THEORY OF THREE-DIMENSIONAL/SPACE INTERACTION OF PURIFYING BLADE/VANE WITH THE HEAD OF THE ROOT-CROPS

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Work [1] presents the results of the studies of cleaning the heads of root-crops on the root by flexible purifying blade/vane in the case of motion by the latter, established/installed on the vertical drive shaft. In this case is analytically investigated the plane motion of the purifying operating unit. However, in practice purifying blade/vane accomplishes the spatial motion, which at present theoretically actually not is investigated and there are no scientific works, which would describe analytically this form of the motion of the purifying operating unit.

Let us examine the case, when the plane of the rotation/revolution of whip/scourge around the axis Ox is arranged/located at a certain angle β to the row of root-crops, i.e. purifying blade/vane actually moves in the space. In this case the velocity vector of forward motion of purifier $\overline{V}_P$ will compose angle β with the plane of the rotation/revolution of the whip/scourge. Let us depict on Fig. 1 velocity vector $\overline{V}_M$ of point M on the whip/scourge and velocity vector of forward motion of purifier $\overline{V}_P$ and projection of this speed on the coordinate axis before beginning the impact/shock at contact point K_1, and also velocity vectors $\overline{U}_1$, $\overline{U}_2$ and $\overline{U}$ after the impact/shock. Let us depict also on Fig. 1 three-dimensional natural system of coordinates $\overline{\tau} K_1 \overline{n} \overline{b}$, where the binormal $\overline{b}$ is directed oppositely the direction of the axis Ox.

Let us decompose vector $\overline{V}_P$ along the coordinate axes Ox and Oy.Then under the action of the speed $\overline{V}_1 = \overline{V}_P \cos\beta + \overline{V}_M$,whip/scourge achieves an impact/shock on the head of root-crops in the vertical plane yOx, and under the action of speed $V_P \sin\beta$ whip/scourge achieves an impact/shock in the horizontal plane xOy, it is more precise along the axis Ox.

Let us depict the diagram of the forces, which act on the head of root-crops during the three-dimensional/space impact/shock at contact point K_1 (Fig. 2).

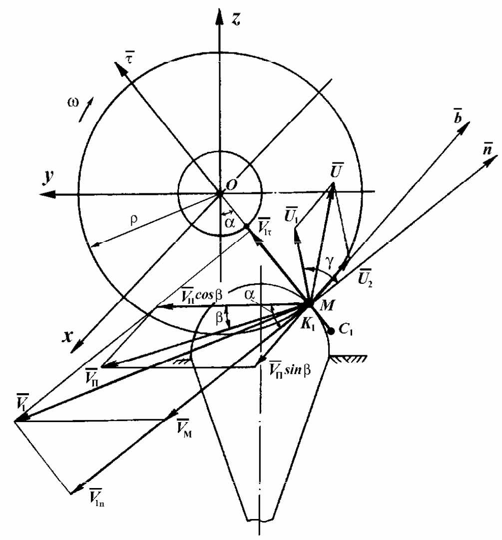

Fig. 1. The diagram of speeds with the impact contact of whip/scourge with the head of root-crops to the impact/shock and after the impact/shock in the case, when the plane of the rotation/revolution of whip/scourge is arranged/located at angle to the row β of the root-crops

The impact/shock in the plane yOz is reduced to case examined above; therefore all obtained analytical expressions for determining the necessary values can be used, but in this case each time instead of $\overline{V_P}$ it is necessary to substitute $V_P \cos\beta$.

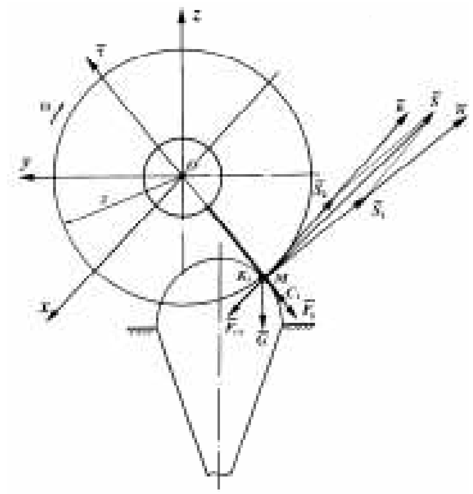

Fig. 2. The diagram of the forces, which act at contact point of whip/scourge with the head of root-crops in the case, when the plane of the rotation/revolution of whip/scourge is arranged/located at angle to the row β of the root-crops

The angle γ of deflection of velocity vector $\overline{U}_1$ after the impact/shock from the standard/normal $\overline{n}$ in the plane yOz is obtained from the expressions [1] $\operatorname{ctg}\gamma = \frac{(V_P \cos\alpha + \omega\rho)\cdot\varepsilon}{V_P \sin\alpha}$ and $\gamma = arc\operatorname{ctg}\left[\frac{(V_P \cos\alpha + \omega\rho)\cdot\varepsilon}{V_P \sin\alpha}\right]$, which were obtained for the case of the plane motion of the purifying blade/vane:

$$\operatorname{ctg}\gamma = \frac{(V_P \cos\beta \cdot \cos\alpha + \omega\rho)\cdot\varepsilon}{V_P \cdot \cos\beta \cdot \sin\alpha} \tag{1}$$

then

$$\gamma = arc\operatorname{ctg}\left[\frac{(V_P \cos\beta \cdot \cos\alpha + \omega\rho)\cdot\varepsilon}{V_P \cdot \cos\beta \cdot \sin\alpha}\right]. \tag{2}$$

We will obtain the modulus/module of speed U_1 after the impact/shock in the plane yOz from the expression $U = \sqrt{V_P^2 \sin^2\alpha + (V_P \cos\alpha + \omega\rho)^2 \cdot \varepsilon^2}$ which was also obtained for the case of the plane motion of the blade/vane:

$$U_1 = \sqrt{V_P^2 \cdot \cos^2\beta \cdot \sin^2\alpha + (V_P \cos\beta \cdot \cos\alpha + \omega\rho)^2 \varepsilon^2}\ , \tag{3}$$

and impact impulse $\overline{S}_1$ along the standard/normal $\overline{n}$ is obtained from the expression $S = -m(1+\varepsilon)\ V_n$ the plane motion of the blade/vane:

$$S_1 = m(1+\varepsilon)(V_P \cdot \cos\beta \cdot \cos\alpha + \omega\rho). \tag{4}$$

For determining the necessary values with the impact/shock along the axis Ox (or also the very, that along the binormal $\overline{b}$), let us design equation $m(\overline{U} - \overline{V}) = \overline{S}$ to the binormal $\overline{b}$, we obtain:

$$m(U_b - V_b) = S \cdot \overline{b}\ , \tag{5}$$

or, as can be seen from Fig. 1,

$$m(U_2 + V_P \cdot \sin\beta) = S_2\ , \tag{6}$$

where:

S_2 – impact impulse from the speed $V_p \cdot \sin\beta$ along the axis $\overline{b}$.

Since U_2 the speed of whip/scourge after impact/shock along the binormal $\overline{b}$, a $V_P \sin\beta$. The speed of whip/scourge to the impact/shock along the binormal $\overline{b}$, then taking into account recovery factor ε, it is possible to write down [2].

$$U_2 = -\varepsilon \cdot V_b = \varepsilon \cdot V_{\Pi} \cdot \sin\beta\ . \tag{7}$$

If we substitute (6) in (7), then we obtain the values of impact impulse along the axis $\overline{b}$:

$$S_2 = m(1+\varepsilon)V_P \cdot \sin\beta \,. \tag{8}$$

Total speed U after impact/shock for the present instance will be equal

$$U = \sqrt{U_1^2 + U_2^2} \,,$$

or

$$U = \sqrt{V_P^2 \cdot \cos^2\beta \cdot \sin^2\alpha + (V_P \cos\beta \cdot \cos\alpha + \omega\rho)^2 \varepsilon^2 + V_P^2 \cdot \sin^2\beta \cdot \varepsilon^2} \,. \tag{9}$$

Total impact impulse will be equal:

$$S = \sqrt{S_1^2 + S_2^2} \,,$$

or

$$S = m(1+\varepsilon)\sqrt{(V_P \cos\beta \cdot \cos\alpha + \omega\rho)^2 + V_P^2 \cdot \sin^2\beta} \,. \tag{10}$$

After the first phase (encounter of whip/scourge with the head of root-crops, or impact/shock of whip/scourge on the head of root-crops) the second phase sets in. The phase of the motion of whip/scourge on the head of root-crops, during which occurs the basic process of **(schesyvaniya)** of remainders/residues from its head. For the analytical description of this process it is necessary to compile the differential equations of motion of the point *K* (arbitrary point of whip/scourge, which it will move) the contact of whip/scourge over the injector face of the root-crops.

The composition of the differential equations of motion of contact point of whip/scourge and head of root-crops for the case, when the plane of the rotation/revolution of whip/scourge around the axis Ox is arranged/located at a certain angle to the row β of the root-crops, is the following stage of a study.

Let us depict, as in the preceding case, the schematic of power interaction of whip/scourge with the head of root-crops during the motion of whip/scourge along the head of root-crops. It is obvious that at contact point *K* will act the same forces, which were examined for the case of the plane motion of blade/vane, nevertheless, in contrast to the foregoing case, the process of schesyvaniya of the remainders/residues of vegetable tops will occur in the space, and therefore structure diagram will be already three-dimensional/space (Fig. 3).

It should also be noted that the obtained above basic kinematic dependences, which describe the motion of whip/scourge along the head of root-crops, occur also for the case, when contact point *K* describes three-dimensional/space trajectory. Thus, in the absolute motion suspension point O_1 of whip/scourge will accomplish displacement/movement according to expression $OO' + O'_1O_2 = V_p \cdot t + \omega rt = (V_p + \omega r)t$ and since whip/scourge can be considered continuous nonductile rod, will be also valid and entire dependences, examined for the case of the plane motion of blade/vane.

Thus, and in this case centrifugal inertial force $\overline{F}_{в}$ at each contact point K approximately remains constant in the value and in the direction; moreover vector $\overline{F}_{в}$ is located in plane yOz, i.e. in the plane of the rotation/revolution of the whip/scourge.

As already mentioned, the given kinematic dependences sufficiently approximately describe the process of moving the whip/scourge along the head root-crops.Therefore for the more precise analytical study of the motion of whip/scourge let us compile the differential equations of motion of the point M of whip/scourge along the head of root-crops. Since tractive propelling power $\overline{P}$ of purifier and rotational moment M_{ob} of whip/scourge enter into the composition of forces of **(schesyvaniya)** $\overline{Q}$, Fig. 3 does not depict they.

Thus, according to the diagram of power interaction, which is depicted in Fig. 3, the differential equation of motion of contact point K along the head of root-crops in the vector form will take the form [3]

$$m\,\overline{a} = \overline{F}_{в} + \overline{G} + \overline{N} + \overline{F}_{mp.} + \overline{Q} \,. \tag{11}$$

Since in this case we have the three-dimensional coordinate system, differential equation (11) is reduced already to the system of three differential equations of the second order of the following form

$$\left.\begin{aligned} m\ddot{x} &= N_x + F_{mp.x} + Q_x, \\ m\ddot{y} &= F_{вy} + N_y + F_{mp.y} + Q_y, \\ m\ddot{z} &= F_{вz} + G + N_z + F_{mp.z} + Q_z, \end{aligned}\right\} \tag{12}$$

where in the right sides of the system of equations (12) the projections of the corresponding forces on the axis i. are written down Ox, Oy and Oz.

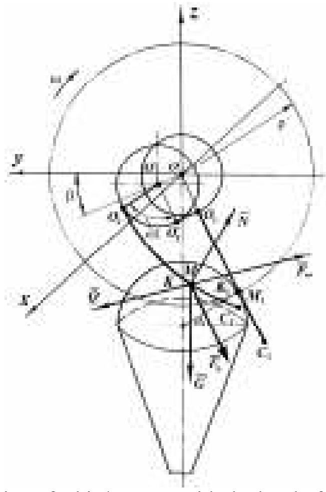

Fig. 3. The schematic of power interaction of whip/scourge with the head of root-crops in the process of schesyvaniya of the remainders/residues of vegetable tops in the case, when the plane of the rotation/revolution of whip/scourge is arranged/located at angle β to the row of the root-crops

Taking into account the values of the projections of the vectors of the forces, which enter into the system of differential equations (12), and expression $F_{в} \approx m\omega^2 \cdot \rho$ and $F_{тр} = f \cdot N$, this system of differential equations acquires the following form

$$\left.\begin{aligned}
&m\ddot{x} = N\cos\left(\widehat{x,\overline{N}}\right) - fN\cos\left(\widehat{\dot{x},\overline{V}}\right) + Q\cos\left(\widehat{\dot{x},\overline{V}}\right),\\
&m\ddot{y} = -m\omega^2\rho\sin\alpha + N\cos\left(\widehat{y,\overline{N}}\right) - fN\cos\left(\widehat{\dot{y},\overline{V}}\right) + Q\cos\left(\widehat{\dot{y},\overline{V}}\right),\\
&m\ddot{z} = -m\omega^2\rho\cos\alpha - mg + N\cos\left(\widehat{z,\overline{N}}\right) - fN\cos\left(\widehat{\dot{z},\overline{V}}\right) + Q\cos\left(\widehat{\dot{z},\overline{V}}\right),
\end{aligned}\right\} \quad (13)$$

where:

$\cos\left(\widehat{x,\overline{N}}\right)$, $\cos\left(\widehat{y,\overline{N}}\right)$, $\cos\left(\widehat{z,\overline{N}}\right)$ – the direction cosines of the force vector $\overline{N}$,

$\cos\left(\widehat{\dot{x},\overline{V}}\right)$, $\cos\left(\widehat{\dot{y},\overline{V}}\right)$, $\cos\left(\widehat{\dot{z},\overline{V}}\right)$ – the direction cosines of the velocity vector of the motion $\overline{V}$ of contact point *K* on the head of root-crops,

$\dot{x}, \dot{y}, \dot{z}$ – the projection of velocity vector $\overline{V}$ on the appropriate coordinate axes.

Direction cosines let us determine accordingly expressions [3]

$$\begin{aligned}
&\cos\left(\widehat{y,\overline{N}}\right) = \frac{\partial f}{\partial y}\cdot\frac{1}{\Delta f}; \quad \cos\left(\widehat{z,\overline{N}}\right) = \frac{\partial f}{\partial z}\cdot\frac{1}{\Delta f};\\
&\cos\left(\widehat{\dot{y},\overline{V}}\right) = \frac{\dot{y}}{V}; \qquad \cos\left(\widehat{\dot{z},\overline{V}}\right) = \frac{\dot{z}}{V},
\end{aligned} \quad (14)$$

moreover the equation of sphere is the equation of relation in this case

$$f(x, y, z) = x^2 + y^2 + z^2 - R^2 = 0 \quad (15)$$

Then the system of differential equations (13) acquires this form:

$$\left.\begin{aligned}
&m\ddot{x} = \frac{N}{\Delta f}\cdot\frac{\partial f}{\partial x} - fN\frac{\dot{x}}{V} + Q\frac{\dot{x}}{V},\\
&m\ddot{y} = -m\omega^2\rho\sin\alpha + \frac{N}{\Delta f}\cdot\frac{\partial f}{\partial y} - fN\frac{\dot{y}}{V} + Q\frac{\dot{y}}{V},\\
&m\ddot{z} = -m\omega^2\rho\cos\alpha - mg + \frac{N}{\Delta f}\cdot\frac{\partial f}{\partial z} - fN\frac{\dot{z}}{V} + Q\frac{\dot{z}}{V},\\
&x^2 + y^2 + z^2 - R^2 = 0.
\end{aligned}\right\} \quad (16)$$

Let us calculate partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$ and the gradient of function Δf let us substitute the obtained expressions into the system of equations (16), we will have

$$\left.\begin{aligned} & m\ddot{x} = \frac{x}{R}\cdot N - fN\frac{\dot{x}}{V} + Q\frac{\dot{x}}{V}, \\ & m\ddot{y} = -m\omega^2\rho\sin\alpha + \frac{y}{R}\cdot N - fN\frac{\dot{y}}{V} + Q\frac{\dot{y}}{V}, \\ & m\ddot{z} = -m\omega^2\rho\cos\alpha - mg + \frac{z}{R}\cdot N - fN\frac{\dot{z}}{V} + Q\frac{\dot{z}}{V}, \\ & x^2 + y^2 + z^2 - R^2 = 0. \end{aligned}\right\} \tag{17}$$

Let us reduce system of equations (17) to the system of two differential equations relative to unknown functions $x(t)$, $y(t)$. For this let us exclude unknown function $z(t)$ from the system of equations (17).

For this purpose we differentiate two times of the equation of relation (15). After a number of conversions we obtain such dependences

$$x\dot{x} + y\dot{y} + z\dot{z} = 0, \tag{18}$$

$$V^2 = -\left(x\ddot{x} + y\ddot{y} + z\ddot{z}\right). \tag{19}$$

Let us multiply further the first equation of system (17) by x, the second to y, and the third on z let us accumulate them piecemeal, we obtain

$$\begin{aligned} & m\left(x\ddot{x} + y\ddot{y} + z\ddot{z}\right) = \frac{\left(x^2 + y^2 + z^2\right)}{R}N - f\frac{N}{V}\left(x\dot{x} + y\dot{y} + z\dot{z}\right) + \\ & + \frac{Q}{V}\left(x\dot{x} + y\dot{y} + z\dot{z}\right) - m\omega^2\rho\left(y\sin\alpha + z\cos\alpha\right) - mgz, \end{aligned} \tag{20}$$

whence, taking into account expressions (18) and (19), we find the normal reaction N, which, as in the preceding case of the plane motion of blade/vane, has form $N = \frac{1}{R}\left[m\omega^2\rho\left(y\cdot\sin\alpha + z\cdot\cos\alpha\right) + mgz - mV^2\right]$ nevertheless, in contrast to the represented form, the value of the square of speed V^2 in this case it is determined accordingly this expression

$$V^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2, \tag{21}$$

and value z is determined accordingly this expression

$$z = \sqrt{R^2 - x^2 - y^2}\,. \tag{22}$$

We obtain from expression (18)

$$\dot z = -\frac{x\dot x + y\dot y}{z}. \tag{23}$$

Let us elevate both parts of equality (23) into the square, we will have

$$\dot z^2 = \frac{(x\dot x + y\dot y)^2}{z^2}, \tag{24}$$

or, taking into account (22),

$$\dot z^2 = \frac{(x\dot x + y\dot y)^2}{R^2 - x^2 - y^2}. \tag{25}$$

Substituting (24) in (21), we obtain

$$V^2 = \dot x^2 + \dot y^2 + \dot z^2 = \dot x^2 + \dot y^2 + \frac{(x\dot x + y\dot y)^2}{R^2 - x^2 - y^2}. \tag{26}$$

Let us substitute expression $N = \frac{1}{R}\left[m\omega^2\rho\left(y \cdot \sin\alpha + z \cdot \cos\alpha\right) + mgz - mV^2\right]$ first two equations (17), we will have

$$\left.\begin{aligned}
m\ddot x &= \frac{1}{R}\left[m\omega^2\rho\,\left(y\sin\alpha + z\cos\alpha\right) + mgz - mV^2\right]\cdot\left(\frac{x}{R} - f\,\frac{\dot x}{V}\right) + Q\,\frac{\dot x}{V},\\
m\ddot y &= -\,m\omega^2\rho\sin\alpha + \frac{1}{R}\left[m\omega^2\rho\left(y\sin\alpha + z\cos\alpha\right) + mgz - mV^2\right]\times\\
&\quad\times\left(\frac{y}{R} - f\,\frac{\dot y}{V}\right) + Q\,\frac{\dot y}{V}.
\end{aligned}\right\} \tag{27}$$

Further, taking into account (22) and (26), we finally obtain the following system of differential equations, which describes the motion of point K on the head of root-crops, in the case, when the plane of the rotation/revolution of whip/scourge is arranged/located at angle β to the row of the root-crops:

$$
\left.
\begin{aligned}
m\ddot{x} = {} & \frac{1}{R}\left[m\omega^2\rho \left(y\cdot\sin\alpha + \sqrt{R^2-x^2-y^2}\cdot\cos\alpha \right) + mg\sqrt{R^2-x^2-y^2} - \right. \\
& \left. - m\dot{x}^2 - m\dot{y}^2 - \frac{m\left(x\dot{x}+y\dot{y}\right)^2}{R^2-x^2-y^2} \right]\cdot\left[\frac{x}{R} - \frac{f\cdot\dot{x}\sqrt{R^2-x^2-y^2}}{\sqrt{\left(R^2-x^2-y^2\right)\left(\dot{x}^2+\dot{y}^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}} \right] + \\
& + \frac{Q\cdot\dot{x}\sqrt{R^2-x^2-y^2}}{\sqrt{\left(R^2-x^2-y^2\right)\left(\dot{x}^2+\dot{y}^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}}, \\
m\ddot{y} = {} & -m\omega^2\rho\sin\alpha + \frac{1}{R}\left[m\omega^2\rho \left(y\cdot\sin\alpha + \sqrt{R^2-x^2-y^2}\cos\alpha \right) + \right. \\
& \left. + mg\sqrt{R^2-x^2-y^2} - m\dot{x}^2 - m\dot{y}^2 - \frac{m\left(x\dot{x}+y\dot{y}\right)^2}{R^2-x^2-y^2} \right]\times \\
& \times\left[\frac{y}{R} - \frac{f\cdot\dot{y}\sqrt{R^2-x^2-y^2}}{\sqrt{\left(R^2-x^2-y^2\right)\left(\dot{x}^2+\dot{y}^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}} \right] + \\
& + \frac{Q\cdot\dot{y}\sqrt{R^2-x^2-y^2}}{\sqrt{\left(R^2-x^2-y^2\right)\left(\dot{x}^2+\dot{y}^2\right)+\left(x\dot{x}+y\dot{y}\right)^2}}.
\end{aligned}
\right\} \quad (28)
$$

System of equations (28) is the system of the nonlinear differential equations of the second order relative to unknown functions $x(t)$ and $y(t)$. The obtained system of differential equations can be solved only by numerical methods on the personal computer under the assigned initial conditions. In this case, as in the case of the plane motion of blade/vane, unknown force Q **(schesyvaniya)** of the remainders/residues of vegetable tops from the head of root-crops must be found from realization conditions for **(schesyvaniya)** of the remainders/residues of vegetable tops from the head of root-crops during the necessary bending strain of the whip/scourge

Let us determine the initial conditions of moving contact point K in this case.With the initial velocity of point K_1 the contact of whip/scourge with the head of root-crops there will be absolute velocity $\overline{U}$ point M_1 whip/scourge after impact/shock. Thus $\overline{V_o} = \overline{U}$, where U it is determined according to expression (9).

As can be seen from diagram in Fig. 1,

$$
\begin{aligned}
\dot{x}_0 &= -U_2, \\
\dot{y}_0 &= U_1\sin\gamma\cdot\sin\alpha - U_1\cos\gamma\cdot\cos\alpha,
\end{aligned} \quad (29)
$$

where:

U_1, U_2 – they are determined according to expressions (9) and (7) respectively,

γ – is determined according to expression (2).

Thus, initial conditions for the system of differential equations (28) take this form:

With $t = 0$:

$$\begin{aligned} x_o &= 0, \\ y_o &= -R\cos\alpha, \\ \dot{x}_o &= -U_2, \\ \dot{y}_o &= U_1 \sin\gamma \cdot \sin\alpha - U_1 \cos\gamma \cdot \cos\alpha. \end{aligned} \tag{30}$$

Let us determine further the force of **(schesyvaniya)** Q on the basis of the conditions of the possibility of the realization of the technological process of the detachment of the cuttings of vegetable tops from the head of root-crops, taking into account the physicomechanical properties precisely of the vegetable tops of the sugar beet

As follows from the data of studies of the institute of the sugar beet of the Ukrainian Academy of agrarian Sciences, the cutting of the vegetable tops of sugar beet in the cross section in the general case close to the triangular form, which has with the base a cavity also of triangular form.This common format of the cross section of cutting, according to [5], it is close to the form of figure, which is depicted in Fig. 4. Thus, the indicated in Fig. 4 sizes/dimensions we use subsequently for the calculations of the cross-sectional area of the cuttings of the vegetable tops.

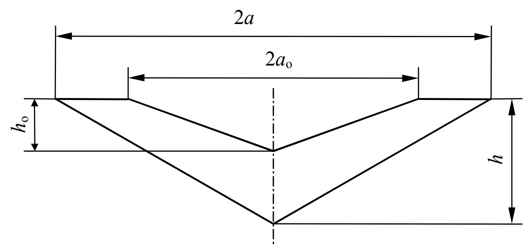

Fig. 4. Diagram of the cross section of the cutting of the vegetable tops of the sugar beet

We will also consider that the process of schesyvaniya occurs directly on the very head of root-crops in the bearing edge of cutting due to the shearing strain of cutting.In that case, it is obvious, that the process of schesyvaniya will be possible with satisfaction of the following condition:

$$\frac{Q}{nF} \geq [\tau] \tag{31}$$

where:

Q – the force of schesyvaniya,
$[\tau]$ – the permissible tangential shear stress for the cutting,
F – the cross-sectional area of one cutting,
n – the number of cuttings, which simultaneously are combed.

We further consider that simultaneously are combed the cuttings of the vegetable tops, which are arranged/located on the head of root-crops in one narrow row by base for this assumption it appears that that schesyvaniye of remainders/residues from the head of root-crops occurs immediately after cutting of its central part by the botvosrezayushchim apparatus. But therefore the cuttings, which are arranged/located from above and in front of the number, which is combed, they are considered as the already cut off, and consequently is not resisted with further schesyvanii of the remainders/residues of vegetable tops by the blade purifier.

Thus, the force of schesyvaniya $\overline{Q}$, which is determined from condition (31), it will be equal to the force, which appears from the side of the deformed whip/scourge, which, in turn, will be approximately equal to force $\overline{Q_k}$, created by torque M_k, transmitting to the whip/scourge, i.e. by torque on the drive shaft of purifier.Therefore it is possible to consider, that $Q = Q_k$.

Let us calculate the further necessary for schesyvaniya of cuttings from the head of root-crops force Q. We will obtain from condition (31)

$$Q \geq nF\left[\tau\right]. \tag{32}$$

As can be seen from Fig. 4, area F the cross section of the cutting of vegetable tops it is equal

$$F = \frac{1}{2}\cdot 2ah - \frac{1}{2}\cdot 2a_o h_o, \tag{33}$$

or

$$F = ah - a_o h_o. \tag{34}$$

Taking into account expressions (32) and (34), we find the force of schesyvaniya Q. It will be equal:

$$Q \geq \left(ah - a_o h_o\right)n\left[\tau\right]. \tag{35}$$

From other side, the force, which appears at contact point K whip/scourge with the head of root-crops, as it was indicated above, it was equal to force Q_k, which it is created by torque M_k on the drive shaft of whip/scourge.It is obvious that in the value force Q_k it will be equal:

$$Q_k = \frac{M_k}{\rho}. \tag{36}$$

Since $Q = Q_k$, that from expression (35) we obtain

$$Q_k \geq \left(ah - a_o h_o\right)n\left[\tau\right] \tag{37}$$

or, taking into account (37),

$$\frac{M_k}{\rho} \geq \left(ah - a_o h_o\right)n\left[\tau\right] \tag{38}$$

Finally from expression (38) we find the value of torque M_k on the drive shaft of the whip/scourge:

$$M_k \geq \rho\left(ah - a_o h_o\right)n\left[\tau\right] \tag{39}$$

Thus, is found the value of the necessary torque on the drive shaft of whip/scourge from the conditions of the possibility of the realization of the technological process of schesyvaniya of the cuttings of vegetable tops from the head of root-crops taking into account their physicomechanical properties

Since we consider the angular rate of rotation of whip/scourge the given one, on the basis (39) it is possible to determine power on the drive shaft $\left(N_{n.в.}\right)$, necessary for the realization of the technological process of schesyvaniya of the remainders/residues of vegetable tops from the head of root-crops.It will be equal:

$$N_{n.в.} = M_k \cdot \omega \geq \rho\left(ah - a_o h_o\right)n\left[\tau\right]\omega, \tag{40}$$

where:

$N_{n.в.}$ – power on the drive shaft of whip/scourge,

ω – the angular rate of rotation of the whip/scourge.

Let us determine force Q, necessary for schesyvaniya of one cutting of vegetable tops, which according to [10] has these linear dimensions: a = 5 mm, a_o = 2 mm, h = 5 mm, h_o = 2 mm. Average/mean value of permissible shearing stress for the material of cutting, also according to [5], it will be equal to: $[\tau] = 1.14 \cdot 10^6$ MPa.

Substituting the value of the given values into expression (36), with n = 1, we obtain: $Q \geq \left(5\cdot 5 - 2\cdot 2\right)\cdot 10^{-6}\cdot 1.14\cdot 10^{-6} = 23.9\ \left(H\right)$.

Taking into account that the cuttings of vegetable tops can have sizes/dimensions of more than averages in the cross section, we assume/take Q = 25 (H).

It must be noted that according to [5], for the realization of the satisfactory process of schesyvaniya of the remainders/residues of vegetable tops from the injector face of root-crops on the root, the necessary tangential forces must be within the limits of 70-120 $\left(\dfrac{H}{cm^2}\right)$ Further, taking into account that the cross-sectional area of the cutting of vegetable tops comprises on the average F = 0.25 cm^2, we obtain, that for schesyvaniya of one cutting of vegetable tops the necessary tangential force must be within the limits of 17.5-30.0 (H). Thus, the obtained in this work value of the force of schesyvaniya Q for one cutting exactly it falls into the interval indicated.

For four cuttings (n = 4)we find from expression (90): $Q \geq 95.6\ \left(H\right)$.

For the practical calculations we can accept Q = 100 (H) and to substitute its value into the system of differential equations (28).

Thus, are found all conditions for solving the system of differential equations (28) with numerical methods with the aid of PEVM.

After solving the system of differential equations (28), further it is possible to find displacements/movements $x(t)$ and $y(t)$ contact point K whip/scourge on the head of root-crops.After this, there is a possibility, using expression (22), to determine the extent of movement $z(t)$.

Further, having the displacements/movements indicated, it is possible to determine the length of the trajectory of the motion of contact point K on the head of root-crops.In this case the length of the trajectory of motion will be the arc length space curve, whose equation in the parametric form will take this form:

$$\left.\begin{array}{l} x = x(t), \\ y = y(t), \\ z = z(t), \end{array}\right\} \tag{41}$$

where:

$x(t)$, $y(t)$ and $z(t)$ – they are found from system of equations (83) as it was shown higher.

Directly arc length L by space curve it is possible to determine, according to [4], with the aid of this expression:

$$L = \int_{o}^{t_1} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt, \tag{42}$$

where:

t_1 – the time of the contact of whip/scourge with the head of root-crops, which can be determined, using an expression.

$$t_1 = \frac{l_1}{V_{\Pi} + \omega r}. \tag{43}$$

This will make possible subsequently to calculate the area of schesyvaniya $F_{сч.}$ in the time of the contact of whip/scourge with the head of root-crops.Actually, if width is known b the seizure of whip/scourge (actually, this it can be the width of whip/scourge itself) during its motion on the head of root-crops, then the area of schesyvaniya by one whip/scourge will be equal:

$$F_{сч.} = L \cdot b \tag{44}$$

Thus, we have all bases in order to analytically estimate the effectiveness of the work of a purifier of this type.

Since the force of **(schesyvaniya)** Q it is equal to the force, which appears from the side of whip/scourge as a result of its bending strain, it is necessary to calculate amount of deflection of whip/scourge, which ensures the creation of this force of schesyvaniya Q. For this we can examine whip/scourge as cantilever beam with the fixed end at suspension point. Accordingly [1] it is possible to compile the differential equation of bent axle of beam/gully and after the twofold integration differential of the equation indicated to obtain the following result for determining the sagging/deflection at any contact point of whip/scourge with the head of the root-crops:

$$\delta = \frac{Q}{3EJ}(l-d)^3, \tag{45}$$

where:

δ – the sagging/deflection of whip/scourge,
EJ – the hardness of whip/scourge,
l – the length of whip/scourge,
$d = MC_1$ distance from the free end of the whip/scourge to contact point (Fig. 3).

In particular, the sagging/deflection of whip/scourge at the free end of the whip/scourge (point C_1 on Fig. 3) with $d = 0$, it will have this value:

$$\delta = \frac{Q \cdot l^3}{3EJ}. \tag{46}$$

Obtained sagging/deflection of whip/scourge, with its hardness EJ, it will ensure the necessary force of **(schesyvaniya)** Q, which is determined according to expression (36).

Thus, the obtained new theoretical dependences give all bases to conducting of the calculations of the basic parameters of technological process and purifier of the heads of root-crops on the root, which has horizontal drive shaft with the hinge attached flexible purifying blades/vanes (whips/scourges). But numerical simulation on PEVM of data of analytical dependences will make possible to also determine the optimum values of the design and kinematic parameters of this operating unit, which widely is adapted in the beet-harvesting technology.

BIBLIOGRAPHY

1. **Bulgakov V., Holovach I.**: Theory of cleaning the heads of root-crops by purifier with the vertical rotational axis. Collection of the scientific works of Kerch sea technological institute “mechanization of production processes of the fisheries, industrial and agrarian enterprises”, Kerch, 2002. 209-226.
2. **Butienin N. W., Łunc. J. L., Merkin D. R.**: Course of theoretical mechanics.T. 2. M.: Science, 1985. 496 s.
3. **Wasilenko P. M.**: Wwiedienije w zemledelczeskuju mechaniku. Silhosposwita 1996. s. 252.
4. **Wygodskij M. J.**: Sprawocznik po wysszej matiematikie. Nauka 1973. s. 870
5. **Pogorielyj L. W., Tatianko J. W., Brej W. W.**: Swiekłouborocznyje masziny (Konstruowanije i rasczet) Technika 1983. s. 168.

SUMMARY

Is proposed the improved theory of interaction of flexible purifying blade/vane with the injector face of root-crops on the root, in the case, when blade/vane moves in the space. On the basis of the obtained differential three-dimensional equations of motion of blade/vane the new mathematical dependences, which base the basic parameters of this interaction, are given.