A THEORY OF PLANE INTERACTION OF A PURIFYING BLADE WITH THE HEAD OF ROOT-CROPS

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The problem of the removal of the remainders of vegetable tops from the heads of root-crops after cutting off its basic mass is considered in many design developments both in the theoretical and experimental research [1-11, etc.].In the scientific works, basically, the results of the studies of the purifiers of the heads of root-crops on the root of different kinds of particular constructions are presented: blade, annular, sectorial, drum, etc.

Nevertheless, a strict theoretical substantiation of this technological process does not exist there so far, since neither the initial process of the impact of the purifying element (flexible blade) on the head of root-crops nor its further motion over the cleaned surface, which, by the way, is achieved in the space, nor the differential equations of this motion are thoroughly or exhaustively examined. In [8] are represented the differential equations of motion of flexible purifying blade for the head of root-crops in the case, when the blade accomplishes motions in the transverse - horizontal plane, i.e. it is installed on the purifier with the vertical rotational axis.

In this work a new theory of the interaction with the head of root-crops on the root of the purifying blade, installed on the drive shaft with the horizontal rotational axis, is examined. In this case are examined the cases, when the plane of the rotation of blades is strictly located along the axis of the row of root-crops and when it placed at the certain angle β to the row of root crops.

Let us build a design mathematical model of the process of interaction of a purifying blade with the head of root-crops, for which let us compose the first equivalent diagram, in which there is examined the spherical head of the rootcrops, located (actually rigidly fixed) in the soil, above which forward moves drive a shaft with the horizontal rotational axis, which bears the flexible purifying blades, which accomplish an impact on the head and the subsequent motion over its surface (Fig. 1). We consider that the blades on the shaft, which has a rotational axis *O*, are hinged on a pin; moreover the plane of their rotation is perpendicular to the axis of the shaft, the direction of the rotation during the motion of the purifier. The drive shaft revolves with the angular velocity ω , the direction of which is shown by the pointer. Let us examine the process of interaction with the head of root-crops of one of the blades of the purifier, by considering that others will be in analogous state. The hinge of the suspension of the blade on the shaft is located at a distance r from the axis O of the shaft itself, the conditional length of the blade in this case is equal to l + r, where l - distance from the axis of suspension to the point M, and the contact of the blade with the head of root-crops at the initial moment (end of the blade is designated by point C_1). Let us also designate the speed of the forward motion of the purifier along the rows of root-crops by $\overline{V_P}$ and the mass of the blade – by m.

Let us examine at first the case, when the plane of the rotation of the purifier blades is located strictly along the row of root-crops (this exactly is precisely such a case, when the conditional axis of the blade will coincide with the axis of root-crops). In this case the schematic of the rates and power interactions of the blade with the head of root-crops will be plane.



Fig. 1. The diagram of the speeds with the impact contact of the blade with the head of root-crops in the case when the plane of the rotation of blades is located along the row

Let us select a fixed coordinate system xOyz, the beginning of which penetrates the rotational axis of the blade, i.e. the axis OxO and coincides with the horizontal axis of the drive shaft. The axis Oy of this coordinate system is directed along the row of root-crops, axis z is vertically upward. Let us show in Fig. 1 the diagram of speeds to the impact of the blade on the head of root-crops and after the impact. Let us again emphasize that in this case the plane of the rotation of the blade coincides with the plane *yOyz*, and the axis of the shaft O is located along the axis Ox.

During the forward motion of the purifier along the row of root-crops and the rotation of the blade around the axis Ox the impact contact of the blade with the head occurs. Let us divide the process of removal of remainders of vegetable tops from head of root-crops into two phases:

1-the contact phase of the blade with the head of root-crops;

2-the phase of further motion of the blade on the head of root-crops.

It is obvious that the impact of the blade on a certain part of the head first occurs upon the encounter of the blade with the head of root-crops (phase 1). Let us designate through M the point on the blade, which at the moment of impact coincides, with the point K_1 , the contact of the blade with the head of root-crops. We will consider that impact impulse \overline{S} will be directed along the standard \overline{n} to the head of root-crops and carried out through the contact point K_1 (Fig. 2). At the speed \overline{V} , at the point M on the blade, the impact/ will be equal to

$$\overline{V} = \overline{V}_P + \overline{V}_M,\tag{1}$$

where:

 \overline{V}_{p} – the speed of the forward motion of the purifier along the row of root-crops (the velocity of following the blade);

 \overline{V}_M – the peripheral speed of point *M* on the blade during the rotation around the axis Ox (relative speed of point *M*).

The peripheral speed V_M of point M will be equal to

$$V_M = \omega \rho = \omega (r+l). \tag{2}$$

Vector \overline{V}_M will be directed tangentially toward the circumference of the radius ρ , which is shown in Fig. 1.

Since we consider that the impact occurs on the common standard \overline{n} to the spherical head of root-crops and the blade, and by the common standard it is precisely tangent to the circumference of the radius ρ , therefore vector \overline{V}_M will be directed along the standard \overline{n} (i.e. actually along the radius of the spherical head of root-crops).

Let us separately depict the diagram of the forces, which act on the head of root-crops during the impact at the contact point K_1 (Fig. 2). The force $\overline{F}_{yd.}$ – the impact force, which appears in the process of the impact and is directed along normal \overline{n} to the surface of the head of root-crops; \overline{G} – the gravitational force of the blade; \overline{F}_w – the centrifugal inertial force, which appears during the rotation of the blade around the axis Ox (this force is directed along the blade, it contributes to the rectification of the blade along the straight line, i.e., the radius ρ of the gyration of point M around the axis Ox).



Fig. 2. The diagram of the forces, which act at the contact point of the blade with the head of root-crops if the plane of the rotation of blades is located along the row

Let us determine further the absolute velocity \overline{U} of point M after the impact, the angle of deflection γ of vector \overline{U} from the standard \overline{n} and the impact impulse \overline{S} .

According to [12], a change in the momentum of point with mass *m* with the impact is equal to:

$$m(\overline{U} - \overline{V}) = \overline{S} , \qquad (3)$$

where:

 \overline{S} – impact impulse;

 \overline{U} – the rate of contact point M after impact;

 \overline{V} – the rate of contact point M to the impact

According to the definition, the impact impulse \overline{S} is determined with the aid of this expression

$$\overline{S} = \int_{0}^{t} \overline{F}_{yd} dt , \qquad (4)$$

where:

 \overline{F}_{yd} – impact force;

 τ - the duration of the impact.

Since the duration of impact τ is a very small time interval, the momenta of all the other forces acting on the head of root-crops at the moment of impact are in effect equal to zero.

Actually, the power impulse \overline{G} will be equal to:

$$\overline{S}_G = \int_0^{\tau} \overline{G} dt = \overline{G} \int_0^{\tau} dt = \overline{G} \cdot \tau ,$$

then

$$\lim_{\tau \to 0} \overline{S}_G = \lim_{\tau \to 0} \overline{G} \cdot \tau = \overline{G} \cdot \lim_{\tau \to 0} \tau = 0.$$

Analogously the power impulse \overline{F}_{e} will be equal to:

$$\overline{S}_{F_w} = \int_0^\tau \overline{F}_w dt = \overline{F}_w \int_0^\tau dt = \overline{F}_w \cdot \tau ,$$

then

$$\lim_{\tau \to 0} \overline{F}_w = \lim_{\tau \to 0} \overline{F}_w \cdot \tau = \overline{F}_w \cdot \lim_{\tau \to 0} \tau = 0.$$

Thus, the action of forces \overline{G} and \overline{F}_w with the impact of the blade on the head of root-crops may be left unconsidered.

Since when the impact impulse \overline{S} is different from zero values, with $\tau \to 0$, $F_{vd} \to \infty$, since otherwise $\overline{S} \to 0$. This results from the expression (4).

For describing the process of the impact of the blade on the head of rootcrops let us select a new system of coordinates $\overline{\tau}K_1\overline{n}$, where the axis $\overline{\tau}$ is directed tangentially toward the head of root-crops at the point K_1 ,

Taking into account that the impact impulse \overline{S} is directed along the standard \overline{n} , the expression (3) can be represented in this form

$$m\overline{U} - m\overline{V} = S\,\overline{n}\,\,,\tag{5}$$

where:

S– the module of the vector of the impact impulse \overline{S} .

Let us write down an equation (5) in the projections on the axis $\overline{\tau}$ and \overline{n} . We will have

$$mU_{\tau} - mV_{\tau} = 0,$$

$$mU_{n} - mV_{n} = S,$$

$$U_{\tau} - V_{\tau} = 0,$$

$$U_{n} - V_{n} = \frac{1}{m}S.$$
(6)

or

Since $\overline{V} = \overline{V}_P + \overline{V}_M$, as can be seen from Fig. 1, the projections of velocity vectors on the axis $\overline{\tau}$ and \overline{n} will be equal to:

$$V_{\tau} = V_P \sin \alpha, \qquad V_n = -(V_P \cos \alpha + \omega \rho), U_{\tau} = U \sin \gamma, \qquad U_n = U \cos \gamma,$$
(7)

where:

 α - the angle between axis Oz and the direction of the blade at the moment of impact; γ - the angle between vectors of speed \overline{U} after impact and standard \overline{n} .

From the first equation of the system (6) we obtain

$$U_{\tau} = V_{\tau}$$
,

then, taking into account (7), we will have

$$U\sin\gamma = V_P\sin\alpha ,$$

whence we find the absolute velocity \overline{U} of the point M after the impact

$$U = V_P \cdot \frac{\sin \alpha}{\sin \gamma} \,. \tag{8}$$

In accordance with the concept of recovery factor \mathcal{E} with the impact/shock [12], it is possible to write down

$$\varepsilon = \frac{U_n}{|V_n|} = \frac{U\cos\gamma}{V_P\cos\alpha + \omega\rho}.$$
(9)

Let us substitute in (9) for U its value, which is determined according to expression (8), we have

$$\varepsilon = \frac{V_P \sin \alpha}{V_P \cos \alpha + \omega \rho} \cdot \operatorname{ctg} \gamma \,. \tag{10}$$

We find from expression (10)

$$\operatorname{ctg} \gamma = \frac{\left(V_P \cos \alpha + \omega \rho\right) \cdot \varepsilon}{V_P \sin \alpha}, \qquad (11)$$

then

$$\gamma = \operatorname{arc}\operatorname{ctg}\left[\frac{\left(V_P \cos \alpha + \omega \rho\right) \cdot \varepsilon}{V_P \sin \alpha}\right].$$
(12)

For enumerating the module of speed U after impact we will use the trigonometric identity

$$\sin \gamma = \frac{1}{\sqrt{1 + \operatorname{ctg}^2 \gamma}}.$$
(13)

From equation (8), taking into account (13), we obtain

$$U = V_P \sin \alpha \cdot \sqrt{1 + \operatorname{ctg}^2 \gamma} \tag{14}$$

Let us substitute in the last expression (14) for $\operatorname{ctg} \gamma$ its value according to expression (11). We have

$$U = \sqrt{V_P^2 \sin^2 \alpha + (V_P \cos \alpha + \omega \rho)^2 \cdot \varepsilon^2}$$
(15)

Thus, are obtained the values of the module of the speed U of the point M of the blade after impact and the angle of deflection γ of vector \overline{U} from normal \overline{n} to the surface of the head of the root-crops

Let us determine further impact impulse S. Using the determination of the recovery factor with the impact ε , it is possible to write

$$U_n = \varepsilon |V_n|,$$

$$U_n = -\varepsilon V_n.$$
(16)

Let us substitute into the second equation of the system (6) for U_n its expression according to (16), we will obtain

$$S = -m(1+\varepsilon)V_n$$

or, by taking into account (7), let us finally find the impact impulse S

or

$$S = m(1+\varepsilon) \cdot (V_P \cos \alpha + \omega \rho). \tag{17}$$

After the first phase (encounter of the blade with the head of root-crops, or impact of the blade on the head of root-crops), the second phase sets in, i.e. the motion of the blade on the head of root-crops, during which occurs the basic process of the combing of the remainders from the head. For the analytical description of this process it is necessary to compile the differential equations of the motion of point K (arbitrary point of the moving blade), the contact of the blade over the injector face of the root-crops.

Let us first examine the case, when the plane of the rotation the blade is located along the row of root-crops, i.e., when coplanar force system occurs.

One should immediately note that the diagram of power interaction at contact point K during the motion of the blade in the injector face of root-crops will differ from the diagram of power interaction, which occurs during the impact of the blade on the head of root-crops (Fig. 2), since in this case, actually, another system of forces will act at the indicated point. Let us depict the power interaction of the blade with the head of root-crops with the fulfillment of the blade over the injector face of root-crops (Fig. 3). At contact point K the following forces will act:

 \overline{F}_{w} – the centrifugal inertial force, which is directed along the radius *OK* of gyration of the blade around the axis *O*;

 \overline{G} – the gravitational force of the blade, which is directed vertically downward;

 \overline{N} – the normal reaction of interaction of the blade with the head of rootcrops, directed along the normal \overline{n} to the head of root-crops, carried out through this position of contact point;

 $\overline{F}_{mp.}$ – the frictional force, which appears during the motion of the blade along the head of root-crops, directed to the side, opposite to the direction of the vector of the absolute velocity of the point *M* of the blade which coincides with the contact point *K*;

 \overline{Q} – the force of the combing of the remainders of vegetable tops from the injector face of root-crops, which is directed to the side of the vector of the absolute velocity of the point M of the blade.

Let us find the values of the indicated forces. For the determination of the centrifugal inertial force \overline{F}_w at any contact point *K* it is necessary to examine the kinematics of the motion of the blade O_1C_1 on the head of root-crops after the impact contact at point K_1 . Since the impact occurs for a very small time interval, in the torque of the impact of the blade there is actually no displacement over the head of root-crops. Therefore for the initial position of the blade on the head of root-crops after impact it is possible to count the position of the impact contact K_1 .



Fig. 3. Schematic of power interaction of the blade with the head of root-crops in the process of the combing of the remainders of the vegetable tops

After impact only the absolute velocity U of the blade was changed, which was determined above according to the expression (15) with the initial velocity of the motion of the blade after impact. Let M_1 be the point on the blade, which at the initial moment t = 0 after impact coincides with the point of the initial contact K_1 . At this moment the blade O_1C_1 is still directed along a radius ρ , since the impact of the blade has been considered above as an absolutely constant value.

Let us examine further the contact of the blade with the head of root-crops at point K at any moment of time t after impact. It is quite obvious that in time t the point O of the rotational axis of the blade with the forward motion of the purifier will move into the point O', moreover

$$OO' = V_P \cdot t \,. \tag{18}$$

Analogously point O_1 will move into the point O'_1 with the forward motion of the purifier, moreover $O_1 O'_1 = OO'$ (see Fig. 3).

Nevertheless, in the rotary motion of the blade around the axis O(O'), point $O_1(O'_1)$ will turn in time t to the angle ωt (see Fig. 5). Thus, in the rotary motion, the suspension point of the blade $O_1(O'_1)$ will describe the arc O'_1O_2 , which will be equal to:

$$O_1'O_2 = \omega r t . \tag{19}$$

Thus, in the absolute motion, the suspension point O_1 of the blade will be displaced to the value

$$OO' + O_1'O_2 = V_{\Pi} \cdot t + \omega r t = (V_P + \omega r)t.$$
⁽²⁰⁾

Since we consider that the blade is a continuous nonductile rod, as a result of the displacement of the suspension point O_1 to the value $(V_P + \omega r)t$, any other point of the blade for the time t interval will move also to the same value. Certainly, each point of the blade moves along its absolute trajectory, which can differ in the form from the trajectories of other points of the blade, nevertheless, along its trajectory the point will move for the value $(V_P + \omega r)t$.

This is the major portion of the displacement of the points of the blade, which is caused by the kinematic, geometric and designs parameters of a purifier of this type.

In this case the contact point M of the blade with the head of root-crops will move along the blade from one point M_1 to the next C_1 . Since the point Mbelongs to the blade, its movement for the time t interval also constitutes $(V_P + \omega r) t$.

If it passes $M_1C_1 = l_1$ distance in time t_1 , then it is possible to write down

$$l_1 = (V_P + \omega r) t_1, \qquad (21)$$

whence we find the time of the contact of the blade with the head of root-crops

$$t_1 = \frac{l_1}{V_P + \omega r} \,. \tag{22}$$

Now let us trace the kinematics of the motion of the contact point K along the head of root-crops. It is obvious that the major portion of the movement of the contact point K occurs due to the forward motion of the purifier. Because of the forward motion of the purifier, the blade as if runs foul of the head of rootcrops, moving on it until the blade slips throughout the entire length due to the rotary motion or if rotational axis O is displaced beyond the boundaries of the arrangement of root-crops. Certainly, a certain displacement of point K occurs due to the bending strain of the blade, nevertheless, it is very insignificant.

Therefore in the first approximation, it is possible to consider that the displacement of the contact point K for the time interval t will be equal to:

$$KK_1 \approx V_P \cdot t \,. \tag{23}$$

This expression will be necessary during the determination of the centrifugal inertial force F_w of the point M, which acts on the head of root-crops at the contact point K.

From the expressions (22) and (23) it is possible to determine the movement of the blade over the head of root-crops in the time of contact t_1 in the first approximation. Namely

$$K_2 K_1 \approx V_P \frac{l_1}{V_P + \omega r}.$$
(24)

It must be noted that the given kinematic dependencies sufficiently approximately describe the process of moving the blade on the head of root-crops, and the obtained values of the time t_1 of the contact of the blade with the head of root-crops and the movement l_1 of the blade over the head of root-crops are also approximate.

For the more precise study of the motion of the blade on the head of rootcrops it is necessary to compile the differential equations of the motion of the point M for the head of root-crops, since during this study the forces, which cause this motion, are considered.

It must be noted that the dominant role in the formation of the force of the combing \overline{Q} play the centrifugal inertial force \overline{F}_w , thrust \overline{P} and rotational moment M_{ob} of the blade. Specifically, because of the action of these forces, the pressing of the blade occurs to the head of root-crops and the bending strain of the blade. Actually, immediately after the impact, the centrifugal inertial force \overline{F}_w is directed along the blade and attempts to straighten the blade along the radius ρ . If this force was absent, then under the action of the forward motion of the purifier and of the rotary motion of the blade around the axis O of the blade, the butt contact of root-crops would simply be deflected by a certain angle to the side, opposite to the rotary

motion, and without any effort it would slip on the head of root-crops, without changing its rectilinear form, since the point of its suspension O_1 would hinge.

However, under the action of the centrifugal force F_w towards the head of root-crops the blade remains directed along the radius ρ , and therefore, as a result of further forward and rotary motion, the blade will slip on the head of root-crops, experiencing in this case the specific bending strains, which create the effect of the combing of the remainders of the vegetable tops.

The amount of centrifugal inertial force \overline{F}_w at the initial contact point K_1 (point M_1) will be equal to:

$$F_{w1} = m\omega^2 \cdot \rho \,, \tag{25}$$

where:

m – the mass of the blade.

Let us determine the centrifugal inertial force \overline{F}_w of the point *M* at any contact point *K* of the blade with the head of root-crops. This force will be equal to

$$F_w = m\omega^2 \cdot O'K , \qquad (26)$$

where:

O'K – distance from the point K to the point O'.

As can be seen from the diagram in Fig. 3, the given distance will be approximately equal to

$$O'K \approx OK_1 - K_1 K + OO', \qquad (27)$$

where:

 $OK_1 = \rho$.

Then, taking into account that $OO' = V_P \cdot t$ and $KK_1 \approx V_P \cdot t$, and also expression (27), we obtain:

$$O'K \approx \rho$$
. (28)

Thus, the centrifugal inertial force \overline{F}_w at each contact point K remains approximately constant in the value and the direction and it will be equal to

$$F_{w} \approx m\omega^{2} \cdot \rho \,. \tag{29}$$

In this case we consider the mass of the blade *m* concentrated in the working part of the blade. The centrifugal inertial force, which appears from the rotation of the mass of the blade, concentrated nearer to the axis of suspension O_1 , will cause the tension of the blade and is balanced by the reaction in the hinge O_1 .

The bending strain of the blade appears as a result of the clamp of the blade at the contact point K by the forces of inertia \overline{F}_w and the weight of the blade \overline{G} under the action of the tractive \overline{P} propelling power of the purifier and the rotational moment of the blade M_{ob} .

The force of the bending strain will be equal to the force of the combing \overline{Q} . Since the propelling power \overline{P} of the purifier and the rotational moment of the blade M_{ob} enter into the composition of forces \overline{Q} , they are not shown in Fig. 3.

The frictional force, as is known, is equal to

$$F_{mp.} = f \cdot N \,, \tag{30}$$

where:

f- the coefficient of the friction of the surface of the blade over the injector face of root-crops; N-a normal reaction at the contact point K of the blade with the head of the root-crops.

Thus, the differential equation of motion of the contact point K for the head of root-crops in the vector form will take this form:

$$m\,\overline{a} = \overline{F}_w + \overline{G} + \overline{N} + \overline{F}_{mp.} + \overline{Q} , \qquad (31)$$

where:

 \overline{a} – the absolute acceleration of the motion of the contact point K on the head of root-crops; m – the mass of the blade.

Since in this case the coplanar force system occurs, which is located in plane yOz, he differential equation of motion (31) is reduced to the system of two differential equations of the second order of the following form

$$m \ddot{y} = F_{wy} + G_{y} + N_{y} + F_{mp.y} + Q_{y}, m \ddot{z} = F_{wz} + G_{z} + N_{z} + F_{mp.z} + Q_{z},$$
(32)

where:

 $F_{wy}, G_y, N_y, F_{mpy}, Q_y$ - the projection of the force vectors $\overline{F}_w, \overline{G}, \overline{N}, \overline{F}_{mp.}, \overline{Q}$ - on the axis *Oy* respectively;

$$F_{wz}$$
, G_z , N_z , F_{mpz} , Q_z – the projection of the vectors of the mentioned forces on the axis respectively.

Taking into account the values of the projections of the vectors of the forces, which enter into the system of differential equations (32), and expression (29) and (30), the mentioned system of equations acquires this form

$$m\ddot{y} = -m\omega^{2}\rho\sin\alpha + N\cos(\hat{y},\overline{N}) - fN\cos(\hat{y},\overline{V}) + Q\cos(\hat{y},\overline{V}),$$

$$m\ddot{z} = -m\omega^{2}\rho\cos\alpha - mg + N\cos(\hat{z},\overline{N}) - fN\cos(\hat{z},\overline{V}) + Q\cos(\hat{z},\overline{V}),$$
(33)

where

 $\cos(y,\overline{N}), \cos(z,\overline{N})$ – the direction cosines of the force vector \overline{N} to the axes Oy and Oz respectively;

 $\cos(\dot{y}, \overline{V})$, $\cos(\dot{z}, \overline{V})$ – the direction cosines of the velocity vector \overline{V} of the motion of contact point *K* on the head of root-crops to the axes *Oy* and *Oz* respectively;

 \dot{y}, \dot{z} – the projection of the velocity vector \overline{V} on the coordinate axis *Oy* and *Oz* respectively. It is known from [13] that the mentioned direction cosines will be equal to

$$\cos(y,\overline{N}) = \frac{\partial f}{\partial y} \cdot \frac{1}{\Delta f}; \quad \cos(z,\overline{N}) = \frac{\partial f}{\partial z} \cdot \frac{1}{\Delta f};$$

$$\cos(\dot{y},\overline{V}) = \frac{\dot{y}}{V}; \quad \cos(\dot{z},\overline{V}) = \frac{\dot{z}}{V},$$
(34)

where:

f(y, z) = 0 – the equation of the relation (surface, over which it moves material point);

 Δf – the module of the gradient of function f(y, z);

V – the module of the velocity vector of the point.

Since it was at first accepted that the head of root-crops has a spherical form, the sphere, which can be represented by this equation , is the equation of relation

$$f(x, y, z) = x^{2} + y^{2} + z^{2} - R^{2} = 0, \qquad (35)$$

where:

R – a radius of the spherical head of the root-crops for the plane yOz x = 0, and therefore the equation of sphere (45) passes into the equation of circumference

$$f(y, z) = y^{2} + z^{2} - R^{2} = 0.$$
(36)

According to [13], the module of the gradient of function and the module of speed will be equal to

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}, \qquad (37)$$

$$V = \sqrt{\dot{y}^2 + \dot{z}^2} .$$
 (38)

Let us substitute (34) in (33) and will add to the system of differential equations (33) the equation of relation (36), then we obtain the following system of differential equations:

$$m \ddot{y} = -m\omega^{2}\rho \sin\alpha + \frac{N}{\Delta f} \cdot \frac{\partial f}{\partial y} - fN \frac{\dot{y}}{V} + Q \frac{\dot{y}}{V},$$

$$m \ddot{z} = -m\omega^{2}\rho \cos\alpha - mg + \frac{N}{\Delta f} \cdot \frac{\partial f}{\partial z} - fN \frac{\dot{z}}{V} + Q \frac{\dot{z}}{V},$$

$$y^{2} + z^{2} - R^{2} = 0.$$
(39)

Let us calculate partial derivatives and gradient of the functions, which enter into the system of equations (39). We will have:

$$\frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 2z.$$
 (40)

Then, according to (37)

$$\Delta f = \sqrt{(2y)^2 + (2z)^2} = 2R.$$
(41)

Let us substitute expressions (40), (41) in (39). Then the system of differential equations (39) acquires this form

$$m \ddot{y} = -m\omega^{2}\rho \sin\alpha + \frac{y}{R} \cdot N - fN\frac{\dot{y}}{V} + Q\frac{\dot{y}}{V},$$

$$m \ddot{z} = -m\omega^{2}\rho \cos\alpha - mg + \frac{z}{R} \cdot N - fN\frac{\dot{z}}{V} + Q\frac{\dot{z}}{V},$$

$$y^{2} + z^{2} - R^{2} = 0.$$
(42)

The system of equations (42) is the system of three equations with three unknown values y, z and N. Therefore it is specific and has unique solution.

Let us exclude from the obtained system of equations (42) the unknown values N and z, thus bringing together this system to one differential equation with one unknown function y(t). For this, t should be differentiated two times on the equation of relation (36). If we differentiate this equation one time, then we obtain

$$2y\dot{y} + 2z\dot{z} = 0$$
, (43)

whence we find

$$y\dot{y} + z\dot{z} = 0. \tag{44}$$

If we differentiate equation (44), then we will have

$$y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0, \qquad (45)$$

or

$$(y\ddot{y} + z\ddot{z}) + (\dot{y}^2 + \dot{z}^2) = 0.$$
 (46)

Since $\dot{y}^2 + \dot{z}^2 = V^2$, we will obtain

$$V^2 = -(y\ddot{y} + z\ddot{z}). \tag{47}$$

Let us multiply the first equation of the system (42) by y, the second one by z and let us accumulate them one by one, then we will obtain

$$m(y\ddot{y}+z\ddot{z}) = -(m\omega^{2}\rho \cdot y\sin\alpha + m\omega^{2}\rho \cdot z\cos\alpha) - mgz + + \frac{N}{R}(y^{2}+z^{2}) - f\frac{N}{V}(y\dot{y}+z\dot{z}) + \frac{Q}{V}(y\dot{y}+z\dot{z}),$$
(48)

whence, taking into account the expressions (44) and (47), we will have

$$-mV^{2} = -m\omega^{2}\rho(y \cdot \sin\alpha + z \cdot \cos\alpha) - mgz + RN.$$
⁽⁴⁹⁾

From the expression (49) we find the normal reaction N. It will be equal to

$$N = \frac{1}{R} \Big[m\omega^2 \rho \left(y \cdot \sin \alpha + z \cdot \cos \alpha \right) + mgz - mV^2 \Big]$$
(50)

Let us carry out further conversions. From the expression (44) we obtain

$$\dot{z} = -\frac{y\dot{y}}{z},\tag{51}$$

then

$$\dot{z}^2 = \frac{(y\dot{y})^2}{z^2},$$
(52)

or

$$\dot{z}^2 = \frac{(y\dot{y})^2}{R^2 - y^2}.$$
(53)

Thus, for the value of the square of the speed of motion V we can obtain this expression

$$V^{2} = \dot{y}^{2} + \dot{z}^{2} = \dot{y}^{2} + \frac{(y\dot{y})^{2}}{R^{2} - y^{2}}.$$
(54)

Substituting the expression (50) into the first equation of the system (52), we obtain

$$m\ddot{y} = -m\omega^{2}\rho \cdot \sin\alpha + \left(\frac{y}{R} - f\frac{\dot{y}}{V}\right) \left[m\omega^{2}\rho\left(y\sin\alpha + z\cos\alpha\right) + mgz - mV^{2}\right] \frac{1}{R} + Q\frac{\dot{y}}{V}.$$
(55)

Since $z = \sqrt{R^2 - y^2}$, taking into account the expression (54), we will finally have

$$m\ddot{y} = -m\omega^{2}\rho \cdot \sin\alpha + \left(\frac{y}{R} - \frac{f \cdot \dot{y}\sqrt{R^{2} - y^{2}}}{\sqrt{(R^{2} - y^{2})\dot{y}^{2} + (y\dot{y})^{2}}}\right) \times \\ \times \left\{m\omega^{2}\rho\left(y \cdot \sin\alpha + \sqrt{R^{2} - y^{2}} \cdot \cos\alpha\right) + mg\sqrt{R^{2} - y^{2}} - (56)\right. \\ \left. - \frac{m\left[\left(R^{2} - y^{2}\right)\dot{y}^{2} + (y\dot{y})^{2}\right]}{R^{2} - y^{2}}\right] \frac{1}{R} + Q \cdot \frac{\dot{y}\sqrt{R^{2} - y^{2}}}{\sqrt{(R^{2} - y^{2})\dot{y}^{2} + (y\dot{y})^{2}}}.$$

Thus, the differential equation of the second order is obtained, in which only one function y is unknown, i.e., is obtained by the differential equation in the so-called normal form, since the higher derivative is expressed as the lowest derivatives and the unknown function.

The unknown force Q, which enters into the equation (56), must be found from the realization conditions for the process of the combing of the remainders of vegetable tops from the head of root-crops during the necessary bending strain of the blade.

Therefore, to solve this equation it is necessary to first find the force Q, or to express it through the known values.

Since equation (56) is nonlinear, it is possible to solve it only by numerical methods on the personal computer under the assigned initial conditions. Let us find such conditions. The initial velocity of the contact point K_1 of the blade with the head of root-crops will be the absolute velocity of the point M_1 of the blade after impact, since at this moment the point M_1 coincides with the point K_1 . Thus, as can be seen from Fig. 1,

$$\dot{y}_{o} = U\sin\gamma \cdot \sin\alpha - U\cos\gamma \cdot \cos\alpha .$$
⁽⁵⁷⁾

Thus, the initial conditions for the differential equation (56) take this form: with t = 0

$$y_o = -R\cos\alpha,$$

$$\dot{y}_o = U\sin\gamma \cdot \sin\alpha - U\cos\gamma \cdot \cos\alpha,$$
 (58)

where:

U – determined according to the expression (15), γ – according to the expression (12).

The differential equation of the motion of point K in the projection on the axis Oz can be found by analogous conversions, nevertheless, for this there is no need, since, knowing y and \dot{y} , from equation (36) it is possible to determine z and \dot{z} .

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SUMMARY

A new theory is proposed of the interaction of a flexible purifying blade with the injector face of rootcrops in the case when the blade installed on the driving side shaft accomplishes plane motion. On the basis of the obtained differential equations of the blade motion some new mathematical dependencies, describing the basic parameters of this interaction , are given.