ON THE LINEARITY RANGE OF THE FERROPROBE SENSOR OF MOVEMENT WITH THE MAGNETIC SYSTEM IN THE FORM OF A RECTANGULAR ONE-CORED MAGNET

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Summary. The principle of design of the ferroprobe sensor of movement with the magnetic system in the form of a rectangular one-cored magnet has been described. The analytical expressions of the orthogonal components of the intensity of the external magnetic fields have been derived. The linearity range of the sensor has been determined.

Key words: magnet, model, potential, relationship, linearity range, ferroprobe sensor

INTRODUCTION

Highly sensitive ferroprobes with consistent metrological characteristics are often used as primary transducers of magnitometric sensors of movement. Current coils, the external fields of magnetic markers as well as the external fields produced by magnets with different pattern can be a source of the information magnetic field.

The topical problem is to derive the analytical expressions of the components of the intensity of the spatial external fields at the end face of the rectangular magnet and to determine the linearity ranges of the sensor depending on the relative position of the ferroprobe sensitive element and the magnet, its shape being taken into account.

The purpose of the article is to determine the linearity range of the ferroprobe sensor of movement on the basis of analyzing the mathematical model of the external fields of the rectangular one-cored magnet used as a magnetic system of the sensor.

RESULTS AND THEIR ANALYSIS

For the analysis of the external fields of the rectangular one-cored magnet its length will be taken as h and the thickness of the pole as 2Δ . Besides it can be considered with high accuracy that magnet is made of an isotropic material with constant relative magnetic permeability μ_h . The ideal model of the ferroprobe sensor with the rectangular one-cored magnet is shown in Fig. 1. A tiny ferroprobe (FP) is

supposed to be situated in the area of the end face of the magnet parallel to its surface and to move along axis x. It is assumed that the magnetic field is created by the residual magnetization the value of which is constant along the magnet and the vector of which is directed along axis y.



Fig. 1. The model of the ferroprobe sensor of movement with the rectangular one-cored magnet

The method described in [1] is used for analyzing the external fields of the magnet. Let the residual magnetization of the core of the magnet be equal to

$$\vec{M}_0(x) = \vec{j}M_{0m}p_\Delta(x), \qquad (1)$$

Where M_{0m} is the peak value of the residual magnetization

$$p_{\Delta}(x) = 1, \quad |x| \le \Delta$$
$$0, \quad |x| > \Delta$$

The expansion of $\vec{M}_0(x)$ into a Fourier integral is shown as

$$\vec{M}_0(x) = \vec{j}M_{0m} \frac{2}{\pi} \int_0^\infty \frac{\sin \Delta\omega}{\omega} \cos x\omega d\omega \quad .$$
 (2)

For the arbitrary harmonic component of the spatial spectrum $\omega = \Omega \neq 0$

$$\vec{M}_{0\Omega} = \vec{j}M_{m\Omega}\cos\Omega x\,,\tag{3}$$

Where $M_{m\Omega} = \frac{2}{\pi} M_{0m} \frac{\sin \Delta \omega}{\omega}$ is the peak value of the magnetization of the harmonic of frequency III, the boundary problem is solved in the following way (Fig. 2).



Fig. 2. The ideal two-dimensioned model of the harmonic component of the spatial spectrum of magnetization

Differential equation concerning scalar potential of the magnetic field is as follows:

$$\Delta \varphi_{k\Omega} = \begin{cases} \frac{div \, \overrightarrow{M}_{r\Omega}}{\mu_r} = -\frac{\Omega M_{rm\Omega} \sin \Omega x}{\mu_r}, & k = 2\\ 0, & k = 1, 3 \end{cases}.$$
(4)

The solution of the different equation (4) is the following [2]

$$\begin{cases} \varphi_{1\Omega} = A_1 e^{\Omega y} \cos \Omega x \\ \varphi_{2\Omega} = (A_2 e^{\Omega y} + B_2 e^{-\Omega y}) \cos \Omega x \\ \varphi_{2\Omega} = (A_2 e^{\Omega y} + B_2 e^{-\Omega y}) \cos \Omega x \end{cases}$$
(5)

It is to satisfy the boundary conditions:

$$\varphi_n = \varphi_{n+1}; \qquad \mu_n \frac{\partial \varphi_n}{\partial y} - M_{ryn} = \mu_{n+1} \frac{\partial \varphi_{n+1}}{\partial y} - M_{ry(n+1)}. \tag{6}$$

The expressions for constants of integration are determined by substituting (5) into (6). The constant of integration for the area 3 where FP is located is equal to:

$$A_{3} = \frac{M_{0\Omega}}{\Omega} \frac{1}{(\mu_{h}+1)[1-(\frac{\mu_{h}-1}{\mu_{h}+1})^{2}e^{-2h\Omega}]} (1-\frac{2\mu_{h}}{2\mu_{h}+1}e^{-h\Omega} + \frac{\mu_{h}-1}{\mu_{h}+1}e^{-2h\Omega}).$$
(7)

After transformation the expression reduces to the following form:

$$A_{3} = \frac{M_{0\Omega}}{\Omega} \frac{1}{\mu_{h}+1} (1 - \frac{2\mu_{h}}{\mu_{h}+1})^{2} e^{-h\Omega} + \frac{\mu_{h}-1}{\mu_{h}+1} e^{-2h\Omega}) \sum_{\alpha=0}^{\infty} (\frac{\mu_{h}-1}{\mu_{h}+1})^{2\alpha} e^{-2h\alpha\Omega}.$$
 (8)

By substituting (8) into (5) the scalar potential of the magnetic field in area 3 corresponding to the harmonic of frequency is found $\omega = \Omega$. The potential caused by all the components of the spectrum of the assumed magnetization is determined from the expression:

$$\varphi_{3} = \frac{2}{\pi} \frac{M_{0m}}{\mu_{h} + 1} \left[\int_{0}^{\infty} \frac{\sin \Delta \omega}{\omega} \sum_{\alpha=0}^{\infty} m^{\alpha} e^{-(y+2h\alpha)\omega} \frac{\cos x\omega}{\omega} d\omega - \frac{2\mu_{h}}{\mu_{h} + 1} \int_{0}^{\infty} \frac{\sin \Delta \omega}{\omega} \sum_{\alpha=0}^{\infty} m^{\alpha} e^{-(y+2h\alpha+h)\omega} \frac{\cos x\omega}{\omega} d\omega + \frac{\mu_{h} - 1}{\mu_{h} + 1} \int_{0}^{\infty} \frac{\sin \Delta \omega}{\omega} \sum_{\alpha=0}^{\infty} m^{\alpha} e^{-(y+2h\alpha+2h)\omega} \frac{\cos x\omega}{\omega} d\omega \right],$$
(9)
where: $m = \left(\frac{\mu_{h} - 1}{\mu_{h} + 1}\right)^{2}$.

Bringing the integrals into the forms which are tabulated and considering the relationship $\vec{H} = -grad\phi$, the expressions for horizontal and vertical components of the intensity of the field from the magnet pole are derived:

$$H_{3x} = \frac{1}{2\pi} \frac{M_{0m}}{\mu_h + 1} \left[\sum_{\alpha=0}^{\infty} m^{\alpha} \ln \frac{(y + 2h\alpha)^2 + (x + \Delta)^2}{(y + 2h\alpha)^2 + (x - \Delta)^2} - \frac{2\mu_h}{\mu_h + 1} \sum_{\alpha=0}^{\infty} m^{\alpha} \ln \frac{(y + 2h\alpha + h)^2 + (x + \Delta)^2}{(y + 2h\alpha + h)^2 + (x - \Delta)^2} +$$
(10)
+ $\frac{\mu_h - 1}{\mu_h + 1} \sum_{\alpha=0}^{\infty} m^{\alpha} \ln \frac{(y + 2h\alpha + 2h)^2 + (x + \Delta)^2}{(y + 2h\alpha + 2h)^2 + (x - \Delta)^2} \right], \quad x \ge 0 \quad ;$
$$H_{3y} = \frac{1}{\pi} \frac{M_{0m}}{\mu_h + 1} \left[\sum_{\alpha=0}^{\infty} m^{\alpha} (arctg \frac{x + \Delta}{y + 2h\alpha} - arctg \frac{x - \Delta}{y + 2h\alpha}) - \frac{2\mu_h}{y + 2h\alpha} \right], \quad x \ge 0 \quad ;$$

$$H_{3\mu} = \frac{1}{\pi} \sum_{\alpha=0}^{\infty} m^{\alpha} (arctg \frac{x + \Delta}{y + 2h\alpha + h} - arctg \frac{x - \Delta}{y + 2h\alpha + h}) +$$
(11)
$$+ \frac{\mu_h - 1}{\mu_h + 1} \sum_{\alpha=0}^{\infty} m^{\alpha} (arctg \frac{x + \Delta}{y + 2h\alpha + 2h} - arctg \frac{x - \Delta}{y + 2h\alpha + 2h}) \right] \quad .$$

when: $h \rightarrow \infty$ the expressions (12) and (13) look like

$$H_{x} = \frac{1}{2\pi} \frac{M_{0m}}{\mu_{h} + 1} \ln \frac{y^{2} + (x + \Delta)^{2}}{y^{2} + (x - \Delta)^{2}} \quad x \ge 0;$$
(12)

$$H_{y} = \frac{1}{\pi} \frac{M_{0m}}{\mu_{h} + 1} \left(\operatorname{arctg} \frac{x + \Delta}{y} - \operatorname{arctg} \frac{x - \Delta}{y} \right) . \tag{13}$$



Fig. 3. The curves of the horizontal component of the intensity of magnetic field H_{3x} when distances from the surface of the pole of the one-cored magnet are different

It should be noted that expressions (11) and (12) are respectively the vertical and horizontal components of the intensity of the magnetic field of an ideal ring-shaped recording head with the rectangular running clearance with the width 2Δ which testifies to the duality of the data of the magnetic systems.

Figure 3 shows the calculated curves of the horizontal component of the intensity of the magnetic field with different distances in proportions Δ from the surface of the pole of the one-cored magnet for values $h = 4\Delta$, $\mu_h = 1000$, $M_{0m} = 10^4 A/m$.

An algorithm has been developed and the corresponding computer program has been implemented in order to calculate the values of the horizontal component of the intensity of magnetic fields H_{3x} in the range of linearity with the given coefficient of non-linearity ε_3 depending on the distance to the surface of the pole of the magnet.

The algorithm of determining the maximum length of the line section H_{3x} is the following: with the fixed original value of coordinate y_0 the current value of coefficient ε is calculated with every change of x and is compared with the given ε_e . If $\varepsilon < \varepsilon_e$ coordinate x gets increment Δx and ε and ε_e are compared again. If $\varepsilon \ge \varepsilon_e$ the values of the local maximum length of the line section $2x_{\Delta}$ as well as the value of the horizontal component of the intensity of the magnetic field at the boundary of the interval of linearity are derived and remembered $H_{3x\Delta}$.

Then coordinate y gets increment Δy and the procedure $2x_{\Delta}$ is repeated. Then the maximum maximorum of the linearity range $H_{3x\Delta}$ is selected from the mass of the local peak values.

Fig. 4 shows the dependence of the given length of the line section $\frac{2 x_{\Delta}}{\Delta}$ on the distance to the surface of the pole of the magnet according to the result of the calculation when $\varepsilon_{\rho} = 0.01$. It is seen from the curve that the maximum range of

linearity is equal to $\frac{2x_{\Delta}}{\Delta} = 1,68$ when the distance between the sensitive element and the magnet is $0,41\Delta$. In these conditions $H_{3x\Delta} = 415\frac{A}{m}$.

The experiments on measuring the external field of the permanent magnet with the dimensions $20 \times 10 \times 20 \text{ mm}^3$ showed the nature of the changes of the horizontal component of the intensity of magnetic fields $H_{3x\Delta}$ and the order of its value with an error not exceeding 3-5 %.

CONCLUSIONS

1. It has been determined on the basis of the derived analytical expressions of the intensity of the external fields of the rectangular one-cored magnet that it is possible to ascertain some areas on the curves of distributions of the horizontal component of intensity of the magnetic field H_{3x} where practical linear change of the intensity of the magnetic fields takes place.

2. In order to obtain the maximum possible range of linearity equal to $\frac{2x_{\Delta}}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to place the ferroprobe sensitive element at a distance of $\frac{2}{\Delta} = 1,68$ it is advisable to p

 $0,41\Delta$ from the end face of the magnet.



Fig. 4. The dependence of the linearity range $2x_{\Delta}$ on the distance to the surface of the pole of the magnet

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О ДИАПАЗОНЕ ЛИНЕЙНОСТИ ФЕРРОЗОНДОВОГО ДАТЧИКА ПЕРЕМЕЩЕНИЯ С МАГНИТНОЙ СИСТЕМОЙ В ВИДЕ ПРЯМОУГОЛЬНОГО ОДНОСТЕРЖНЕВОГО МАГНИТА

Смирный М.Ф.

Аннотация. Описан принцип построения феррозондового датчика с магнитной системой в виде прямоугольного одностержневого магнита. Получены аналитические выражения ортогональных составляющих напряженности внешнего поля магнита. Определен диапазон линейности датчика. Рис. 4. Ист. 2.

Ключевые слова: феррозондовый датчик, постоянный магнит, двумерная модель, напряжённость магнитного поля, диапазон линейности.