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Study of mathematical model of dynamics combined machine-tractor unit

R. Antoshchenkov, L. Tishchenko, A. Lebedev

Kharkiv Petro Vasylenko National technical university of agriculture,
e-mail: roman.tiaxntusg@gmail.com

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Summary. Agricultural machinery and tractor units studied as multi-element mobile machines in this article. The combined sowing unit consists of three elements, such as a tractor, the capacity for seed and sowing machines that move in succession. Known layout diagrams sowing units that have the capacity and the drill can change the sequence of location or capacity of the seed can be on the tractor and be rigidly connected with it. The dynamics of multi-machine data remains under-investigated. To study the dynamics of multi-machine use the Lagrange equation of the 2nd kind. Mathematical model of spatial movement of mobile machines are complex, and the study of the dynamics of multi-machine requires significant computing resources. The paper reviewed and investigated the spatial dynamic model of the combined sowing machine and tractor unit. For a mechanical system with a spatial movement of the units of dynamic equations are represented in the matrix form. The kinematic parameters are generated automatically by the software and differential and kinematic structures. The dynamic equations of a nonholonomic system can be obtained by a linear combination of the equations of the dynamics of holonomic systems with coefficients taken from the linear form. For numerical integration obtained in the system of ordinary differential equations convert them to normal Cauchy form in generalized coordinates or pseudo coordinates. The results of theoretical research of mathematical models of the dynamics of the combined machine-tractor unit as an example of the unit John Deere8345R + John Deere 1910 + John Deere 1895. Modes of motion, velocity components of the unit, and the path of movement, speed and dynamic wheel radius are study in this article.

Key words: mathematical model, dynamics, tractor, hopper, seeder.

INTRODUCTION

As studied in previous researches an agricultural machinery and tractor units are multi-element mobile machines. The combined sowing unit consists of three elements, such as a tractor, the capacity for seed and sowing machines that move one after the other [1]. Basic layout diagrams sowing units that have the capacity and the drill can change the sequence of location [2], or the capacity for seed can be on the tractor and be rigidly connected with it. The dynamics of multi-machine data remains not sufficiently studied.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

To study the dynamics of multi-machine apply the principle of d'Alembert-Lagrange equation [3] or the Lagrange equation of the 2nd kind. [4] It is known [5], in which the movement of the mobile machine read in conjunction with the semi-trailer with the help of Lagrange equations of the 2nd kind. A mathematical model of the motion of one machine [6, 7, 8] has been repeatedly investigated. In [9-12] the dynamics and stability of the mobile machine. Agricultural machines and units in studies of the dynamics presented in the form of one, two and three-mass model in the works [13-16]. Mathematical model of spatial movement of mobile machines are complex [17] and the study of the dynamics of multi-machine requires significant computing resources [18].

In these works mathematical model of the machine is an integrated multi-element and change the structure or internal communications, you need to rebuild it anew, resulting in increased labor costs and time to study. It follows that for the correct solution of problems of the dynamics of nonholonomic multiple systems is necessary to form the basic equations of dynamics [19] and justify the equation of communication, as is done in the example in [20] for the plane-parallel movement of the machine-tractor unit.

OBJECTIVES

The aim of this work is a theoretical study of the spatial mathematical model of dynamics of multi-element combination of machine and tractor unit.

THE MAIN RESULTS OF THE RESEARCH

On the sidelines of the Ukrainian widespread sowing combined machine and tractor units of production John Deere (Fig. 1).

Fig. 1. Combined sowing machine tractor unit
John Deere 8345R + John Deere 1910 + John Deere 1895

To select the optimal modes of motion and aggregation need to study its dynamics. To solve the problem, consider the spatial dynamic model combined seeding machine-tractor unit, which is shown in Fig. 2 and use the following notation: n – upper index value indicates the receiving affiliation variable element of the unit $T, Б, C$, respectively the tractor, hopper, seeder; $XOYZ$ – global coordinate system; $xoyz^n$ – associated coordinate system; p. o^n – the center of gravity; p. O – the center of the global coordinate system; $\alpha^n, \beta^n, \gamma^n$ – rotation angles about respective axes x, y, z; m^n – the mass of the unit cell; J_x^n, J_y^n, J_z^n – given the moments of inertia to the respective axes; $\overline{\upsilon}$ – forward speed of movement; D_{hf}^n, D_{hr}^n – front and rear hinge point (accession process equipment); $P\kappa_{ij}^n$, $M\kappa_{ij}^n$ $N\kappa_{ij}^n$ – tangential thrust, torque and normal reaction to the appropriate wheel assembly; $\omega\kappa_{ij}^n$ – wheels speed; $C\text{ш}_{ij}^n$, $k\text{ш}_{ij}^n$ – given tire stiffness and compliance elements of the unit.

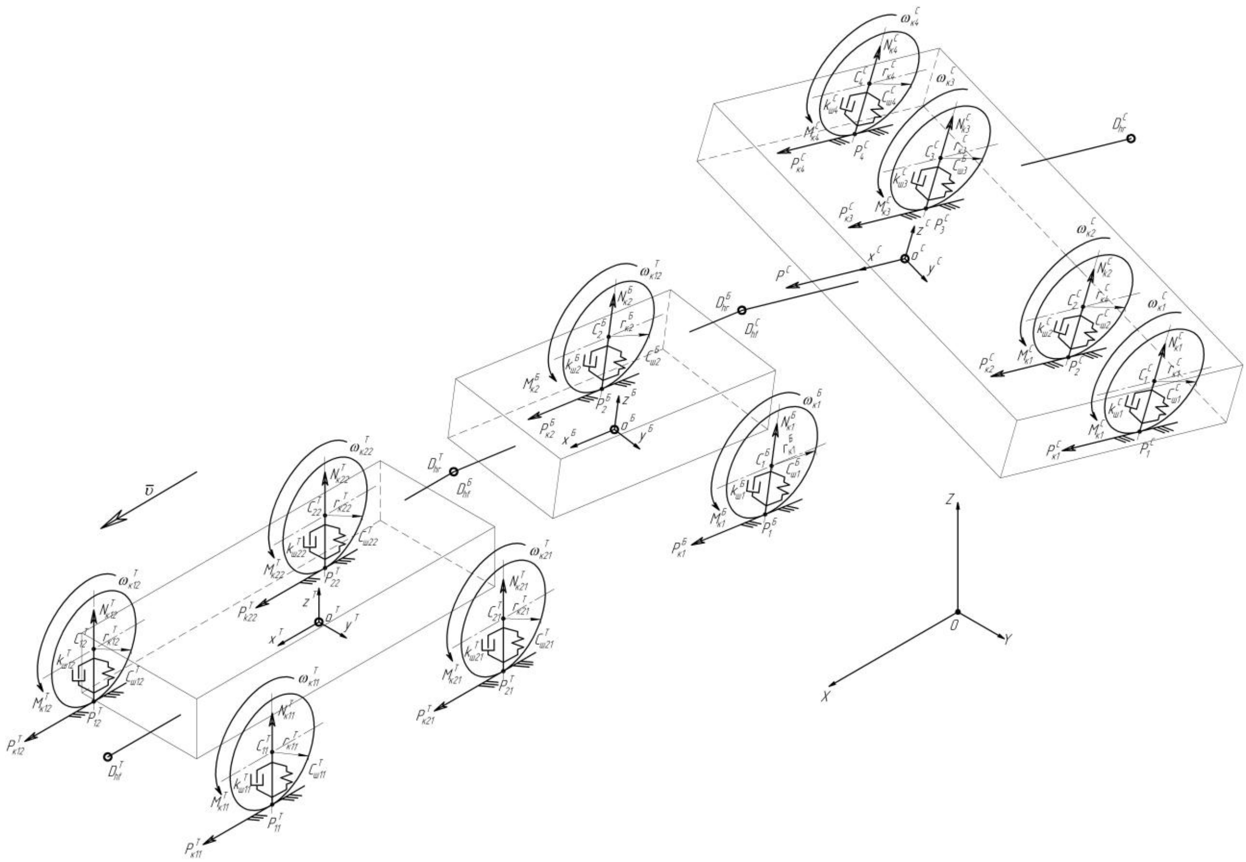

Fig. 2. Dynamic multi-element model of combined machine-tractor unit.

Formation of mathematical models of the dynamics is performed according to the following methodology. For a mechanical system with a spatial movement of the units of dynamic equations are presented, in the form [19]:

$$\mathbf{U} = \sum_{i=1}^{n} \left\{ \tilde{\mathbf{W}}_{C_i}^T m_i \vec{a}_{C_i} + \tilde{\mathbf{W}}_{\omega_i}^T \left(\left[\vec{J}_i\right] \cdot \vec{\varepsilon}_i + \vec{\omega}_i \times \left[\vec{J}_i\right] \cdot \vec{\omega}_i \right) \right\} - \quad (1)$$
$$- \tilde{\mathbf{W}}_P^T \mathbf{P} = 0,$$

where: n – number of solid bodies in the model, m_i, $\left[\vec{J}_i\right]$, $\vec{a}_{C_i}$, $\vec{\omega}_i$, $\vec{\varepsilon}_i$ – mass, inertia tensor, acceleration of the center of mass, angular velocity and angular acceleration of the i-th body, $\tilde{\mathbf{W}}_{C_i}$, $\tilde{\mathbf{W}}_{\omega_i}$ - structural matrix the radius vector of the center of mass and angular velocities of the bodies, the formulas for which are given below.

Vectors $\vec{a}_{C_i}$ are set in the absolute coordinate system, and vectors $\vec{\omega}_i$, $\vec{\varepsilon}_i$ – a body-related coordinate system (usually the main axes of inertia axes). The structural matrix $\tilde{\mathbf{W}}_{C_i}$, $\tilde{\mathbf{W}}_{\omega_i}$ may be generated either through the matrix $\mathbf{G}$ of expression (2), or by direct differentiation pseudorange similarly flat case:

$$\tilde{\mathbf{W}}_{C_i}^u = \frac{\partial \vec{r}_{C_i}}{\partial \boldsymbol{\pi}} = \frac{\partial \vec{v}_{C_i}}{\partial \dot{\boldsymbol{\pi}}} = \mathbf{W}_{C_i}^u \mathbf{G},$$

$$\tilde{\mathbf{W}}_{\omega_i}^u = \frac{\partial \vec{\omega}_i^{(i)}}{\partial \dot{\boldsymbol{\pi}}} = \mathbf{W}_{\omega_i}^u \mathbf{G}.$$

Thus, the kinematic parameters $\vec{a}_{C_i}, \vec{\omega}_i$, $\vec{\varepsilon}_i$ are also generated automatically by geometric and differential structures:

$$\ddot{\vec{r}}_{C_i} = \frac{d}{dt}\left(\frac{\partial \vec{r}_{C_i}}{\partial \mathbf{q}} \dot{\mathbf{q}} + \frac{\partial \vec{r}_{C_i}}{\partial t} \right) = \frac{d}{dt}\left[\mathbf{W}_{C_i}^u \left(\mathbf{G} \dot{\boldsymbol{\pi}} + \boldsymbol{\gamma} \right) + \frac{\partial \vec{r}_{C_i}}{\partial t} \right] = ,$$
$$= \tilde{\mathbf{W}}_{C_i}^u \ddot{\boldsymbol{\pi}} + \ldots$$

$$\vec{\omega}_i^{(i)} = \mathbf{W}_{\omega_i}^u \dot{\mathbf{q}} = \widetilde{\mathbf{W}}_{\omega_i}^u \dot{\boldsymbol{\pi}} + \ldots,$$

$$\vec{\varepsilon}_i^{(i)} = \frac{d}{dt}\left(\widetilde{\mathbf{W}}_{\omega_i}^u \dot{\boldsymbol{\pi}} + \ldots\right) = \widetilde{\mathbf{W}}_{\omega_i}^u \ddot{\boldsymbol{\pi}} + \ldots.$$

The summands in (1) generated by the force elements are formed similarly to the case of flat formulas, depending on how you defined their structure.

When using the matrix $\mathbf{G}$ in the above formulas, one (in the form of a transposed) is a common factor, and which can be taken from the left. In the three-dimensional case we obtain the equations of the form:

$$\mathbf{G}^T \left(\begin{array}{l} \sum_{i=1}^{n} \left\{ \begin{array}{l} \mathbf{W}_{C_i}^{u\,T} m_i \vec{a}_{C_i} + \\ + \mathbf{W}_{\omega_i}^{u\,T} \left(\begin{array}{l} \left[\vec{J}_i^{(i)}\right] \cdot \vec{\varepsilon}_i^{(i)} + \\ + \vec{\omega}_i^{(i)} \times \left[\vec{J}_i^{(i)}\right] \cdot \vec{\omega}_i^{(i)} \end{array} \right) \end{array} \right\} - \\ - \mathbf{W}_P^T \mathbf{P} \end{array} \right) = 0. \qquad (2)$$

It follows that the dynamic equations of a nonholonomic system can be obtained by a linear combination of the equations of the dynamics of holonomic systems with coefficients taken from the linear form, expressing generalized velocities through independent generalized velocities (pseudo velocity) (2). The resulting equations are actually a vector-matrix form known in the analytical mechanics “equations of nonholonomic systems with speed-dependent excluded” [19, 20].

Note also that equation (2) up to notation structural matrices coincide with the equations for holonomic systems, and differ only in the formation of the structural matrix, they included. There are two algorithms for automatic generation of equations of motion of nonholonomic systems for solids.

In the first foundation laid by the fact that the dynamic equations of nonholonomic system represents a certain combination of equations drawn up for a holonomic system. Therefore, at first an equation of motion of the system without taking into account the kinematic structures (1). Followed by their linear combination with non-zero coefficients of **G** formed by equation (8).

The second algorithm [19, 21] is based on the direct calculation of the coefficients of the structural matrix for a nonholonomic system by differentiating the structures and the formation of equations pseudovelocity replacement operations there with generalized speeds transactions with pseudovelocity.

In both algorithms for the system of equations must be supplemented by the equations (2) and the kinematic parameters instead of each of the linear and angular acceleration and angular velocity substitute expression obtained by differentiating the structures in time with regard to (2).

Direct problem of the dynamics of a mechanical system is to determine the motion (in generalized or pseudo-coordinates) under the action of the applied forces. The problem is reduced to the integration of systems of ordinary differential equations (lime) together with (1) or (2) for nonholonomic mechanical systems with given initial conditions.

For numerical integration obtained in the SODE consider algorithms for their transformation to the normal form of Cauchy in generalized coordinates or pseudo coordinates. For holonomic systems introduce the vectors of generalized acceleration and velocity $\mathbf{w} = \dot{\mathbf{v}} = \ddot{\mathbf{q}}$, $\mathbf{v} = \dot{\mathbf{q}}$ – and rewrite the equation (2) in the form of:

$$\mathbf{M}\mathbf{w} = \mathbf{F}, \qquad (3)$$

where: $\mathbf{M} = \sum_{i=1}^{n} \left\{ \mathbf{W}_{C_i}^T m_i \mathbf{W}_{C_i} + \mathbf{W}_{\omega_i}^T \left[\vec{J}_i\right] \mathbf{W}_{\omega_i} \right\}$ – matrix of inertia of the system, $\mathbf{F}$ – the vector-matrix generalized forces of the system minus the terms of the inertial terms on the left side, non-generalized acceleration, which can be obtained by substituting the equations of motion of analytical expressions pseudo acceleration zero and taking the results with the opposite sign, i.e.:

$$\mathbf{F} = -\mathbf{U}\big|_{\ddot{\boldsymbol{\pi}}=0}.$$

After allowing the system (3) with respect to the generalized accelerations – $\mathbf{w} = \mathbf{M}^{-1}\mathbf{F}$ finally we get the ash in the form of Cauchy:

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{M}^{-1}\mathbf{F} \end{cases}. \qquad (4)$$

Similarly, for the systems described in pseudocoordinates (in generalized coordinates and pseudovelocity) and for nonholonomic systems form a Cauchy obtain:

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{G}\mathbf{v} + \mathbf{g} \\ \dot{\mathbf{v}} = \mathbf{M}^{-1}\mathbf{F} \end{cases}. \qquad (5)$$

The first equation (1), when $\mathbf{v} = \dot{\boldsymbol{\pi}}$, the same as the expression of the dependence of the generalized velocities through pseudovelocity.

The initial conditions for the system are the values of generalized coordinates and generalized velocities independent (pseudovelocity) at the initial time:

$$\mathbf{q}\big|_{t=0} = \mathbf{q}_0, \qquad \boldsymbol{\pi}\big|_{t=0} = \boldsymbol{\pi}_0.$$

Build in an analytical form inverse matrix of inertia $\mathbf{M}^{-1}$ is not possible. In the numerical integration at every step by the time the matrix of inertia is calculated from the values of generalized coordinates in the previous step and to calculate the left side of the bottom of vector equations (4) and (5) solve systems of linear equations by Kraut.

A dynamic model of a multi-element of combined machine-tractor unit (Fig. 2) has eight generalized coordinates, ie, eight degrees of freedom:

$$\dot{\mathbf{q}} = \left[X^T, Y^T, Z^T, \beta^T, \alpha^Б, \beta^Б, \alpha^C, \beta^C\right]^T. \qquad (6)$$

As an independent coordinate with dependent variations selected:

$$\dot{\mathbf{v}} = \begin{bmatrix} \alpha^T, \gamma^T, \varphi\kappa_{11}^T, \varphi\kappa_{12}^T, \varphi\kappa_{21}^T, \varphi\kappa_{22}^T, \gamma^Б, \\ \varphi\kappa_{11}^Б, \varphi\kappa_{12}^Б, \gamma^C, \varphi\kappa_1^C, \varphi\kappa_2^C, \varphi\kappa_3^C, \varphi\kappa_4^C \end{bmatrix}^T . \qquad (7)$$

Then a mathematical model of dynamics of multi-element machine-tractor unit has the form:

$$\begin{cases} \dot{X}^T = f_1(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{Y}^T = f_2(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{Z}^T = f_3(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \beta^T = f_4(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{\alpha}^Б = f_5(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{\beta}^Б = f_6(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{\alpha}^C = f_7(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{\beta}^C = f_8(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}), \end{cases} \qquad (8)$$

where: f_i – a function of the vector-matrices $\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}$; $i = 1,\ldots 8$ – ordinal number generalized coordinates.

Equations independent coordinates with dependent variations has the form:

$$\begin{cases} \dot{\alpha}^{\mathrm{T}} = \dfrac{a^{\mathrm{T}}\dot{X}^{\mathrm{T}} + b^{\mathrm{T}}\dot{Y}^{\mathrm{T}} + c\dot{Z}^{\mathrm{T}} - \beta^{\mathrm{T}}\left(d^{\mathrm{T}}\cos\gamma^{\mathrm{T}} + \sin\gamma^{\mathrm{T}}\right)}{\cos\gamma^{\mathrm{T}} - d^{\mathrm{T}}\sin\gamma^{\mathrm{T}}}; \\ \dot{\gamma}^T = \alpha^T\beta^T + vB_x^T\dfrac{\mathrm{tg}\,\gamma^T}{l^T}; \\ \dot{\varphi\kappa}_{11}^T = \dfrac{vC_{11_{xy}}}{Z_{C_{11}}^T};\ \dot{\varphi\kappa}_{12}^T = \dfrac{vC_{12_{xy}}}{Z_{C_{21}}^T}; \\ \dot{\varphi\kappa}_{21}^T = \dfrac{vC_{21_{xy}}}{Z_{C_{21}}^T};\ \dot{\varphi\kappa}_{22}^T = \dfrac{vC_{22_{xy}}}{Z_{C_{22}}^T}; \\ \dot{\gamma}^Б = \dfrac{vyA^Б}{l_1^Б - hf_x^Б} + \alpha^Б\beta^Б; \\ \dot{\varphi\kappa}_1^Б = \dfrac{vC_{1x}^Б}{Z_{C_1}^Б};\ \dot{\varphi\kappa}_2^Б = \dfrac{vC_{2x}^Б}{Z_{C_1}^Б}; \\ \dot{\gamma}^C = \dfrac{vyA^C}{l_1^C - hf_x^C} + \alpha^C\beta^C; \\ \dot{\varphi\kappa}_1^C = \dfrac{vC_{1x}^C}{Z_{C_1}^C};\ \dot{\varphi\kappa}_2^C = \dfrac{vC_{2x}^C}{Z_{C_2}^c}; \\ \dot{\varphi\kappa}_3^C = \dfrac{vC_{3x}^C}{Z_{C_3}^C};\ \dot{\varphi\kappa}_4^C = \dfrac{vC_{4x}^C}{Z_{C_4}^C}. \end{cases} \qquad (9)$$

Thus a dynamic model of spatial movement machine-tractor unit consists of equations (8) and (9), which are formed using methodology (1) - (5).

RESULTS SOLVING OF MATHEMATICAL MODEL

Consider the results of theoretical studies of the mathematical model of the dynamics of the combined machine-tractor unit as an example of the unit John Deere 8345R + John Deere 1910 + John Deere 1895. The results of the simulation are shown in fig. 3 - 9.

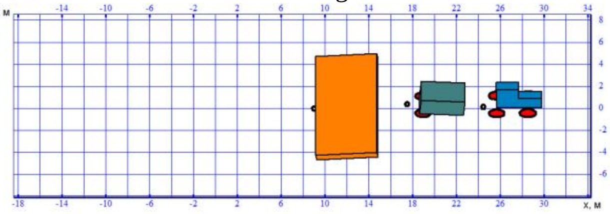

Fig. 3. Simulation of the movement of the unit

Modeling the spatial movement of the unit volume primitives are shown in Fig. 3. In the straight running of the tractor calculate the coordinates of the centers of mass of the unit (Fig. 4), respectively, the following elements of the tractor moving in a straight line.

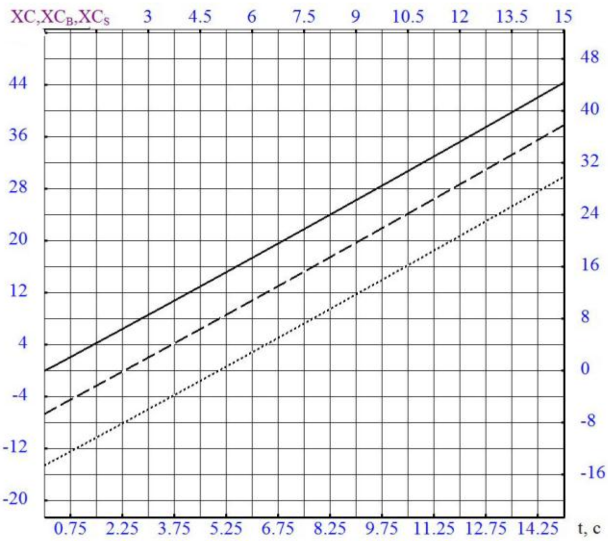

Fig. 4. The coordinates of the centers of mass of the unit in straight running (—— X^T, – – – $X^Б$, · · · · X^C)

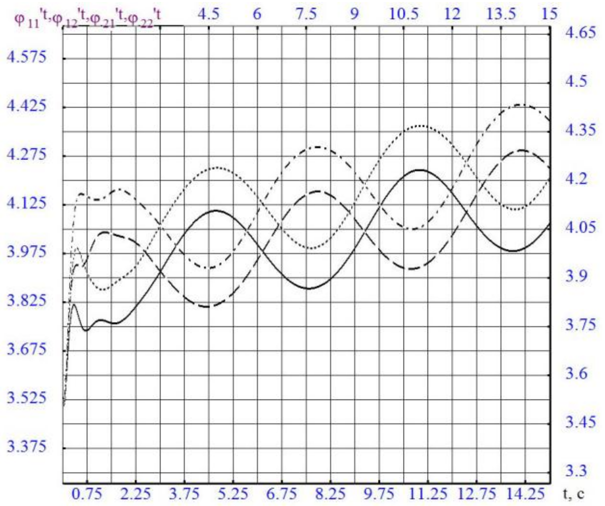

Fig. 5. Wheels speed^ (—— $\dot{\varphi\kappa}_{11}^T$, – – – $\dot{\varphi\kappa}_{12}^T$, · · · · $\dot{\varphi\kappa}_{21}^T$, – · – · – $\dot{\varphi\kappa}_{22}^T$).

Consider the case of motion of the machine and tractor unit in the field to manage the movement of mechanic hand and a constant speed. The speed of rotation of wheels of the tractor shown in fig. 5. In accordance with the manipulated variable speed mechanic wheels are in the form of harmonic oscillations (Fig. 6).

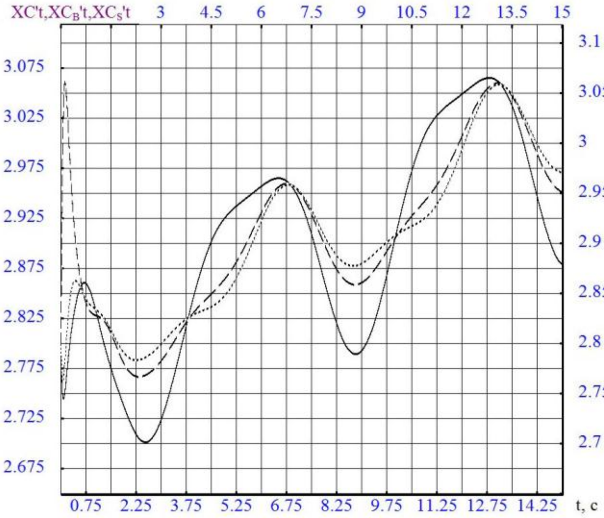

Fig. 6. Velocity of the center of mass of the unit under control the movement (—— $\dot{X}^T$, – – – $\dot{X}^Б$, · · · · $\dot{X}^C$).

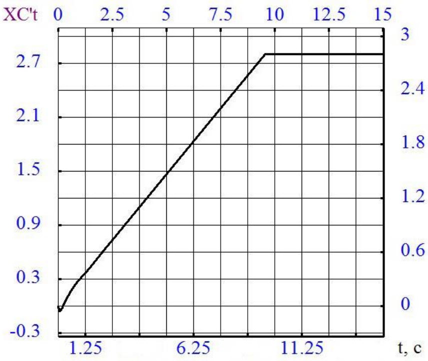

Fig. 7. The translational speed of the tractor during acceleration ($\dot{X}^T$).

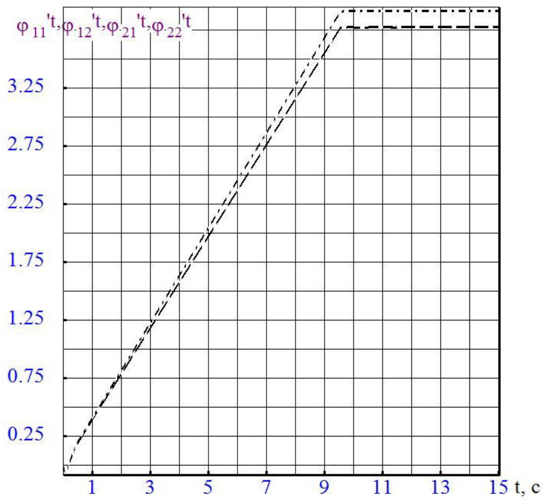

Fig. 8. Tractor wheel speed during acceleration (—— $\dot{\varphi}\kappa^T_{11}$, – – – $\dot{\varphi}\kappa^T_{12}$, · · · · $\dot{\varphi}\kappa^T_{21}$, – · – · – $\dot{\varphi}\kappa^T_{22}$)

Acceleration of the unit is carried out up to speed of 2.8 m/s, which corresponds to agrotechnical requirements of 10 km/h (fig. 7).

The speed of rotation of wheels of the tractor are different for the front and rear axles, but equally on the sides $\dot{\varphi}\kappa^T_{11} = \dot{\varphi}\kappa^T_{12}$, $\dot{\varphi}\kappa^T_{21} = \dot{\varphi}\kappa^T_{22}$ (Fig. 8).

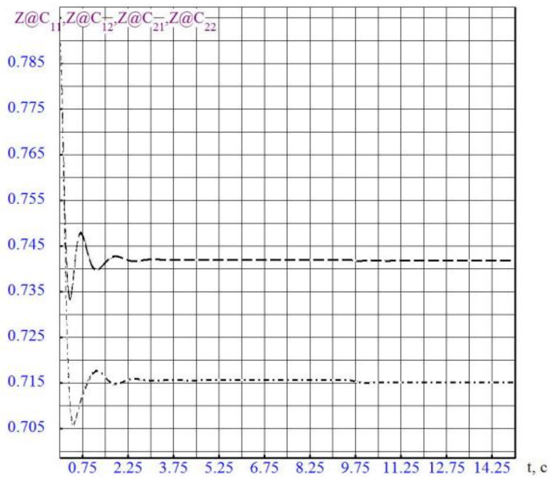

Fig. 9. Dynamic radius of the tractor wheels during acceleration (—— $r^T_{д11}$, – – – $r^T_{д12}$, · · · · $r^T_{д21}$, – · – · – $r^T_{д22}$).

The mathematical model allows the simulation process to determine the deformation of the tire and the dynamic radius of the wheel (Fig. 9). During the movement of the tractor tires deform at 0.06-0.09 m.

CONCLUSIONS

1. The approach proposed in this paper can reduce labor expenditure and time for modeling the spatial movement of multi-mobile machines. This methodology allows you to build a mathematical model with minimal resources to make changes in the mathematical formalism of the test process. If you change the structure of the investigated multiple-mobile machine only change constraint equation, which reduces the cost of developing a mathematical model.

2. The results of theoretical studies of the mathematical model of the dynamics of the combined machine-tractor unit as an example of the unit John Deere 8345R + John Deere 1910 + John Deere 1895. Certain elements of speed and deflection unit allow further investigation of the stability of motion.

3. The leading wheels of the tractor are deformed under the influence of the traction force. The deformation of the wheels was 0.06-0.09 m, which is necessary for the subsequent simulation of slipping.

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ИССЛЕДОВАНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ДИНАМИКИ КОМБИНИРОВАННОГО МАШИННО-ТРАКТОРНОГО АГРЕГАТА

Р. Антощенков, Л. Тищенко, А. Лебедев

Аннотация. В работе сельскохозяйственные машинно-тракторные агрегаты исследуются как многоэлементные мобильные машины. Комбинированные посевные агрегаты состоят из трёх элементов, таких как трактор, ёмкость для посевного материала и сеялки, которые располагаются последовательно друг за другом. Известны компоновочные схемы посевных агрегатов, у которых ёмкость и сеялка могут менять последовательность расположения или ёмкость для посевного материала может находиться на тракторе и быть жёстко связана с ним. Динамика таких многоэлементных машин остаётся недостаточно исследованной.

Для исследования динамики многоэлементных машин применяют уравнения Лагранжа 2-го рода. Математические модели пространственного движения мобильных машин являются сложными, а исследование динамики многоэлементных машин требует значительных вычислительных ресурсов. В работе рассмотрена и исследована пространственная динамическая модель комбинированного посевного машинно-тракторного агрегата. Для механической системы с пространственным движением звеньев уравнения динамики представляются в матричном виде. Кинематические параметры программно формируются автоматически по кинематическим и дифференциальным структурам. Уравнения динамики неголономной системы могут быть получены линейной комбинацией уравнений динамики голономной системы с коэффициентами, взятыми из линейной формы. Для численного интегрирования полученной в работе системы обыкновенных дифференциальных уравнения их преобразовывают к нормальной форме Коши в обобщенных координатах или псевдокоординатах.

Приведены результаты теоретических исследований математической модели динамики комбинированного машинно-тракторного агрегата на примере агрегата John Deere 8345R + John Deere 1910 + John Deere 1895.

Определены режимы движения, скорости элементов агрегата, траектории движения, скорости вращения и динамические радиусы колёс.

Ключевые слова: математическая модель, динамика, трактор, бункер, сеялка.