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LONGITUDINAL VIBRATIONS OF THE SUGAR BEET ROOT CROP BODY AT VIBRATIONAL EXTRACTION FROM SOIL

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Summary. The longitudinal vibrations theory of a continuous elastic body with one fixed extremity is designed. The Hamilton principle of a stationary operation is applied. By Ritz's method the Ritz's equation of frequencies for viewed oscillatory process is obtained. In particular, analytical expressions for definition of the first and second fundamental frequencies of body vibrations and forced vibrations amplitude of its any cross-section are obtained.

Key words: vibrating lifter, sugar beet, mathematical model

INTRODUCTION

In the present paper, the introduced in [Babakov 1968] common vibration theory of direct rods of a variable cross-section vibrations of a continuous elastic body with one timbered extremity is examined. An example of such a body can be the sugar beet root crop located in soil and, what is crucial, the soil enclosing a root crop is also an elastic medium.

Let's consider a case when vibrating motions will be applied to a particular body in longitudinal – vertical plane (the example will be in the position when to a root crop which is in undestroyed soil, force from the two sides of a vibration digging out end-effector will be applied at its extraction from soil).

The principle of Hamilton stationary operation applies to the research of holonomic systems vibrations with the infinite number of freedoms degree [Babakov 1968]. In the theory, longitudinal, torsional and transverse vibrations of Hamilton direct rods function are applied, which in the most common shape accept the following aspect [Babakov 1968]:

$$S = \int_{t_1}^{t_2} \int_{o}^{l} L\left(t, x, y, \frac{\partial y}{\partial t}, \frac{\partial y}{\partial x}, \frac{\partial^2 y}{\partial t^2}, \frac{\partial^2 y}{\partial t\, \partial x}, \frac{\partial^2 y}{\partial x^2} \right) dx\, dt \,. \tag{1}$$

where: $L = T - V$ – lagrangian;
T – system kinetic energy;
V – system potential energy.

The principle of Hamilton for research of the continuous elastic body longitudinal vibrations occurring under vertical disturbing force operation which varies under such sort harmonic law

$$Q_0(t) = H \sin \omega t \,, \tag{2}$$

where: H – forced vibrations amplitude,
ω – forced vibrations frequency.

As it shows from the presented scheme (Fig. 1), continuous elastic body – the root crop having the cone-shaped shape (whose apex angle is equal 2γ, and the upper is a little bit above the surface soil level), is modeled as a variable cross-section rod with the timbered lower extremity (point O). In a barycentre which is marked out by a point C, force $\overline{G}$ – a body weight is applied. Its general length – h. Body (root crop) link with soil is defined by the soil common response $\overline{R}_x$ which is located along an axis x.

The above-mentioned disturbing force $\overline{Q}_0$ is affixed to a body at once from its two sides, therefore on the scheme it is presented by two components $\overline{Q}_{01}$ and $\overline{Q}_{02}$. These forces are applied apart x_1 from coordinates origin (point O) and they give rise to vibrations of a body (root crop) in a longitudinal – vertical plane which destroy its links with soil and create conditions of extraction.

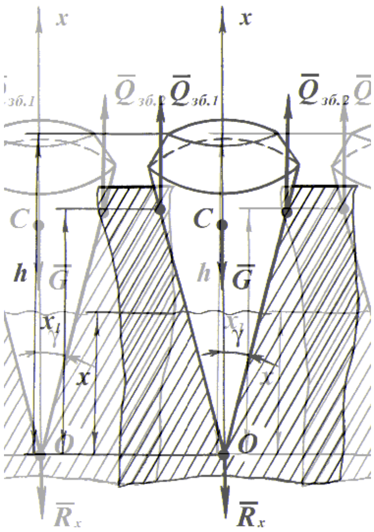

Fig. 1. Action of forces on sugar beet roots during lifting with vibrating lifter

According to [Babakov 1968] Hamilton, the function for longitudinal direct rods vibrations looks like:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\mu(x)\left(\frac{\partial y}{\partial t}\right)^2 - EF(x)\left(\frac{\partial y}{\partial x}\right)^2 + Q(x,t)\,y\right]dx\,dt \,. \tag{3}$$

where: $F(x)$ – body cross-sectional area in any point which is apart x from the lower extremity (m^2),

E – Young's modulus for a body material $(N \cdot m^{-2})$,

$y(x,t)$ – any body cross-section longitudinal bias in an instant t (m),

$Q(x,t)$ – longitudinal exterior loading intensity, directional along a body axis $(N \cdot m^{-1})$,

$\mu(x)$ – body running mass $(kg \cdot m^{-1})$.

Let's find magnitudes expressions which are included in a function (3). Taking into account that the body has the shape of a cone, we discover that its cross-section area $F(x)$ in a point being on the arbitrary distance x from a point O will be equal to

$$F(x) = \pi x^2\, tg^2\gamma \,. \tag{4}$$

It is obvious that a body's running mass can be defined with the help of such expression

$$\mu(x) = \rho \cdot \pi x^2\, tg^2\gamma \,, \tag{5}$$

where: ρ – body density $(kg \cdot m^{-3})$.

As the magnitude $Q(x,t)$ which is included in the function (3), is the distributed load intensity, which is measured in $(N \cdot m^{-1})$, the disturbing force should have dimensionality of loading intensity. With the help of the first order impulsive function $\sigma_1(x)$ [Babakov 1968] it is possible to determine concentrated load intensity and thus to include concentrated forces as a component of the loading distributed on length.

So, if $Q_0(t)$ – the concentrated disturbing force which is affixed in a point x_1 and is measured in Newton function

$$Q_0(x,t) = Q_0(t) \cdot \sigma_1(x - x_1) \tag{6}$$

has dimensionality $(N \cdot m^{-1})$ and expresses concentrated load intensity in a point x_1.

Function $\sigma_1(x - x_1)$ is equated to null for everything x, except for $x = x_1$, where it becomes infinite.

Let the disturbing force operating under the law (2) be affixed on a body apart x_1 from a reference point (a point O on Fig. 1). Then according to (6) it is possible to write

$$Q_0\left(x,t\right)=H\sin\omega t\cdot\sigma_1\left(x-x_1\right). \tag{7}$$

As the continuous elastic body is interlinked to soil which also is an elastic medium at an operation on it of a disturbing force (2) there is a soil reaction force to body migration at its vibrations. This force also influences the process of body extraction from soil especially at the beginning of oscillatory process while its links with soil are not dislocated yet.

It is obvious that resistance soil force (for all body) is a distributed load on a contacting area of the body with soil and therefore its intensity is definable as soil reaction force to body migration length unity.

Let c – coefficient of soil elastic deformation, referred to a contacting area which is measured $\left(N\cdot m^{-2}\right)$. We shall consider that soil enclosing a body under a disturbing force operation $H\sin\omega t$ realizes forced vibrations according to the same harmonic law with amplitude which is defined by soil elastic properties. Then soil intensity $P\left(x,t\right)$ reaction to body migration to a point x will be equated

$$P\left(x,t\right)=2\pi\, cx\cdot tg\gamma\cdot\sin\omega t. \tag{8}$$

Thus we shall have such relation for a longitudinal exterior loading:

$$Q\left(x,t\right)=Q_0\left(x,t\right)-P\left(x,t\right).$$

Taking into account expressions (4), (5), (7) and (8) the Hamilton function (3) will get such aspect:

$$S=\frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left\{\rho\cdot\pi x^2 tg^2\gamma\left(\frac{\partial y}{\partial t}\right)^2-E\pi x^2\cdot tg^2\gamma\left(\frac{\partial y}{\partial x}\right)^2+\left[H\sin\omega t\cdot\sigma_1\left(x-x_1\right)-\right.\right.$$
$$\left.\left.-2\pi\, cx\cdot tg\gamma\cdot\sin\omega t\right]y\left(x,t\right)\right\}dx\,dt. \tag{9}$$

For a body in soil natural shapes and frequency in pitch determination the Ritz's method [Babakov 1968] is usable. According to this method we shall search for body harmonic longitudinal vibrations in such aspect:

$$y\left(x,t\right)=\varphi\left(x\right)\sin\left(pt+\alpha\right), \tag{10}$$

where: $\varphi\left(x\right)$ – natural shape of principal vibrations,

p – fundamental frequency of principal vibrations.

As natural shapes and fundamental frequencies are interlinked to system free vibrations, it is necessary to select that part which features the system free vibrations in the function (9). It is obvious that it will be the function of such aspect

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\rho\pi x^2 tg^2\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\pi x^2 tg^2\gamma\left(\frac{\partial y}{\partial x}\right)^2\right]dxdt. \tag{11}$$

Substituting expression (10) in a function (11) we shall receive:

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left\{\rho\cdot\pi x^2\cdot tg^2\gamma\cdot\varphi^2(x)\cdot p^2\cdot\cos^2(p\,t+\alpha) - E\pi x^2\cdot tg^2\gamma[\varphi'(x)]^2\sin^2(p\,t+\alpha)\right\}dx\,dt. \tag{12}$$

Integrating expression (12) on t in limits of one period $T = \frac{2\pi}{p}$, we shall have:

$$S_2 = \frac{\pi}{2p}\int_{o}^{h}\left\{\rho\pi\,x^2 tg^2\gamma\varphi^2(x)p^2 - E\,\pi\,x^2 tg^2\gamma\,[\varphi'(x)]^2\right\}dx. \tag{13}$$

According to the Ritz's method the function's (13) values are considered on plurality of functions linear combinations, that is the expressions which are looking as follows:

$$\varphi(x) = \sum_{i=1}^{n}\alpha_i\cdot\psi_i(x), \tag{14}$$

where: α_i – parameters whose variations we obtain as the necessary class of admissible functions,

$\psi_i(x)$ – basis functions which are specially selected and are known functions, satisfying the problem geometrical boundary conditions.

Thus, substituting expression (14) for expression (13), after the relevant transformations we shall receive:

$$S_2 = \frac{\pi}{2p}\int_{0}^{h}\left[\rho\cdot\pi x^2\cdot tg^2\gamma\cdot p^2\sum_{i,k=1}^{n}\psi_i(x)\cdot\psi_k(x)\,\alpha_i\cdot\alpha_k - E\pi x^2\cdot tg^2\gamma\sum_{i,k=1}^{n}\psi_i'(x)\cdot\psi_k'(x)\alpha_i\cdot\alpha_k\right]dx. \tag{15}$$

Let's introduce such labels for further:

$$\begin{aligned}
&\int_{o}^{h}\rho\cdot\pi\,x^2\cdot tg^2\gamma\cdot\psi_i(x)\cdot\psi_k(x)\,d\,x = T_{ik},\\
&\int_{o}^{h}E\pi\,x^2\cdot tg^2\gamma\cdot\psi_i'(x)\cdot\psi_k'(x)\,d\,x = U_{ik},\\
&(i,k = 1,2,...,n).
\end{aligned} \tag{16}$$

Substituting (16) for (15), we shall receive the following as function from parameters $\alpha_1, \alpha_2, ..., \alpha_n$:

$$S_2(\alpha_1, \alpha_2, ..., \alpha_n) = \frac{\pi}{2p} p^2 \sum_{i,k=1}^{n} T_{ik} \alpha_i \alpha_k - \frac{\pi}{2p} \sum_{i,k=1}^{n} U_{ik} \alpha_i \alpha_k . \qquad (17)$$

We examine the extremum of the function (17). For this purpose we differentiate it with respect to expression (17) to α_i, $(i = 1,2,...,n)$ we shall also equate to null the obtained partial derivatives. In the outcome we shall receive the linear homogeneous system equations to unknowns $\alpha_1, \alpha_2, ..., \alpha_n$ from which, in turn, we discover the equation of Ritz's frequencies for longitudinal vibrations of a continuous elastic body, timbered in soil:

$$\begin{vmatrix} U_{11} - p^2 T_{11} & U_{12} - p^2 T_{12} & ... & U_{1n} - p^2 T_{1n} \\ U_{21} - p^2 T_{21} & U_{22} - p^2 T_{22} & ... & U_{2n} - p^2 T_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ U_{n1} - p^2 T_{n1} & U_{n2} - p^2 T_{n2} & ... & U_{nn} - p^2 T_{nn} \end{vmatrix} = 0 \qquad (18)$$

In practice, as a rule, only more often the first and the second one, the lowest frequencies are defined which influence most significantly the considered process. Therefore it is possible to define the first and second frequencies of the viewed body vibrations.

For the definition of the first and second frequency the equation (18) will get such aspect:

$$\begin{vmatrix} U_{11} - p^2 T_{11} & U_{12} - p^2 T_{12} \\ U_{21} - p^2 T_{21} & U_{22} - p^2 T_{22} \end{vmatrix} = 0 . \qquad (19)$$

As a result of the given equation’s solution we obtain expressions for the determination of the first (basic) frequency value:

$$p_1 = \frac{0,66}{h} \sqrt{\frac{E}{\rho}} , \qquad (20)$$

and the second frequency:

$$p_2 = \frac{27,93}{h} \sqrt{\frac{E}{\rho}} . \qquad (21)$$

Let's calculate the first and second frequency value for a continuous elastic body whose example can be a root crop of a sugar beet with the following parameters [Pohorelyy et al 1983]: $h = 250$ *[mm]*; $E = 18,4 \cdot 10^6$ *[N·m⁻²]*; $\rho = 1300$ *[kg·m⁻³]*. As evaluations result we shall receive:

$$p_1 = \frac{0,662422}{250 \cdot 10^{-3}} \sqrt{\frac{18,4 \cdot 10^6}{1300}} = 315 \quad \left(s^{-1}\right),$$

$$p_2 = \frac{27{,}931592}{250 \cdot 10^{-3}} \sqrt{\frac{18{,}4 \cdot 10^6}{1300}} = 13292 \quad \left(s^{-1}\right).$$

Let's transfer further to forced vibrations research of a continuous elastic body. Cleanly forced vibrations will occur according to the law

$$y(x,t) = \varphi(x) \sin \omega t \,, \tag{22}$$

where: $\varphi(x)$ - the shape of forced vibrations.

For the definition of the body's forced vibrations shape we shall substitute expression (22) in the function (9), receiving the following function:

$$\begin{aligned} S_3 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h} \Big\{ &\rho \pi x^2 tg^2 \gamma \omega^2 \varphi^2(x) \cos^2 \omega t - E\pi x^2 tg^2 \gamma \left[\varphi'(x)\right]^2 \sin^2 \omega t + \\ &+ \left[H\sigma_1 (x - x_1) - 2\pi c x tg\gamma\right] \varphi(x) \sin^2 \omega t \Big\} dx\, dt. \end{aligned} \tag{23}$$

Integrating expression (23) on t in limits of one period $T = \dfrac{2\pi}{\omega}$, we shall have:

$$\begin{aligned} S_4 = \frac{\pi}{2\omega} \int_{o}^{h} \Big\{ &\rho \pi x^2 tg^2 \gamma \varphi^2(x) \omega^2 - E\pi x^2 tg^2 \gamma \left[\varphi'(x)\right]^2 + \\ &+ H\sigma_1 (x - x_1) \varphi(x) - 2\pi c\, x\, tg\gamma \varphi(x) \Big\} dx. \end{aligned} \tag{24}$$

According to the Ritz's method we shall consider the function's (24) value on linear combinations plurality of the following aspect

$$\varphi(x) = \alpha \psi(x) , \tag{25}$$

where: α – parameter whose variation we obtain of the admissible functions class,
$\psi(x)$ – basis function.

Substituting expression (25) in the function (24), we shall receive:

$$\begin{aligned} S_4 = \frac{\pi}{2\omega} \int_{o}^{h} \Big\{ &\rho \pi x^2 tg^2 \gamma \alpha^2 \psi^2(x) \omega^2 - E\pi x^2 tg^2 \gamma \alpha^2 \left[\psi'(x)\right]^2 + \\ &+ H\sigma_1 (x - x_1) \alpha \psi(x) - 2\pi c\, x\, tg\gamma\, \alpha\, \psi(x) \Big\} dx. \end{aligned} \tag{26}$$

Let's inject such labels:

$$\int_0^h \rho \pi x^2 \cdot tg^2 \gamma \cdot \psi^2(x) dx = T , \tag{27}$$

$$\int_0^h E \pi x^2 \cdot tg^2 \gamma \cdot [\psi'(x)]^2 dx = U , \tag{28}$$

$$\int_0^h \left[H\sigma_1 (x - x_1) \cdot \psi(x) - 2\pi c x \cdot tg\gamma \cdot \psi(x) \right] dx = L . \tag{29}$$

Substituting expressions (27–29) in (26), we shall have

$$S_4(\alpha) = \frac{\pi}{2\omega} \left(\omega^2 T \alpha^2 - U\alpha^2 + L\alpha \right). \tag{30}$$

So, on functions (25), the plurality (26) turns to function from an explanatory variable α, looking like (30).

A necessary requirement for the function (30) stability (that is extremum existence) are equality to null of its first variation, namely:

$$\frac{\partial S_4}{\partial \alpha} \cdot \delta\alpha = 0 , \tag{31}$$

whence we obtain the following equation:

$$2\omega^2 T\alpha - 2U\alpha + L = 0 , \tag{32}$$

from whose intensity the necessary parameter value α is discovered. It will be equated:

$$\alpha = \frac{L}{2\left(U - \omega^2 T\right)} . \tag{33}$$

Let's accept for the basic function $\psi(t)$ the shape of a fixed cross-section rod forced longitudinal vibrations with one rigidly timbered extremity which originate under a longitudinal harmonic force operation of the frequency ω affixed in the point $x = x_1$.

According to [Babakov 1968], the shape of evocative rod forced vibrations has the following aspect:

$$\psi(x) = D_1 \sin ax \qquad \text{for } x \le x_1 , \tag{34}$$

$$\psi(x) = D_2 \cos a(h - x) \qquad \text{for } x > x_1 , \tag{35}$$

where:

$$D_1 = -\frac{1}{a\,EF}\cdot\frac{\cos a\left(h - x_1\right)}{\cos\,a\,h}, \tag{36}$$

$$D_2 = -\frac{1}{a\,EF}\cdot\frac{\sin a x_1}{\cos\,a\,h}, \tag{37}$$

$$a = \omega\sqrt{\frac{\mu}{EF}}, \tag{38}$$

and

μ – rod running mass,
F – rod cross-sectional area,
E – Young's modulus for a rod material,
h – rod length,
ω – frequency of rod forced vibrations.

Having calculated parameters T, U and L, by expressions (27), (28) and (29), we shall receive the necessary parameter value α according to expression (33) at which the functional (26) will have a steady-state value:

$$\alpha = \frac{\begin{gathered}-HD_1\sin a\,x_1 + H\,D_2\left[\cos a\left(h - x_1\right) - 1\right] - 2D_1\,\pi c\cdot tg\gamma\left(\frac{\sin a x_1}{a^2} - \frac{x_1\cos a\,x_1}{a}\right) - \\ - 2D_2\,\pi c\cdot tg\gamma\left[\frac{x_1}{a}\,\sin a\left(h - x_1\right) - \frac{1}{a^2}\,\cos a\left(h - x_1\right) + \frac{1}{a^2}\right]\end{gathered}}{\begin{gathered}2E\,\pi\,tg^2\gamma\left[D_1^2\left(\frac{a^2x_1^3}{6} + \frac{x_1^2 a\cdot\sin 2ax_1}{4} + \frac{x_1\cos 2\,a\,x_1}{4} - \frac{\sin 2a\,x_1}{8a}\right) - D_2^2\left(\frac{a^2\left(x_1^3 - h^3\right)}{6} + \frac{x_1^2 a\sin\left(2ah - 2ax_1\right)}{4} + \frac{h}{4} - \right.\right. \\ \left.\left. - \frac{x_1\cos\left(2ah - 2ax_1\right)}{4} - \frac{\sin\left(2ah - 2ax_1\right)}{8a}\right)\right] - 2\omega^2\rho\,\pi\,tg^2\gamma\left[D_1^2\left(\frac{x_1^3}{6} - \frac{x_1^2\sin 2\,a\,x_1}{4a} - \frac{x_1\cos 2\,a\,x_1}{4a^2} + \frac{\sin 2\,a\,x_1}{8a^3}\right) + D_2^2\left(\frac{h^3 - x_1^3}{6} + \right.\right. \\ \left.\left. + \frac{x_1^2\sin\left(2ah - 2ax_1\right)}{4a} + \frac{h}{4a^2} - \frac{x_1\cos\left(2ah - 2ax_1\right)}{4a^2} - \frac{\sin\left(2ah - 2ax_1\right)}{8\,a^3}\right)\right]\end{gathered}}. \tag{39}$$

Taking into account expressions (25), (34) and (35), we shall receive expressions for the forced vibrations shape of a continuous elastic body, timbered in soil. They have the following aspect:

$$\varphi(x) = \alpha \cdot D_1 \sin a\,x\,, \qquad \text{for } x \le x_1\,, \tag{40}$$

$$\varphi(x) = \alpha \cdot D_2 \cos a\,(h - x), \qquad \text{for } x > x_1\,, \tag{41}$$

where α is determined according to (39).

Having substituted expressions (40-41) for (22), we shall finally receive the law of continuous elastic body forced vibrations, timbered in soil:

$$y(x,t) = D_1 \alpha \sin ax \cdot \sin \omega t\,, \qquad \text{for } x \le x_1\,, \tag{42}$$

$$y(x,t) = D_2\, \alpha\, \cos a\,(h - x) \cdot \sin \omega\, t\,, \qquad \text{for } x > x_1\,. \tag{43}$$

By results of forced vibrations theoretical researches of a continuous elastic body timbered in soil concrete calculation of specified vibrations amplitude is carried out.
For an example we use a sugar beet root crop with the following parameters:

length $h = 250 \cdot 10^{-3}\ (m)$,
cone angle $\gamma = 14^o$,
Young's modulus $E = 18{,}4 \cdot 10^6\ (N \cdot m^{-1})$,
density $\rho = 1300\ (kg \cdot m^{-3})$,
coefficient of a soil elastic deformation $c = 1 \cdot 10^5 (N \cdot m^{-2})$.

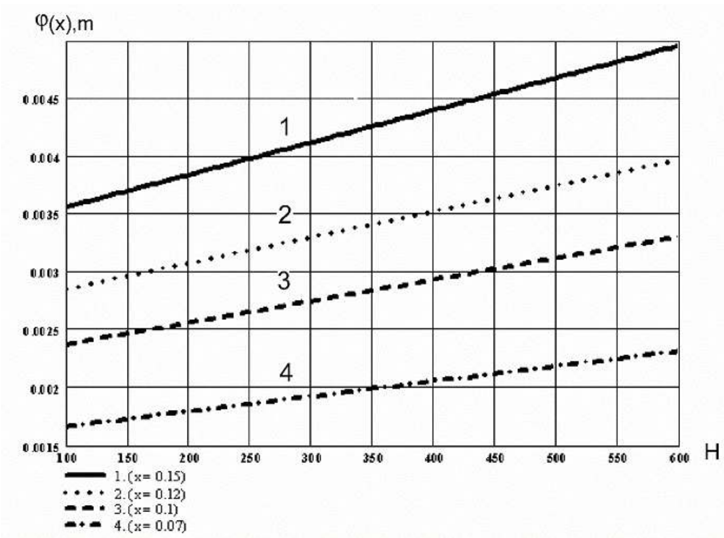

Fig. 2. Relationship between amplitude of forced longitudinal vibration of sugar beet root and excavation force

Disturbing force amplitude H it is selected in limits 100...600 N. Disturbing force frequency ω, according to [Pohorelyy *et al*. 1983], we shall accept equal $\omega = 20{,}00\ (Hz)$.

Calculation is carried out in program MathCAD with the purpose of definition of amplitude dependence of forced longitudinal vibrations of a root crop body from change of a disturbing force in a gamut 100...600 N for different body cross-sections.

The calculation results are presented in the graph, in Fig. 2.

As it can be seen from the reduced graph, with magnification of disturbing force magnitude the amplitude of continuous elastic body longitudinal forced vibrations increases under the linear law. And with the distance of a root crop body cross-sectional area from the coordinates origin O, the amplitude also increases. So, at $x = 0{,}07\ m$ the amplitude is in the range $1{,}7...2{,}3\ mm$, at $x = 0{,}1\ m$ – in the range $2{,}3...3{,}5\ mm$, at $x = 0{,}12\ m$ – in the range $2{,}8...3{,}9\ mm$, at $x = 0{,}15\ m$ (capture point) – in the range $3{,}2...4{,}8\ mm$.

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