$$

or:

$$\frac{Q}{Q_0} = \left[\frac{D}{D_0}\right]^{1.5} \tag{36}$$

From economical point of view this is very unfavourable, and should only be applied in very special situations.

SIMILARITY IN RESIDENCE TIMES

Equal residence time can be achieved if the volume divided by the throughput remains constant, or, if we define Z as the average residence time:

$$Z = \text{const}\frac{HLW}{Q} \tag{37}$$

which yields for screws with equal helix angle:

$$z = h + 1 + l - q \tag{38}$$

or with:

$$q = 2 + n + h \tag{39}$$

we find that:

$$z = l - n - 1 \tag{40}$$

For screws with geometric similarity, this means that (l = -1, h = 1 and z = -n), equal residence times are only possible if the rotation speed is constant. In other cases equal residence times can only be obtained by changing the screw length, according to:

$$l = 1 + n \tag{41}$$

GUIDELINES FOR SCALING

In extrusion of thermoplastic starches generally both heat of conduction and heat of dissipation are important in the process. In small machines the Brinkmann number is relatively small but in larger machines the dissipation becomes more dominant and the process becomes more adiabatic. Because the thermal problems are predominant the basis for the guide lines are equation 29, this equation can be combined with various other (less strict) requirements. Application of equation 29 gives a variety of possibilities for scaling rules, which give for screws with equal length to diameter ratio for instance:

N	H	q
-1.0	0.5	1.5
-0.6	0.7	2.1
-0.4	0.8	2.4
0.0	1.0	3.0

Equal end temperatures with adiabatic operation still leave the degrees of freedom to scale according to similar temperature profiles (of course!) with q = 1.5 or to scale kinematically with q = 3 and with values in between. For the design of extruders for thermoplastic starches this means that the thermal stability of material and of the process are important. It can be envisioned that for the compounding process the preparation of the starch kinematic scale-up is preferred because temperature effects are still mildly important but kinematic similarity is important to obtain the same mixing mechanism (and therefore the same material) in the small and large scale process. On the other hand, for processes like film blowing, thermal similarity is extremely important leading to thermal scale-up. Profile extrusion and sheet extrusion are 'in between' processes and could be designed with n = -0.4 and h = 0.8 leading to q = 2.4.

In the examples above the L/D ratio remains constant but the screw length can also be changed to retain extra degrees of freedom. This leads to a three dimensional matrix of parameters, but is outside the scope of this work.

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