

Dynamics Model of a Vehicle with DC Motor

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Summary. The torque of passive forces was determined, dependent on aerodynamic drag, rolling resistance and slope of the ground. To determine the course of active (driving) forces torque as a function of angular velocity, Kirchhoff's voltage equation was used for the armature circuit. Susceptibility and attenuation of motion transmission systems were taken into account. Kinematic motion parameters were obtained from a solution of equations of drive systems motion. To solve the equations, the Runge-Kutta method was used. The authors designed a simulation program, which allows for the determination of temporary displacement waveforms, velocity and acceleration of the vehicle as well as the consumption of electricity and mechanical work performed depending on the involved motion resistance and input voltage.

Exemplary calculation has been provided. There are formulated conclusions and recommendations resulting from the performed simulation.

Key words: auto, dynamics, kinematics, DC motor, theoretical model.

INTRODUCTION

The first cars with steam powered engines with belt transmission and the wheels with wire-spokes were designed in the second half of the eighteenth century. After nearly a hundred years there was designed a car with a petrol engine suitable for everyday use. The consequences of the rapid development of automotive industry in the twentieth century, such as environmental pollution, increasing prices of liquid and gaseous fuels, have become the main reason for the search of new technical solutions. Currently, hybrid cars are now being produced, powered with a combination of internal combustion and electric engines [11], as well as vehicles with electric motor only.

Publications in the field of automotive industry in general are numerous, and a range of issues discussed is huge. They are considered in many aspects. The subjects of anal-

yses include safety, reliability, cost and range of batteries and even customer reviews on new drive technologies [16]. New designs and materials are proposed and analyzed. The computer simulation method is used in analyzing the geometry and dynamics of motion, dynamics of suspension systems, human-machine relationship systems. The impact of automotive industry on the environment is considered, as well as alternative methods of technical solutions concerning drive systems and their power supply methods. The issues related to the cost-effectiveness of the use of innovative solutions are discussed.

The use of electric vehicles is seen as a way to reduce the liquid fuels consumption and carbon dioxide emissions [18]. Electric cars powered by aluminum-air batteries have zero CO₂ emissions. They are made of materials that can be recycled, and vehicles can cover a distance up to 1600 km [4]. However, as long as solid fuels are used to generate electricity, as is the case in Poland, greenhouse gas emissions resulting from the use of electric vehicles, in which one needs to recharge the batteries – will be greater than the emissions generated by modern internal combustion engines [17].

There are works underway on the construction of high performance and lightweight electric battery. Previously used heavy lead-acid batteries are replaced with alkaline, NiCd and NiMH, and Li-ion and Li-polymer batteries. Being in the research phase, the metal-air batteries, instead of the conventional cathode have air electrode. The use of such solutions can substantially reduce the overall weight of the vehicle [7]. In the hybrid electrical energy storage systems in an electric vehicle supercapacitors are also used [9]. A light and high-performance graphene supercapacitor can cause revolution in the field of energy storage. In more expensive solutions fuel cells are used that do not require charging and supporting photovoltaic cells. It is difficult to predict which method of energy storage in future will be the best. Gołębowski et al. [3] propose an analysis of patents to predict

new trends in the development of technology of manufacturing batteries used in electric cars. Simultaneously, work is underway on the use of “mechanical battery” – the energy of a flywheel speeding up to speed of 30-40 thousand rev./min, levitating in a magnetic field in a hermetic vacuum casing.

The operating cost of electric cars currently produced is not competitive in relation to the cost of typical hybrid cars. It will be competitive when the battery price drops to approx. 400 €/kWh and their efficiency and number of charge cycles will allow them for operation throughout the operation of the vehicle [15]. However, the analysis of Yang et al. [19] carried out nine years earlier proved that the most promising candidates in relation to the car with a combustion engine in terms of the coverage, purchase price, fuel costs and life-cycle costs are electric cars powered by aluminum-air batteries.

Weight reduction of the car directly affects the improvement of all its parameters. The electric vehicle will always be lighter than the equivalent combustion one. The electric motor is much lighter than the combustion engine. It can be directly integrated with the wheels of the car and does not load the body. The gearbox, differential, fuel tank, fuel and oil pumps, alternator, engine cooling system, brake, and clutch are redundant. Energy demand to overcome frictional resistance in the drive system of electric vehicle is by half less than in vehicles with internal combustion engines, and is approximately 14% [6]. Due to lack of components that are heated to high temperature, it is possible to use light and non-corrosive materials, plastics and advanced composites. In the electric drive solutions, an electronic control system does not require any special transducers. It is easier to control the maximum current intensity, voltage and engine rotational speed [13]. The use of controllers allows optimal energy consumption depending on the required dynamic performance of electric vehicles [10]. The advantages of electric cars are important and relevant in urban areas where driving conditions are good. In the difficult driving conditions, electric drive is not necessarily economic due to the high-energy consumption. A comparison of the energy consumption of two conventional internal combustion vehicles with diesel and electric counterparts in poor driving conditions indicates that with the use of moderate acceleration and deceleration, maximum range of electric vehicle is 21.8% smaller than given by the manufacturer, and with aggressive driving style even by 26.9% [14].

For computer modeling and simulation such software tools are used as: Powertrain System Analysis Toolkit (PSAT), Advanced Vehicle Simulator (ADVISOR), PSIM and the Virtual Test Bed [2]. In the computing techniques, the methods of artificial intelligence and fuzzy models are also used [12]. Kołodziejczyk and Moćko [8] presented a simulation program allowing calculating, among other things, energy consumption and the level of discharge of lead-acid batteries, supplying electric vehicle, moving in a certain way. Guenther et al. [5] studied the estimated useful life of traction batteries for different load scenarios. The developed simulation models are used to study new solutions for drive systems and the prototypes of electric cars [1].

Thanks to the carried out experimental and theoretical research, electric vehicles are still being improved and the

demands placed on today are more sophisticated than a few years ago. The measure of the quality of cars produced today is comfort, reliability and low operating costs. Therefore, fatigue strength of car elements, resistance to shock overload of drive assembly and suspension system, durability and efficiency of the batteries, as well as low vehicle weight, are essential.

At each stage of technology development, depending on the current capabilities it is necessary to appropriately seek a compromise between mutually dependent requirements such as maximum speed, battery capacity and “range” – the distance traveled without recharging, taking into account the landform.

This study aims to develop a mathematical model of the car with an electric DC motor considering the kinematic, dynamic and electrical parameters and determine the effect of these parameters on the “range” of the vehicle.

A computing algorithm has been developed and the authors’ own simulation program that links the geometric (dimensions of the car, gear ratio), kinematic (distance traveled, speed, acceleration) and dynamic parameters (the waveform of passive and active forces of the drive system, the expended energy). In the algorithm, there were used simplifications and many important phenomena addressed by the authors of the cited works were disregarded. By using the software, the temporal waveforms of mechanical work performed and mechanical energy consumed while driving at a specified time were determined.

MATERIALS AND METHODS

THE TORQUE OF PASSIVE FORCES

The torque of passive forces M_B loading powertrain depends primarily on the shape and nature of the ground and the drag, which are functions of speed. The loading of system is influenced also by rolling resistance of tires on the ground, resistance to motion of powertrain (gearbox, bearings), and even way of controlling the motor. The driving torque in the wheel axle of a moving vehicle must be greater than torque of passive forces described by the correlation:

$$M_B(\omega_2, \alpha_t) = M_A + M_F + M_T + M_C, \quad (1)$$

where:

M_A – the load torque dependent on drag [N·m],
 M_B – cumulative torque of passive forces [N·m],
 M_C – the torque of constant friction forces in the drive system (bearings, gearbox) [N·m],

M_F – the load torque dependent on the rolling resistance [N·m],

M_T – the load torque dependent on slope [N·m],

where:

$$M_A(\omega_2) = 0,5c_x \rho_p A \omega_2^2 R_o^3, \quad (2)$$

$$M_F(\alpha_t) = m_a g f \cos \alpha_t, \quad (3)$$

$$M_T(\alpha_t) = m_a g R_o \sin \alpha_t, \quad (4)$$

where:

- A – front area of the car [m²],
- c_x – drag coefficient,
- f – coefficient of friction of the tires with the ground [m],
- g – acceleration due to gravity [m·s⁻²],
- m_a – gross vehicle weight [kg],
- R_o – tread radius [m],
- α_i – the angle of slope [rad],
- r_p – air density [kg·m⁻³],
- w_2 – wheel angular velocity [rad·s⁻¹].

THE TORQUE OF THE ACTIVE FORCES, THE DYNAMICS OF THE VEHICLE MOVEMENT

Simplified diagram of a vehicle drive system with two engines is shown in Figure 1.

It consists of two identical separately excited DC motors (2) with a constant current supply of the coil winding. The currents are regulated by regulators (1). The driving torques of motors M_S are transmitted via the gearboxes (3) with a gear ratio i_p and then through the elastic shafts of torsional elasticity coefficients k_φ and damping l_φ to the driven wheels of the vehicle (4) having load torque $0.5M_B$, assuming equal loading of both drives.

After adopting certain simplifications, Kirchhoff voltage equation for the armature circuit can be applied to the armature circuit – Figure 1:

$$u_a(t) = c_s i_p \frac{d\varphi_1(t)}{dt} + R_a i_a(t) + L_a \frac{di_a(t)}{dt}, \quad (5)$$

where:

- c_s – motor constant [V·s],
- i_a – the armature current intensity [A],
- i_p – gear ratio,
- L_a – the inductance of armature circuit [H],
- R_a – the resistance of armature circuit [Ω],
- t – time [s],
- u_a – power supply voltage of the armature [V],
- j_1 – angular displacement of the transmission output shaft [rad].

Electric DC motor torque M_S , for a steady stream of excitation and a wide range of changes to the armature current can be expressed by the equation:

$$M_S(t) = c_s i_a(t), \quad (6)$$

where:

- M_S – electric motor torque [N·m].

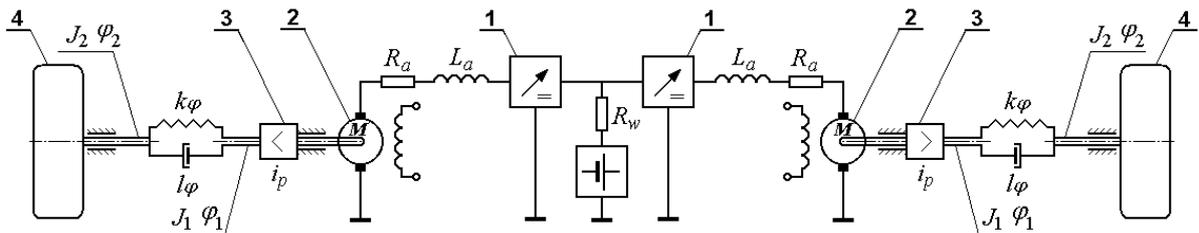


Fig. 1. The electromechanical system with two DC motors; 1 – voltage regulator, 2 – DC motor, 3 – reduction gear, 4 – wheel of a vehicle

Mechanical system of powertrains has been examined as a system with two degrees of freedom. Inertia of the first and second mass is determined by the values J_1 and J_2 . Kinetostatic mass balance equation for the reduced inertia J_1 in accordance with the D'Alembert's law is in the form:

$$J_1 \frac{d^2\varphi_1(t)}{dt^2} = M_N(t) - M_{KL}(t), \quad (7)$$

where:

$$M_N(t) = M_S(t) i_p, \quad (8)$$

$$M_{KL} = k_\varphi [\varphi_1(t) - \varphi_2(t)] + l_\varphi \left[\frac{d\varphi_1(t)}{dt} - \frac{d\varphi_2(t)}{dt} \right], \quad (9)$$

$$J_1 = J_w i_p^2, \quad (10)$$

J_1 – mass moment of inertia of the rotor and the rotating gear elements reduced to the axis of rotation of the gear output shaft [kg·m²],

J_w – mass moment of inertia of the rotor and rotating components with angular velocity of the rotor [kg·m²],

k_j – reduced torsional stiffness coefficient of the powertrain [N·m·rad⁻¹],

l_j – reduced torsional damping coefficient of the powertrain [kg·m·s⁻¹],

M_N – drive torque to the wheel axle [N·m],

j_2 – angular displacement of the vehicle wheel [rad].

Assuming that the power and inertia load are distributed half-half on both drives, so with a load of the output shaft system $0.5M_B$, Kinetostatic equilibrium equation of the second mass of inertia $0.5J_2$ takes the form:

$$\frac{1}{2} J_2 \frac{d^2\varphi_2(t)}{dt^2} = M_{KL}(t) - \frac{1}{2} M_B(t), \quad (11)$$

where:

J_2 – the mass moment of inertia of the vehicle and wheels reduced to the rotation axis of the wheels [kg·m²].

Mass moment of inertia J_2 while disregarding components having little effect on its value is determined by the relationship:

$$J_2 = m_a R_o^2 + 4m_o [(R_o - r_o)^2 + 0,75r_o^2], \quad (12)$$

where:

m_o – wheel weight [kg],

r_o – the mean radius of the tire [m].

In order to carry out numerical solution of a set of differential equations consisting of the equation of the first order (5) and two second-order equations (7) and (12), there are

introduced the state coordinates, which are represented by the following figures: $\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_2, i_a$. After making substitutions:

$$Y_1 = \dot{\varphi}_1, Y_2 = \dot{\varphi}_2, Y_3 = \varphi_1, Y_4 = \varphi_2, Y_5 = i_a, \quad (13)$$

five first-order differential equations in the coordinates of the state are obtained

$$\frac{dY_1}{dt} = \frac{1}{J_1}(M_N - M_{KL}) = \frac{1}{J_1}[c_s i_p Y_5 - k_\varphi(Y_3 - Y_4) - l_\varphi(Y_1 - Y_2)], \quad (14)$$

$$\frac{dY_2}{dt} = \frac{2}{J_2}\left(M_{KL} - \frac{1}{2}M_B\right) = \frac{2}{J_2}\left[k_\varphi(Y_3 - Y_4) + l_\varphi(Y_1 - Y_2) - \frac{1}{2}(C_2 Y_2^2 + C_1)\right], \quad (15)$$

$$\frac{dY_3}{dt} = Y_1, \quad (16)$$

$$\frac{dY_4}{dt} = Y_2, \quad (17)$$

$$\frac{dY_5}{dt} = \frac{1}{L_a}(u_a(t) - R_a Y_5 - c_s i_p Y_1), \quad (18)$$

where:

$$C_1 = m_a g (R_o \sin \alpha_i + f \cos \alpha_i) + M_{b,c}, \quad C_2 = 0,5 c_s \rho_p A R_o^3. \quad (19)$$

Typical initial conditions are as follows:

$$Y_1(0) = 0, Y_2(0) = 0, Y_3(0) = 0, Y_4(0) = 0, Y_5(0) = 0. \quad (20)$$

To solve the set of equations (14) – (18) the Runge-Kutta optimized method of fourth order was used. As a result of solution of equations there were obtained values of angular displacement φ_2 , angular velocity $\omega_2 = d\varphi_2/dt$, angular acceleration $\varepsilon_2 = d\omega_2/dt^2$ of vehicle wheels and the currents flowing through the power supply circuits i_a as a function of time.

Using the obtained values, the displacement $s_a(t)$, the speed $v_a(t)$ and acceleration $a_a(t)$ of the vehicle can be determined:

$$s_a(t) = \varphi_2(t) R_o, \quad (21)$$

$$v_a(t) = \omega_2(t) R_o, \quad (22)$$

$$a_a(t) = \varepsilon_2(t) R_o. \quad (23)$$

Mechanical work done by the motors of the vehicle $W_a(t)$ while driving at a given time can be determined according to the relationship:

$$W_a(t) = 2i_p \int_{\varphi_{2p}}^{\varphi_{2k}} M_S(t) d\varphi_2, \quad (24)$$

where:

j_{2p}, j_{2k} – initial and final angular position of the transmission output shaft [rad], wherein $(j_{2k} - j_{2p})$ – angular displacement of the transmission output shaft [rad].

While the electricity consumed within the specified time of vehicle movement can be calculated according to the relationship:

$$E_e(t) = 2 \int_{t_p}^{t_k} u_a(t) i_a(t) dt, \quad (25)$$

where:

$(t_k - t_p)$ – the time of the vehicle movement [s].

NUMERICAL EXAMPLE

There were performed simulating calculations in which the waveforms of kinematic and dynamic parameters were determined during the starting phase for the most adverse variant of motor power supply involving switching at time $t_0 = 0$, the maximum value of the supply voltage.

More important data for calculations were assumed as follows: the value of constant friction torque $M_{b,c} = 0.5 \text{ N}\cdot\text{m}$, angle of slope $\alpha_i = 0$, values of coefficients: $c_x = 0.65$, $f = 0.012 \text{ m}$, values of constants $g = 9.81 \text{ m}\cdot\text{s}^{-2}$, $r_p = 1.226 \text{ kg}\cdot\text{m}^{-3}$. Other data for the calculation are given in Table 1.

Table 1. The numerical values of data adopted in the calculation

Name of item	Designation	Name of parameter	Value	Unit
Vehicle	m_a	total weight	450	kg
		frontal surface area	2.2	m^2
DC motor	N_s	power	1.5	kW
		constant inductivity	0.14	$\text{V}\cdot\text{s}$
		resistance	1.2	mH
		power supply voltage	0.93	W
		power supply voltage	48	V
Wheel	m_o	mass	4	kg
Tire	R_o	tread radius	0.22	m
		radius of the profile	0.10	m
Gear unit	i_p	gear ratio	11	-
Battery		voltage	12	V
		capacity	60	Ah
		number	4	item

RESULTS

The calculation results in the form of temporal waveforms of displacement, velocity and acceleration of the car, the mechanical work performed and electricity consumption are shown in Figures 2 and 3.

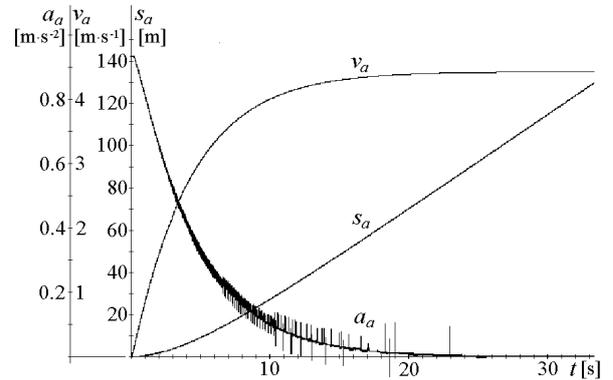


Fig. 2. Temporal waveforms of displacement, velocity and acceleration of the car

CONCLUSIONS

In order to reduce the energy requirements and thereby increase the so called range, one should:

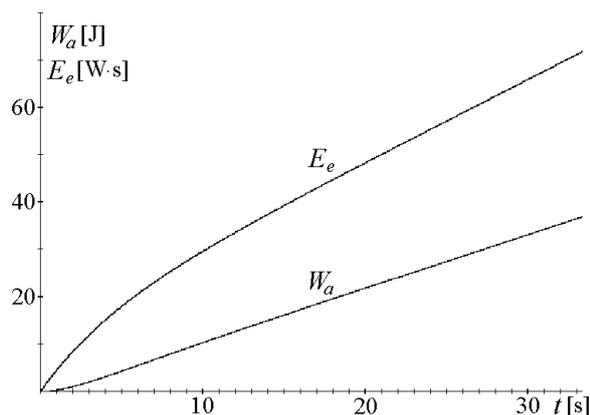


Fig. 3. Temporal waveforms of mechanical work performed and electricity consumption

- strive to reduce the constant resistances e.g. through the use of direct drives to the wheels to eliminate differential gear,
- use a body shape providing a small value c_x ,
- overcome the same distance at a lower speed, and not use high accelerations,
- use energy recovery while driving on descending inclines and while braking.

When driving at a constant speed on a flat level surface, the influence of the masses of car elements is the smallest. The consequence of an increase in weight is the increase mainly of rolling resistances and resistances while driving up the hill. The ability of acceleration decreases then and thus increases the energy consumption during the acceleration of the car, but more energy during engine braking is recovered.

Most energy is consumed during the acceleration phase of the car. During the start lasting 15 s, energy consumption is 2 times greater than during driving with the maximum speeds at the same time. In order to save energy, it is advisable to use electronic controls that prevent the engine powering with the maximum voltage during start-up.

REFERENCES

- 1 **Adamczyk D., Michna M., Ronkowski M. 2012.** Uniwersalny model symulacyjny układu napędowego prototypu samochodu elektrycznego „EL V001”. *Prace Naukowe Instytutu Maszyn Napędów i Pomiarów Elektrycznych Politechniki Wrocławskiej*. 66 *Studia i Materiały* 32, 319-324 (in Polish).
- 2 **Gao, D.W.; Mi, C; Emadi, A. 2006.** Modeling and Simulation of Electric and Hybrid Vehicles. *Browse Journals&Magazines*. Volume 95, Issue 4, 729-745.
- 3 **Golembiewski B., vom Stein N., Sick N., Wiemhofer H.-D. 2015.** Identifying trends in battery technologies with regard to electric mobility: evidence from patenting activities along and across the battery value chain. *Journal of Cleaner Production* 87, 800-810.
- 4 **Grolms M. 2014.** Electric car with aluminium-air battery can go the distance. Internet: <http://www.materialsvIEWS.com/electric-car-aluminium-air-battery-can-go-distance>, dostęp 14.01.2015.
- 5 **Guenther C., Schott B., Hennings W., Waldowski P., Danzer M.A. 2013.** Model-based investigation of electric vehicle battery aging by means of vehicle to grid scenario simulations. *Journal of Power Sources*, 239, 604-610.
- 6 **Holmberg K., Andersson P., Erdemir A. 2012.** Global energy consumption due to friction in passenger cars. *Tribology International*. 47, 221-234.
- 7 <http://www.phinergy.com>.
- 8 **Kołodziejczyk J., Moćko W. 2014.** Zastosowanie symulacji komputerowej do analizy zużycia energii przez samochód elektryczny. *Transport samochodowy*. 1, 69-79 (in Polish).
- 9 **Korzybski W., Rode H. 2012.** Wybrane sposoby produkcji i magazynowania energii elektrycznej. *Zeszyty Naukowe Instytutu Pojazdów*. 1(87), 145-156 (in Polish).
- 10 **Kraa O., Aboubou A., Becherif M., Ayad M.Y., Sadi R., Bahri M., Ghodbane H. 2014.** Fuzzy Logic Maximum Structure and State Feedback Control Strategies of The Electrical Car. *Energy Procedia*, 50, 178-185.
- 11 **Menes M. 2011.** Współczesne kierunki rozwoju techniki samochodowej. *Transport samochodowy*, 4, 19-36 (in Polish).
- 12 **Rahman A., Hossain A., Zahirul A.A.H.M., Rashid M. 2012.** Fuzzy knowledge-based model for prediction of traction force of an electric golf car. *Journal of Terra-mechanics*, 49, 13-25.
- 13 **Tarkowski P., Siemionek E. 2010.** Układy napędowe pojazdów elektrycznych. *Postępy Nauki i Techniki*. 5, 162-169 (in Polish).
- 14 **Traveset-Baro O., Rosas-Calsas M, Jover E. 2015.** Transport energy consumption in mountainous roads. A comparative case study for internal combustion engines and electric vehicles in Andorra. *Transportation Research Part D*, 34, 16-26.
- 15 **Van Vliet O., Brouwer A.S., Kuramochi T., van den Broek M., Faaij A., 2011.** Energy use, cost and CO₂ emissions of electric cars. *Journal of Power Sources*, 196, 2298-2310.
- 16 **Warth J., von der Gracht H.A., Darkow I.-L. 2013.** A dissent based approach for multi-stakeholder scenario development – The future of electric drive vehicles. *Technological Forecasting & Social Change*, 80, 566-583.
- 17 **Waśkiewicz J., Chłopek Z. 2013.** Projekcje zapotrzebowania nośników energii przez polski park samochodów osobowych w latach 2015- 2030. *Transport samochodowy*, 3, 5-16 (in Polish).
- 18 **Wu X., Freese D., Cabrera A., Kith W.A. 2015.** Electric vehicles' energy consumption measurement and estimation. *Transportation Research Part D*, 34, 52-67.
- 19 **Yang S., Knickle H. 2002.** Design and analysis of aluminium/air battery system for electric vehicles. *Journal of Power Sources*. 112, 162-173.

MODEL DYNAMIKI POJAZDU Z SILNIKIEM ELEKTRYCZNYM PRĄDU STAŁEGO

Streszczenie. Wyznaczono wartość momentu sił biernych, zależnych od oporów powietrza, oporów tocnych i nachylenia terenu. Do wyznaczenia przebiegu momentu sił czynnych (napędowych) jako funkcji prędkości kątowej wykorzystano napięciowe równanie Kirchhoffa dla obwodu twornika. Uwzględniono podatność i tłumienie układów transmisji ruchu. Kinematyczne parametry ruchu otrzymano z rozwiązania równań ruchu układów napędo-

wych. Do rozwiązania równań zastosowano metodę Rungego-Kutty. Napisany został własny program symulacyjny, który umożliwia wyznaczenie czasowych przebiegów przemieszczenia, prędkości i przyspieszenia pojazdu oraz zużywanej energii elektrycznej i wykonanej pracy mechanicznej w zależności od uwzględnionych oporów ruchu i wartości napięcia zasilającego. Zamieszczono przykład obliczeniowy. Sformułowano wnioski i sugestie wynikające z przeprowadzonej symulacji.

Słowa kluczowe: pojazd, dynamika, kinematyka, silnik prądu stałego, model teoretyczny.