Mathematical models of rheological transformations during transfer of potential of mass, energy and momentum pulse

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Received November 13.2017: accepted December 12.2017

Summary. The processes of convection-diffusion transfer of the potential of the determining parameter, which can be mass, energy or momentum pulse, are being considered. It is shown that the existing methods of mathematical modelling generally consider the transfer processes of mass, heat energy and momentum from a source of infinite cardinality to certain unlimited volume under the initial conditions of the first, second or third kind. In addition, it is assumed that the physical transition zone of the potential is infinitesimally small. At the same time, very complex mathematical models are obtained, which are often unfit for practical usage. It is shown that the rheological transition zone can be a technological device, for example, a reactor, an evaporation plant, an absorber, a rectifying column, etc., in which not only transformation of the main determining parameter occurs, but also creation of new ones. That is, the process of transformation under convection-diffusion transfer can be multistage with simultaneous output (drain) of the determining parameters - concentration, heat energy, pressure, etc. For mathematical description of the transfer processes of the potential the known laws of the transfer phenomena are used, which are supplemented by the corresponding laws of drain of the created potential. It is shown that in this case transformation of the input potential is described by nonlinear differential equations, which under certain conditions can be solved analytically. Key words: potential, transfer, transformation, drain, model.

INTRODUCTION

The study and methods of solving differential equations describing the transfer phenomena are given considerable attention in a large number of papers in mathematical physics [1-4]. In all cases, the basic transfer laws of heat (the Fourier law), mass (Fick's laws) and momentum (Newton's law) were taken as a basis. In addition, it was believed that transfer of heat, mass and momentum is carried out from a source to a certain medium either only diffusively or convectively, or in a mixed way, i.e. convection-diffusion under various initial and boundary conditions [5]. In addition, it was assumed that the potential of heat, mass and momentum of the source is infinite, and the medium is unlimited. In the simplest case, the transfer processes of the potential of

heat, mass, and momentum pulse are described by the following differential equations [7-11]:

- for the diffusion transfer flow with constant velocity of the potential:

$$\frac{\partial \varphi(x,\theta)}{\partial \theta} + D_{\varphi} \frac{\partial^2 \varphi(x,\theta)}{\partial x^2} = 0 ; \qquad (1)$$

- for the convection-diffusion transfer flow with constant velocity of the potential:

$$\frac{\partial \varphi(x,\theta)}{\partial \theta} + D_{\varphi} \frac{\partial^2 \varphi(x,\theta)}{\partial x^2} + v_0 \frac{\partial \varphi(x,\theta)}{\partial x} = 0; \quad (2)$$

- for the convection-diffusion transfer flow with variable velocity of the potential to the medium without acceleration:

$$\frac{\partial \varphi(x,\theta)}{\partial \theta} + D_{\varphi} \frac{\partial^2 \varphi(x,\theta)}{\partial x^2} + v_C(x,\theta) \frac{\partial \varphi(x,\theta)}{\partial x} = 0; \quad (3)$$

- for the convection-diffusion transfer flow with variable velocity of the potential from the source to the medium with acceleration:

$$\frac{\partial \varphi(x,\theta)}{\partial \theta} + \tau_{\partial} \frac{\partial \varphi(x,\theta)}{\partial \theta^{2}} + D_{\varphi} \frac{\partial^{2} \varphi(x,\theta)}{\partial x^{2}} + v_{C}(x,\theta) \frac{\partial \varphi(x,\theta)}{\partial x} = 0$$
(4)

where: $\varphi(x,\theta)$ - the transfer potential of heat, mass, and momentum pulse along the spatial coordinate x and time θ , τ_{∂} - time constant of the potential transfer from the source to the medium, D_{φ} - the transfer coefficient of the transfer potential, $v_C(x,\theta)$ - variable velocity of the transfer convection flow into the medium.

As it is noted in [12], equations (1) - (4) are fair under the condition that velocity of the potential flow transfer from the source to the medium is unlimited. Many scientists [12, 13] call this condition a "paradox" and offer their options for solving it. In fact, equations (1) - (4) are the equations of balance of the potential, which is transferred with unlimited velocity from one medium with infinite potential to another medium, which has infinite volume. As a rule, for the analytical solution of such equations, various assumptions, simplifications and approximations were used; it allowed these equations to be reduced to a form that made it possible to describe such processes by analytic equations [14-15].

ANALYSIS OF PUBLICATIONS

When studying the processes of heat conduction, it was assumed that there were no heat sources, or they had unlimited power. In the scientific literature, it is rarely recalled that there is output (drain) of mass or energy, and nothing was said about flow velocity, its nature and parameters that affect this drain. Methods of mathematical physics, namely, methods of integral transformations allow solving a relatively narrow range of problems in the transfer theory [16-18]. When considering the systems of differential equations with very general boundary conditions, precise methods of solving face great difficulties, which become even greater when considering nonlinear problems. Formally, all the problems of the transfer phenomena are non-linear, since they contain parameters or gradients that are also functions of the transfer parameters, for example, dependence of the heat conductivity coefficient on the temperature, the molecular diffusion coefficient on concentration and the temperature, etc. In such cases, problems of the transfer phenomena are solved by numerical methods. At present, the most common method of the approximate solution of nonlinear equations of the transfer phenomena is the finite difference method [19, 20], which is also called the grid method. As it is shown in [12], the grid method does not give any unambiguity of solution of nonlinear differential equations. And formal transition to the equation in finite differences can automatically lead to loss of one or more solutions of the nonlinear equation. The finite difference method is based on replacement of derivatives by their approximate values, which are expressed by differences in values of the function at some discrete points - the grid nodes. As a result of such transformations, the differential equation is replaced by an equivalent relation in finite differences, the solution of which reduces to performing simple algebraic operations. The main drawbacks of this method are that the solution can not be presented in the analytical form. In most cases, the problem of this or that transport phenomenon reduces to the Fourier, Fick and Newton differential equations, which have the following general form [21, 22]:

$$\frac{\partial U(x,\theta)}{\partial \theta} + \mathbf{K} \frac{\partial^2 U(x,\theta)}{\partial x^2} = 0, \qquad (5)$$

where: U - the determining parameter (temperature, concentration, density, kinematic viscosity), x - the transfer spatial coordinate, θ - the transfer tim, K - the transfer coefficient (the coefficient of thermal diffusivity, diffusion, viscosity).

In this case, the following conditions [2, 3, 5, 23] are imposed on these equations:

1. The boundary conditions of the first kind: $\theta > 0, -\infty < x < +\infty, U(0, \theta) = U_0 = const, U(x, 0) = U_C,$ $\frac{\partial U(+\infty, \theta)}{\partial x} = \frac{\partial U(-\infty, \theta)}{\partial x} = 0$ (the boundary conditions can be supplemented depending on the problem being researched). Then the solution of equation (5) will be described by the following Gaussian error function:

$$\frac{U(x,\theta) - U_0}{U_0 - U_C} = erf\left(\frac{x}{2\sqrt{K\theta}}\right).$$
(6)

2. The boundary conditions of the second kind: $\theta > 0$, $U(x,0) = U_0 = const$, $0 < x < +\infty$, $U(\infty, \theta) = U_0$, $k_{\Pi} \frac{\partial U(0, \theta)}{\partial x} + q_C = 0$ $\frac{\partial U(\infty, \theta)}{\partial x} = 0$ and some others,

which depend on the problem. The solution of equation (5) with the boundary conditions of the second kind has the form:

$$U(\delta,\theta) - U_0 = \frac{2q_C}{k_{\Pi}} \sqrt{\mathbf{K}\theta} \cdot \operatorname{erfc}\left(\frac{x}{2\sqrt{\mathbf{K}\theta}}\right), \quad (7)$$

where: q_C - the transfer source, k_{Π} - the conductivity coefficient (for example, heat conductivity, mass conductivity).

3. The boundary conditions of the third kind: $\theta > 0$,

$$0 < x < +\infty, \ k_{\Pi} \frac{\partial U(0,\theta)}{\partial x} + k_{\alpha} [U_{C} - U(0,\theta)] = 0,$$
$$U(\infty,\theta) = U_{0}, \ U(x,0) = U_{0} = const, \ \frac{\partial U(\infty,\theta)}{\partial x} = 0,$$

where: k_{α} - the emission coefficient (heat emission, mass emission), U_C - the initial value of the determining parameter.

The solution of equation (5) with the boundary conditions of the third kind has the following form:

$$\frac{U(x,\theta) - U_0}{U_C - U_0} = erfc \left(\frac{x}{2\sqrt{K\theta}}\right) - \left[\exp\left(Hx - H^2 K\theta\right)\right]^* \\ * erfc \left(\frac{1}{\sqrt{K\theta}} + H\sqrt{K\theta}\right)$$
(8)

where: $H = k_{\alpha} / k_{\Pi}$.

The general diffusion model with presence of the chemical transformation is described in this form [12]:

$$\frac{\partial Q}{\partial \theta} + w \nabla Q + \nabla (D_{\partial} \nabla Q) = -k_P f(Q), \qquad (9)$$

where: Q - substance concentration in the flow, w-average flow velocity of the phase, θ - the diffusion transfer time, D_{∂} - the diffusion coefficient, k_P -constant of the chemical transformation (reaction) velocity, f(Q) - a function that depends on substance concentration in the flow.

If the diffusion process passes in one direction, for example, in direction x, then equation (9) is reduced to the following:

$$\frac{\partial Q(x,\theta)}{\partial \theta} + D \frac{\partial^2 Q(x,\theta)}{\partial x^2} =$$

$$= -w \frac{\partial Q(x,\theta)}{\partial x} - k_P f[Q(x,t)]$$
(10)

where: t - the chemical transformation time.

Equation (10) is referred to nonlinear, and as it is shown in [5, 12, 13], it is practically impossible to obtain its analytical solution in a general form. If we compare equations (5) and (10), we can see that when:

$$w \frac{\partial Q(x,\theta)}{\partial x} + k_P f[Q(x,t)] = 0$$
(11)

we arrive at Fick equation of the first kind. Equation (11) is nonlinear, as it functionally depends on three variables: the spatial coordinate x, the time of mass transfer θ , and the chemical transformation time t. If we assume that $\theta \approx t$ and f[Q(x,t)] = Q(x,t), then equation (11) is reduced to the following form:

$$\frac{w}{k_P} \frac{\partial Q(x,\theta)}{\partial x} + Q(x,t) = 0.$$
(12)

Equation (8) describes the process of free transfer of concentration Q(x,t) through the spatial coordinate x at the current time t of the chemical transformation. If $\theta = t$, and x = wt, then under the initial conditions $Q(0,t) = Q_0$ and $\frac{\partial Q(\infty,t)}{\partial x} = 0$ the solution of equation (8) is:

$$Q_p(t) = Q(x,\theta) \exp(-k_P t).$$
(13)

Thus, in fact, equality (13) is the analytical solution of nonlinear equation (10), provided that change in the mass transfer time is equal to change in the chemical transformation time. Equation (13) shows that change in concentration of the reacting components along the coordinate x is carried out according to the exponential law and depends on the coefficient of the chemical transformation velocity and the flow velocity of this transformation. Equation (12) shows how concentration Q of the reacting components varies depending on the time t and the spatial coordinate x. Since for the singlestage reaction processes concentration of the newly created (output) substance is proportional to concentration of the reacting components, output concentration of the chemical transformation will be described by the following equation:

$$\tau_p \frac{\partial Q_p(t)}{\partial t} + Q_p(t) = Q(x,\theta), \qquad (14)$$

where: $\tau_p = 1/k_p$ - time constant of the chemical transformation, $Q_p(t)$ - concentration of the output substance during the chemical transformation time t, $Q(x,\theta)$ - concentration of the reacting components in the reaction mass.

When $Q(x,\theta) = const$ the rate of change in output concentration is described by the following equation:

$$\pi_p \frac{\partial^2 Q_p(t)}{\partial t^2} + \frac{\partial Q_p(t)}{\partial t} = k_p f[Q(x,t)].$$
(15)

Taking into account (15), equation (10) is reduced to the following form:

$$\frac{\partial Q(x,\theta)}{\partial \theta} + D \frac{\partial^2 Q(x,\theta)}{\partial x^2} + w \frac{\partial Q(x,\theta)}{\partial x} =$$

$$= \tau_p \frac{\partial^2 Q_p(t)}{\partial t^2} + \frac{\partial Q_p(t)}{\partial t}$$
(16)

Thus, from the viewpoint of rheological transformations theory, we can say that the left-hand side of equation (16) describes velocity of the convectiondiffusion transfer process by a certain flow with concentration $Q(x,\theta)$ of the reacting components in the reaction zone (rheological transformation zone), and the right-hand side is velocity of the output process (drain) of concentration $Q_p(t)$ of the newly created product from this zone. Equation (16) is non-linear, as it contains the following variables: concentration Q of the input material flow, which in its turn depends on the variable spatial coordinate x and its transfer time to the rheological transformation zone θ , and concentration $Q_{p}(t)$ of the created product, which varies with the output (drain) time of the material flow from this zone. Proceeding from this, it is possible to assert that in equation (10) the unknown function of the chemical transformation is the derivative of the aperiodic function of the first order, that is,

$$k_p f[Q(x,t)] = \tau_p \frac{\partial}{\partial t} \left(\frac{\partial Q_p(t)}{\partial t} + Q_p(t) \right),$$

or

$$\int_{0}^{t} k_{p} f[Q(x,t)] = \tau_{p} \frac{\partial Q_{p}(t)}{\partial t} + Q_{p}(t).$$
(17)

As it is shown in [23], in the rheological transfer zone two and more stage transformations are possible, which can occur both according to the aperiodic and oscillatory laws.

3. PURPOSE AND RESEARCH TASK STATEMENT

The purpose of this paper is to justify the methods for solving nonlinear problems of mass, energy, and momentum transfer phenomena with the help of which it is possible to obtain mathematical models of the complex nonlinear transfer phenomena in the analytical form available for use in practical problems of control of technological devices. To achieve this goal, it is necessary to perform the following tasks:

- to justify the possibility of using the Dirac's integral impulse delta function to describe the transfer processes of mass, energy and momentum pulse;

- to justify application of the zero-gradient method for solving non-linear problems of transfer of mass,

energy, and momentum pulse to obtain the mathematical models of various processes of transfer phenomena in the analytical form.

In the further study of the transfer phenomena on the basis of the rheological transitions theory, we will derive from the transfer condition for concentration, energy, and momentum potential. We consider potential as the difference in the pulse of mass (concentrations of substances in solid, liquid and gaseous media), energy (heat, ultrasonic, electrical) and momentum (mechanical and material flows). When considering the problems of the transfer phenomena, we will judge from the fact that there is an object (reactor, absorber, heat exchanger, etc.), which is the irreversible rheological transition zone (IRT). The input flow (material, thermal, electromagnetic or other) of mass, energy or momentum with the initial determining parameter, for example, concentration Q_0 , temperature T_0 , amplitude A_0 or other is supplied to the input of the object. In the object, there is the irreversible transformation of the determining parameter up to a certain current value of it: Q, T, A, creating the corresponding difference of concentration $\Delta Q = Q_0 - Q$, temperature $\Delta T = T_0 - T$, amplitude $\Delta A = A_0 - A$, that is, the transfer potential, which we will call respectively: $\varphi_Q = \Delta Q$, $\varphi_T = \Delta T$, $\varphi_A = \Delta A$ etc. In the object, as in the IRT zone the corresponding transformation takes place, as a result of which a new determining parameter is formed: concentration of new substance Q_P , temperature T_P , and amplitude A_P . If under zero initial conditions $Q_P = 0$, $T_P = 0$ and $A_P = 0$, then potentials of the output flows will be $\varphi_{PQ} = Q_P$, $\varphi_{PT} = T_P$ and $\varphi_{PA} = A_P$ respectively. Potentials φ_{Pi} of the output flows will be called drains of the i – th determining parameter.

DESCRIPTION OF PROCESSES OF TRANSFER POTENTIAL DISTRIBUTION

Let the material or energy flow with constant potential φ_{0i} is supplied to a technological object (apparatus). This flow creates a heterogeneous physical body (gas, liquid or solid) in the object. Non-uniformity of distribution of the transfer potential $\varphi_i(\bar{r},\theta)$ (\bar{r} - the direction vector of the transfer flow, θ - the transfer time) leads to deviation from the equilibrium state and is the cause of the transfer flow due to the diffusion, convection or convection-diffusion flow. In this field macroscopic motion of the substance is observed, which is characterized by the field of velocities $v(\bar{r},\theta)$. Sources for drain of the transfer potential are characterized by volumetric density [24]. The transfer of the potential $\varphi_i(\bar{r},\theta)$ through the surface of the volume consists of transfer due to substance macroscopic motion and flow q[13]. Thus, equation for the total flow F can be presented as: $\overline{F} = \overline{\varphi}\overline{v} + \overline{q}$. In the integral form, the condition for preserving the potential for the given volume V will be as follows:

$$\int_{V} \frac{\partial \varphi(\bar{r}, \theta)}{\partial \theta} dV = \oint_{S} F d\bar{f} + \int_{V} \gamma_{3} dV , \qquad (18)$$

where: *S* - the volume surface; $d\bar{f} = \bar{n}d\sigma$ - the surface element; \bar{n} - the unit vector of the element; $d\bar{f}$, γ_p - drain velocity of the transformed potential.

For a certain volume, transfer equation (18) is reduced to the following form:

$$\frac{\partial \varphi}{\partial \theta} + div \left(\varphi \overline{v} \right) = -div \overline{q} - \gamma_p = 0 \tag{19}$$

If in the considered volume physicochemical processes take place that are the consequence of interaction of two or more substances, then the potential transfer equation takes the following form:

$$\frac{\partial \varphi_j}{\partial \theta} + div \Big(D_j \nabla \varphi_j \Big) + div \Big(\varphi_j v_j \Big) = \gamma_{pi}, \qquad (20)$$

where: D_j - the effective diffusion coefficient of the i – th substance; ∇ - the linear operator.

If the potential is transferred in one spatial direction *x*, then equation (20) simplifies and takes the form:

$$\frac{\partial \varphi_j}{\partial \theta} + D_j \frac{\partial^2 \varphi_j}{\partial x^2} + v_j \frac{\partial \varphi_j}{\partial x} = \gamma_{pi}.$$
 (21)

Potential drain is nothing but a certain function, depending on the way of output of the transformation result, that is, the potential of the main determining parameter of the output flow from the considered volume and the potentials of other determining parameters that are interrelated with the main one. For more detailed understanding of the essence of equation (21), we consider examples of transfer of the material flow concentration in some technological devices.

Example 1. Let the flow of hot liquid with temperature T_{02} and the flow of cold liquid with temperature T_{0x} flow into the liquid mixer simultaneously. As a result of mixing of two flows, a liquid flow with temperature T_p is formed. When mixing two flows, the temperature of the flow of hot liquid decreases, and of the cold one - increases. As a result, an output flow of liquid with temperature T_p is formed. If cold liquid was supplied into the mixer first, the temperature at the mixer output will increase. If hot liquid was supplied into the mixer first, the output temperature will decrease. However, in both cases, the temperature will change according to the same laws. Thus, in the mixer, as a control object, there is a certain volume in which temperature T_{02} and T_{0x} transformation takes place. This transformation occurs due to the convectiondiffusion process between two input flows. Such a process is described by the well-known Fourier heat energy transfer equations. If mixing proceeds in the direction of a certain spatial coordinate x, such a process is described by the left-hand side of equation (21). This conditional liquid volume, in which heat transfer from one flow to another occurs, is called the rheological transition zone (RTZ). As mentioned above, a new flow with temperature T_p is formed in this zone. If the mixing process in the RTZ is determined by the initial temperatures of the input flows, the effective coefficient of diffusion D_j and the linear mixing rate v_j of the flows, then the output flow is determined only by the change rate of temperature T_p , that is, time constant. Since the temperature of the output flow can only increase to temperature T_{0z} or decrease only to temperature T_{0x} , then the heating (or cooling) process will follow the aperiodic law. If the initial temperature is T_{0x} , the process of heating the liquid output flow will be described by the following equation:

$$T_p(t) = T_{0x} \left[1 - \exp(t/\tau_{_H}) \right]$$
(22)

where: *t* - the current time of liquid heating: τ_{H} - heating constant of the output flow.

If the initial temperature is T_{02} , the process of cooling of the output flow will be described by the following equation:

$$T_p(t) = T_{02} \exp(t/\tau_{_H}). \tag{23}$$

Time constants in both cases will be the same. At a certain given volume V of liquids mixing and the flow rate F_p of the output flow, the time constant can be found by the following formula: $\tau_n = V/F_p$. It is not difficult to see from equations (22) and (23) that the output rate of the output flow with temperature $T_p(t)$ is described by the following differential equation:

$$\gamma_{pi}(t) = \tau_{\scriptscriptstyle H} \frac{d^2 T_p(t)}{dt^2} + \frac{d T_p(t)}{dt}, \qquad (24)$$

Thus, the process of rheological transformation of the temperature of the input flows into the temperature of the output flow will be described by the following nonlinear differential equation:

$$\frac{\partial \varphi_j(x,\theta)}{\partial \theta} + D_j \frac{\partial^2 \varphi_j(x,\theta)}{\partial x^2} + v_j \frac{\partial \varphi_j(x,\theta)}{\partial x} =$$

$$= \tau_n \frac{d^2 \varphi_p(t)}{dt^2} + \frac{d \varphi_p(t)}{dt}$$
(25)

where: $\varphi_p(t)$ - change in the output temperature potential in time t.

Example 2. The fresh solution for evaporation with concentration Q_{0j} and superheated steam to ensure the temperature balance of the evaporation process with temperature T_{0j} are supplied to the evaporation plant under vacuum. The evaporated solution is output from the plant with concentration Q_p . As it is shown in [25], the process of evaporation of liquid solutions has four variable output coordinates, which include concentration of the evaporated solution, the boiling temperature of the

solution, the pressure of the secondary steam, and the solution level in the bottom of the plant. All output coordinates are interrelated and in the first approximation they are described by differential equations of the fourth order in the following form:

$$\tau_{4}^{4} \frac{d^{4} \varphi_{pi}(t)}{dt^{4}} + \tau_{3}^{3} \frac{d^{3} \varphi_{pi}(t)}{dt^{3}} + \tau_{2}^{2} \frac{d^{2} \varphi_{pi}(t)}{dt^{2}} + \tau_{1} \frac{d \varphi_{pi}(t)}{dt} + \varphi_{pi}(t) = k_{pi} \varphi_{ji}$$
(26)

where: τ_1 , τ_2 , τ_3 , τ_4 - conversion time constants; φ_{pi} potential of the *i*-th output coordinate; k_i transformation equivalent coefficient; φ_{ji} potential of
the *i*-th input coordinate.

If the processes of transfer of all input flows are characterized by a convective component moving with velocity v_{ii} , then equation (20) takes the following form:

$$\frac{\partial \varphi_{ji}(x,\theta)}{\partial \theta} + D_j \frac{\partial^2 \varphi_{ji}(x,\theta)}{\partial x^2} + v_{ji} \frac{\partial \varphi_{ji}(x,\theta)}{\partial x} = k_{pi} \sum_{i=0}^{4} \tau_i \frac{d^{i+1} \varphi_{pi}(t)}{dt^{i+1}}$$
(27)

In this case, the zone of rheological transitions, and, correspondingly, transformations, will be the evaporation plant. Drain for concentration of the evaporated solution is consumption of bottom liquid; for the temperature consumption of the secondary steam and bottom liquid; for pressure of the secondary steam - consumption of its condensate and still bottom; for the liquid level in the plant - consumption of the fresh solution and bottom liquid. Since all output coordinates of the evaporation plant are interrelated, each corresponding channel will be described by the equation of type (27). Since the evaporation plant belongs to a multidimensional technological control object with two input determining flows - the fresh solution and the primary steam, then its output coordinates can be divided into two groups of determining channels according to the principle of influence of the input flows on them:

group 1:

- concentration of the evaporated solution - consumption of the fresh solution;

group 2:

- the boiling point of the fresh solution – the primary steam consumption;

- the evaporated solution level – the primary steam consumption;

- the secondary steam pressure – the primary steam consumption.

A characteristic feature is that between the output parameters of group 1 and group 2 there is generally an extremum that can be used to optimize the control process. Proceeding from the above, the evaporation plant can be described by a system of the following nonlinear differential equations [25]:

- by concentration of the target component in still bottom:

$$\frac{\partial Q_{cp}(x,\theta)}{\partial \theta} + D_{cp} \frac{\partial^2 Q_{cp}(x,\theta)}{\partial x^2} + v_{cp} \frac{\partial Q_{cp}(x,\theta)}{\partial x} = k_{pQ} \sum_{i=0}^4 \tau_{iQ} \frac{d^{i+1}Q_k(t)}{dt^{i+1}}; \quad (28)$$

- by the boiling point of the solution in the plant still:

$$\frac{\partial T_{\Pi}(x, \theta)}{\partial \theta} + D_j \frac{\partial^2 T_{\Pi}(x, \theta)}{\partial x^2} + v_{ji} \frac{\partial T_{\Pi}(x, \theta)}{\partial x} = k_{pP} \sum_{i=0}^4 \tau_{iT} \frac{d^{i+1}T_k(t)}{dt^{i+1}}; \quad (29)$$

- by the level of the evaporated solution in the plant still:

$$\frac{\partial T_{\Pi}(x, \theta)}{\partial \theta} + D_{j} \frac{\partial^{2} T_{\Pi}(x, \theta)}{\partial x^{2}} + v_{ji} \frac{\partial T_{\Pi}(x, \theta)}{\partial x} = k_{pL} \sum_{i=0}^{4} \tau_{iL} \frac{d^{i+1} L_{k}(t)}{dt^{i+1}}; \quad (30)$$

- by the secondary steam pressure:

$$\frac{\partial T_{\Pi}(x,\theta)}{\partial \theta} + D_{j} \frac{\partial^{2} T_{\Pi}(x,\theta)}{\partial x^{2}} + v_{ji} \frac{\partial T_{\Pi}(x,\theta)}{\partial x} = k_{pP} \sum_{i=0}^{4} \tau_{iP} \frac{d^{i+1} P_{\Pi}(t)}{dt^{i+1}}; \quad (31)$$

During the study of multidimensional control objects, like evaporation plants, absorption and rectification columns, reactors, it is considered that some output parameters can be stabilized by changing the output flows. For example, the level is stabilized by changing consumption of the still bottom, the secondary steam pressure – by its condensation with the refrigerant, the boiling point - by change in the primary steam consumption. Change in the temperature of the primary steam and concentration of the target component in the fresh solution in evaporation plants and rectification columns are generally assumed to be influential parameters.

Thus, in presence of *n* interconnected coordinates that are formed during transfer of the input potential of the determining parameter, the right-hand side of equation (27) will be described by the differential equation of the *n*-th order. The objective of research is to find the analytical solution of the obtained nonlinear differential equation. In most practical cases, it can be assumed that convection-diffusion transfer time θ is equal to drain time *t*, and velocity of the potential transfer process is $v_{ji} = \partial x / \partial t$. Then, having determined that $\partial x^2 = v_{ji}^2 \partial t^2$, equation (25) takes the form:

$$\frac{\partial \varphi_{j}(t)}{\partial t} + \frac{D_{j}}{v_{ji}^{2}} \frac{\partial^{2} \varphi_{j}(t)}{\partial t^{2}} + \frac{\partial \varphi_{j}(t)}{\partial t} =$$

$$= \tau_{\mu} \frac{d^{2} \varphi_{p}(t)}{dt^{2}} + \frac{d \varphi_{p}(t)}{dt}$$
(32)

If the potential of the output flow is proportional to the potential of the determining parameter of the input flow, then having defined that $\varphi_j(t) = k_{\Pi}\varphi_p(t)$, we arrive at the following linear differential equation

$$\tau_{\Pi} \frac{d\varphi_p(t)}{dt} + \varphi_p(t) = k_{\Pi} \varphi_{0j}, \qquad (33)$$

where $\tau_{\Pi} = (k_{\Pi}D_j + \tau_{\mu}v_{ji}^2)/v_{ji}^2(k_{\Pi} + 2)$ - transfer time constant; k_{Π} - the transfer coefficient; φ_{0j} - the nominal

potential of the determining parameter of the input flow. Under initial zero conditions, we obtain the following mathematical model of the transfer process in the

following analytical form:

$$\varphi_{p}(t) = k_{\Pi} \left[1 - \exp\left(-t \frac{v_{ji}^{2} (k_{\Pi} + 2)}{(k_{\Pi} D_{j} + \tau_{\mu} v_{ji}^{2})} \right) \right]. \quad (34)$$

If the transfer time of the potential is not equal to the drain time, then the process of rheological transformation is clearly nonlinear and for the analytical solution of such problems it is necessary to use other boundary conditions and other methods that will be described in the following publications.

CONCLUSIONS

When transferring the potential of mass, energy and momentum pulse, their balance must be struck, that is, the amount of the potential of the determining parameter that came with the input flow into a certain medium should be equal to the amount of the potential of this parameter that is created in this medium.

1. The right-hand side of this equation describes the process of transferring the potential from the source of the determining parameter (mass, energy, momentum) to the medium and shows how this transfer is carried out - by diffusion, convection or convection-diffusion.

2. If we multiply this equation by certain transfer time, then we arrive at the equation of balance of the potential of the determining parameter (mass, energy and momentum), the right-hand side of which describes how much of this parameter was formed during transformation of the input determining parameter.

3. It is shown that for transformation of the input determining parameter, some space is needed, which is called the rheological transformation zone. In this zone, one or several transformations can take place simultaneously, which are interrelated. Such transformations are described by nonlinear equations of the second and higher orders, which in general form do not have a solution. However, for some potential transfer processes, such nonlinear differential equations can have an analytical and fairly simple solution.

4. The methods for solving such equations will be described in subsequent publications.

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МАТЕМАТИЧЕСКИЕ МОДЕЛИ РЕОЛОГИЧЕСКИХ ПРЕОБРАЗОВАНИЙ ПРИ ПЕРЕНОСЕ ПОТЕНЦИАЛА ИМПУЛЬСА МАССЫ, ЭНЕРГИИ И КОЛИЧЕСТВА ДВИЖЕНИЯ

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Аннотация. Рассматриваются процессы диффузионно-конвекционного переноса потенциала определяющего параметра, которым может быть импульс массы, энергии или количества движения. что Показано. существующие методы математического моделирования, как правило рассматривают процессы переноса массы, тепловой энергии и количества движения от источника бесконечной мощности к некоторому неограниченному объему при начальных условиях первого, второго или третьего рода. Кроме того, принимается, что зона физического перехода потенциала бесконечно малая. При этом получаются очень сложные математические модели, зачастую непригодные к практическому использованию. Показано, что зоной реологического перехода может быть технологический аппарат, например, реактор, выпарная установка, абсорбер, ректификационная колонна и др., в которых происходит не только преобразование основного определяющего параметра, но и создание новых. То-есть процесс преобразования при диффузионно-конвекционном переносе может быть многостадийным с одновременным выходом (стоком) определяющих параметров – концентрации. тепловой энергии, лавления И дp. Лля математического описания процессов переноса потенциала используются известные законы явлений переноса, которые дополнены соответствующими законами стока созданного потенциала. Показано, что в этом случае преобразования входного потенциала описывается нелинейными дифференциальными уравнениями, которые при определенных условиях могут быть решены аналитически.

Ключевые слова: потенциал, перенос, преобразование, сток, модель.