TEKA Kom. Mot. i Energ. Roln. – OL PAN, 2010, 10B, 105-113

THE ANALYTICAL PROBLEM SOLUTION OF FINDING THE SEVERAL COALS MIX CHARACTERISTICS

Vitaly Pozhidayev, Oleg Grachyov

Volodymyr Dal East-Ukrainian National University, Lugansk, Ukraine

Summary. The solution method of the problem of finding the several coals mix characteristics is offered. The expression describing relation between an output and boundary ash content is found. Offered method and practical use of the Reinhardt theorem allows to find maximum output with set ash content.

Key words: optimization, ordinary coal, fractional composition, granulometric composition, batch characteristics

INTRODUCTION

At the solution of the questions connected with a mix of coals enrichment, it is necessary to be able to build Anri's curves λ, β, θ for a mix [1-6]. Graphic methods of the solution are labor-consuming also it is impossible to use them as a component of algorithms. The analytical method of a coal's mix cumulative characteristics construction is necessary.

The problem consists in receiving the maximum cumulative output of a concentrate with set by average ash content. The statement that the maximum output of a concentrate in this case will be received if ash content demarcation (boundary) layers will be equal belongs to Reinhardt.

This statement are accepted by experts without the proof. Negation its justice leads to occurrence of bulky and inexact methods of the problems connected with separate enrichment of several coals solution.

CONSTRUCTION OF THE SEVERAL COALS MIX CHARACTERISTICS

Let's consider a mix of two coals. The share of one of them let will be equal in a mix *p*, a share of another $q = 1 - p$. Functions $\lambda_1(\Gamma_1)$, $\beta_1(\Gamma_1)$, $\lambda_2(\Gamma_2)$, $\beta_2(\Gamma_2)$ are accordingly set. Let dependences of an output by density $\Gamma_1 = f_1(\rho)$ and

$\Gamma_1 = f_1(\rho)$ $\Gamma_2 = f_2(\rho)$ are set. Considering that there are Γ_1, Γ_2 functions from ρ, curves λ and β will be considered also as functions from ρ, $\lambda(\Gamma) = \lambda(\Gamma(\rho))$ and $\beta(\Gamma) = \beta(\Gamma(\rho))$.

Let's consider infinitely narrow fraction $(\rho, \rho + \Delta\rho)$. Outputs will be $\rho\left[\Gamma_1(\rho+\Delta\rho) - \Gamma_1(\rho)\right]$, $\rho\left[\Gamma_2(\rho+\Delta\rho) - \Gamma_2(\rho)\right]$. If $\Gamma(\rho)$ is an unknown function of the coals mix distribution by density the output of cumulative mix in the same fraction will be $\Gamma(\rho+\Delta\rho) - \Gamma(\rho)$. Thus, we can write down

$$\Gamma(\rho+\Delta\rho) - \Gamma(\rho) = p\left[\Gamma_1(\rho+\Delta\rho) - \Gamma_1(\rho)\right] + q\left[\Gamma_2(\rho+\Delta\rho) - \Gamma_2(\rho)\right].$$

From this it follows that the output of cumulative mix $\Gamma(\rho)$ is connected with outputs $\Gamma_1(\rho)$, $\Gamma_2(\rho)$ and the equation

$$\Gamma'(\rho) = p\Gamma_1'(\rho) + q\Gamma_2'(\rho). \qquad (1)$$

Through $U(\Gamma_1, \Gamma_2)$ fraction's average ash content with boundary outputs Γ_1, Γ_2 is designated. If function U is set for a mix then average ash content in fraction $(\rho, \rho+\Delta\rho)$ for components will be accordingly. $U_1(\rho, \rho+\Delta\rho)$, $U_2(\rho, \rho+\Delta\rho)$. Then for a mix it is possible to write down average ash content in this fraction through

$$U(\rho, \rho+\Delta\rho) = \frac{p\Gamma_1'(\rho)U_1(\rho, \rho+\Delta\rho) + q\Gamma_2'(\rho)U_2(\rho, \rho+\Delta\rho)}{p\Gamma_1'(\rho) + q\Gamma_2'(\rho)}. \qquad (2)$$

From the equation (1) follows that

$$\Gamma(\rho) = p\Gamma_1(\rho) + q\Gamma_2(\rho), \qquad (3)$$

And from the equation (2), considering that $U(\Gamma, \Gamma+\Delta\Gamma) \to \lambda(\Gamma)$, at $\Delta\Gamma \to 0$, it is possible to receive

$$\lambda(\Gamma(\rho)) = \frac{p\Gamma_1'(\rho)\lambda_1(\Gamma_1(\rho)) + q\Gamma_2'(\rho)\lambda_2(\Gamma_2(\rho))}{p\Gamma_1' + q\Gamma_2'}. \qquad (4)$$

Equalities (3) and (4) in common define a curve $\lambda(\Gamma)$ for a mix of coals, equality (3) defines a curve $\Gamma(\rho)$ or distribution function by density. As the curve $\lambda(\Gamma)$ for a mix is found, to curves $\beta(\Gamma)$, $\theta(\Gamma)$ it is already possible to pass under known formulas.

Let's generalize a problem on a case of coals n. Outputs are connected with ρ by $\Gamma_1(\rho), \Gamma_2(\rho), \ldots, \Gamma_n(\rho)$, elementary ash contents are set by functions from outputs

$\lambda_1(\Gamma_1), \lambda_2(\Gamma_2), ..., \lambda_n(\Gamma_n)$. If shares of coals participation in a mix will be $p_1, p_2, ..., p_n$, then for the fixed value of density we will find boundary ash content $\lambda(\Gamma(\rho)) = \dfrac{1}{\sum p_i \Gamma_i'(\rho)} \sum p_i \Gamma_i'(\rho) \lambda_i(\Gamma_i(\rho))$, and values of a cumulative output Γ and ρ density are connected by a relation $\Gamma(\rho) = \sum p_i \Gamma_i(\rho)$.

RELATION BETWEEN THE OUTPUT AND THE BOUNDARY ASH CONTENT

Let's stop in more details on a variant of the formula (2) selections of the function describing communication between an output and boundary ash content. Let $\dfrac{d\lambda}{d\Gamma} \sim \Gamma^{\alpha}\left(1-\Gamma\right)$.

Then $\int\limits_0^1 \dfrac{d\lambda}{d\Gamma} d\Gamma = \int\limits_0^1 \left(\Gamma^{\alpha} - \Gamma^{\alpha+1}\right) d\Gamma = \dfrac{1}{\alpha+1} - \dfrac{1}{\alpha+2} = \dfrac{1}{\left(\alpha+1\right)\left(\alpha+2\right)}$.

Integral from normalized functions $\int\limits_0^1 \dfrac{d\lambda}{d\Gamma} d\Gamma = 1$, so $\dfrac{d\lambda}{d\Gamma} \sim \left(\alpha+1\right)\left(\alpha+2\right)\Gamma^{\alpha}\left(1-\Gamma\right)$.

We will find the second derivative without constant multiplier and, having equated it to zero, we will receive $\Gamma^* = \dfrac{\alpha}{\alpha+1}$. As $\alpha\left(1-\Gamma^*\right) - \Gamma^* = 0$, that $\alpha = \dfrac{\Gamma^*}{1-\Gamma^*}$, and as $\Gamma^* \in \left(0,1\right)$ that $\alpha = \dfrac{\Gamma^*}{1-\Gamma^*}$ will give $\alpha \in \left(0;+\infty\right)$. We will find $\int\limits_0^{\Gamma} \Gamma^{\alpha}\left(1-\Gamma\right) d\Gamma = \dfrac{\Gamma^{\alpha+1}}{\alpha+1} - \dfrac{\Gamma^{\alpha+2}}{\alpha+2}$.

As on the one hand $\int\limits_0^1 \dfrac{d\lambda}{d\Gamma} d\Gamma = \dfrac{1}{\left(\alpha+1\right)\left(\alpha+2\right)}$, and $\int\limits_0^1 \lambda'\left(\Gamma\right) d\Gamma = \lambda_k - \lambda_0$ - with another we will receive after transformation $\lambda\left(\Gamma\right) = \lambda_0 + \left(\lambda_k - \lambda_0\right)\left(\alpha+1\right)\left(\alpha+2\right)\Gamma^{\alpha+1}\left(\dfrac{\Gamma^{\alpha+1}}{\alpha+1} - \dfrac{\Gamma^{\alpha+2}}{\alpha+2}\right)$.

Having integrated the received expression, we will receive

$$\Lambda(\Gamma)=\lambda_0\Gamma+(\lambda_k-\lambda_0)(\alpha+1)\Gamma^{\alpha+2}\left(\frac{1}{\alpha+1}-\frac{\Gamma}{\alpha+3}\right).$$

Let's consider $\Lambda(\Gamma):\lambda=\Lambda'$; $\lambda'=\Lambda''=f(\Gamma)$

$$\max f=f(\Gamma^*)=(\lambda_k-\lambda_0)(\alpha+1)(\alpha+2)\left(\Gamma^\alpha-\Gamma^{\alpha+1}\right)\Big|_{\Gamma=\Gamma^*}=$$

$$=(\lambda_k-\lambda_0)(\alpha+1)(\alpha+2)(\Gamma^*)^\alpha(1-\Gamma^*).$$

Let's designate through $p=\Gamma^*$ a concentrate share, then $q=1-\Gamma^*$ – a share of a cindery product. Then $\max f=(\lambda_k-\lambda_0)(\alpha+1)(\alpha+2)p^\alpha q$. As far as $\alpha=\frac{\Gamma^*}{1-\Gamma^*}$, that $.\lambda=\frac{p}{q}$, $\alpha+1=\frac{p+q}{q}$, $\alpha+2=\frac{p+2q}{q}$. As $U(\Gamma_1,\Gamma_2)=\frac{\Lambda(\Gamma_2)-\Lambda(\Gamma_1)}{\Gamma_2-\Gamma_1}$, and $0<\gamma_1<\gamma_2<1$, $\beta_0=\lambda(0)=\lambda_0,\lambda(1)=\lambda_k$ we will find expression for Anri’s curves [7-10].

$$\Gamma_1=0,\ \Gamma_2=\Gamma,\ \beta(\Gamma)=\frac{\Lambda(\Gamma)-\lambda_0}{\Gamma},\ \Gamma_1=\Gamma,\ \Gamma_2=1,\ \Theta(\Gamma)=\frac{\lambda_k-\Lambda(\Gamma)}{1-\Gamma}$$

$$\Gamma_1\to\Gamma_2=\Gamma,\ \lambda(\Gamma)=\lambda_0+(\lambda_k-\lambda_0)(\alpha+1)(\alpha+2)\left(\frac{1}{\alpha+1}-\frac{\Gamma}{\alpha+2}\right)\Gamma^{\alpha+1}.$$

$$\frac{\lambda(\Gamma)-\lambda_0}{\lambda_k-\lambda_0}\frac{1}{(\alpha+1)(\alpha+2)}=\Gamma^{\alpha+1}\left(\frac{1}{\alpha+1}-\frac{\Gamma}{\alpha+2}\right).$$

where $F(\lambda)$ – distribution function λ, as random variable; $t=\left(\frac{\lambda-\lambda_0}{\lambda_{\hat{e}}-\lambda_0}\right)^2$ – normalized value of boundary ash content; λ_0, $\lambda_{\hat{e}}$ – accordingly the minimum and maximum value of ash content; a_0, a_1 – parameters of distribution which are from experimental data after a flattening of distribution function $\frac{1/\Gamma-1}{\sqrt{1/t-1}}=a_0+a_1t$.

Let's enter auxiliary function

$$G(\Gamma)=\lambda_0+(\lambda_k-\lambda_0)(\alpha+1)(\alpha+2)\Gamma^{\alpha+1}\left(\frac{1}{\alpha+1}-\frac{\Gamma}{\alpha+2}\right)-\lambda,\quad G(0)<0,$$

$G(1)>0$.

Then for a finding of inverse function it is enough to find an auxiliary function zero. The result of U calculating [11-14] is presented on fig.1.

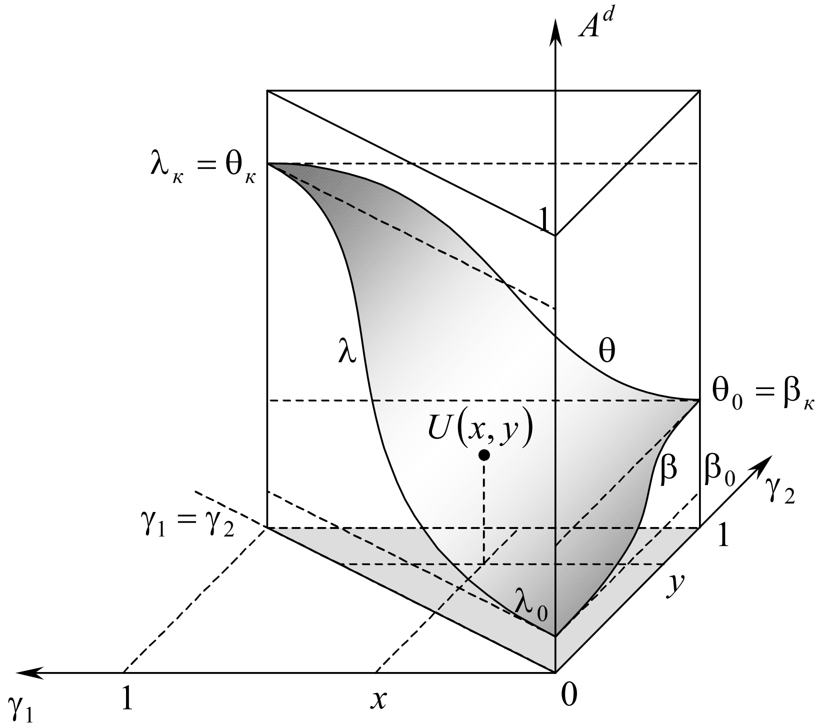

Fig. 1. An enrichment surface

$x = \gamma_1$ – an fractions output which have emerged, corresponding to boundary density ρ_1; $y = \gamma_2$ – an fractions output which have emerged, corresponding to boundary density ρ_2; $A^d = U(\gamma_1, \gamma_2)$ – ash content intermediate fraction $\rho_1 \div \rho_2$ ($\rho_1 \le \rho_2$); $\beta(\gamma) = U(0, \gamma)$ –average ash content fractions which have emerged, with an output γ; $\lambda(\gamma) = U(\gamma, \gamma)$ –infinitely narrow fraction $\rho \div (\rho + d\rho)$ ash content with an output $d\gamma = dx = dy$; $\theta(\gamma) = U(\gamma, 1)$ – average ash content fractions which have sunk, with an output $1 - \gamma$.

THE ANALYTICAL PROOF AND USE IN PRACTICAL RESEARCHES REINHARDT THEOREM

The theorem 1. Let two various coals with characteristics $\lambda_1(\Gamma_1)$, $\beta_1(\Gamma_1)$ are given and $\lambda_2(\Gamma_2)$, $\beta_2(\Gamma_2)$. The mix is made of these coals concentrates, and the share of the first coal in a mix will be p, and a share of the second $q = 1 - p$.

It is required to choose such lines of demarcation of division for each coal, i.e. such values Γ_1 and Γ_2 that the cumulative output of a concentrate was maximum provided that its average ash content is equal to β_C [15-17].

The proof. Values Γ_1, Γ_2 provide a maximum of a concentrate output if and only if in points Γ_1 and Γ_2 values of curves λ are equal, i.e. $\lambda_1(\Gamma_1) = \lambda_2(\Gamma_2)$. Thus average ash content to a concentrates mix is equal to the set constant β_C and the maximum output of a concentrate is reached.

For any two lines of demarcation with outputs Γ_1 and Γ_2, and average ash contents $\beta_1(\Gamma_1)$, $\beta_2(\Gamma_2)$ at a dale of the first coal in a mix p, and the second $q = 1 - p$, we will receive a cumulative output of a concentrate

$$\Gamma = p\Gamma_1 + q\Gamma_2 . \tag{5}$$

So average ash content

$$\beta_C = \frac{\Gamma_1 p\beta_1(\Gamma_1) + \Gamma_2 q\beta_2(\Gamma_2)}{p\Gamma_1 + q\Gamma_2}, \tag{6}$$

Values Γ_1, Γ_2 also should satisfy to equality (6) and provide a maximum of expression (5). We will designate $F(\Gamma_1, \Gamma_2) = \dfrac{\Gamma_1 p\beta_1(\Gamma_1) + \Gamma_2 q\beta_2(\Gamma_2)}{p\Gamma_1 + q\Gamma_2} - \beta_C$. It is required to find a maximum $\Gamma = p\Gamma_1 + q\Gamma_2 \Rightarrow \max$, under a condition we $F(\Gamma_1, \Gamma_2) = 0.$ Will consider auxiliary function $L(\Gamma_1, \Gamma_2) = p\Gamma_1 + q\Gamma_2 + \mu F(\Gamma_1, \Gamma_2)$, where μ – an uncertain multiplier by Lagranzh. Then for Γ_1, Γ_2 we will write down μ system

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial \Gamma_1}=p+\mu\dfrac{\partial F}{\partial \Gamma_1}=0\\ \dfrac{\partial L}{\partial \Gamma_2}=q+\mu\dfrac{\partial F}{\partial \Gamma_2}=0\\ F(\Gamma_1,\Gamma_2)=0\end{array}\right\}. \tag{7}$$

It is necessary to prove that Γ_1, Γ_2 if and so if is a solution of system (7), when $\lambda_1(\Gamma_1)=\lambda_2(\Gamma_2)$. For this purpose it is necessary to find partial derivative functions $F(\Gamma_1,\Gamma_2)$. It is easy to check that the system will take a form

$$\left.\begin{array}{l}\mu\{\Gamma_1[p\Gamma_1+q\Gamma_2]\beta_1'(\Gamma_1)+q\Gamma_2[\beta_1(\Gamma_1)-\beta_2(\Gamma_2)]\}+(p\Gamma_1+q\Gamma_2)^2=0\\ \mu\{x_2[p\Gamma_1+q\Gamma_2]\beta_2'(\Gamma_2)-p\Gamma_1[\beta_1(\Gamma_1)-\beta_2(\Gamma_2)]\}+(p\Gamma_1+q\Gamma_2)^2=0\\ p\Gamma_1\beta_1(\Gamma_1)+q\Gamma_2\beta_2(\Gamma_2)=(p\Gamma_1+q\Gamma_2)\beta_C\end{array}\right\}. \tag{8}$$

Eliminating parameter μ from first two equations of system (8) we will write down equality

$$(p\Gamma_1+q\Gamma_2)[\Gamma_1\beta_1'(\Gamma_1)-\Gamma_2\beta_2'(\Gamma_2)+\beta_1(\Gamma_1)-\beta_2(\Gamma_2)]=0.$$

From last expression dependence follows

$$\frac{d}{d\Gamma_1}\left[\Gamma_1\beta_1(\Gamma_1)\right]=\frac{d}{d\Gamma_2}\left[\Gamma_2\beta_2(\Gamma_2)\right]. \tag{9}$$

Between curves λ and β there is a relation $\Gamma\beta(\Gamma)=\int_0^{\Gamma}\lambda(\tau)d\tau$, so equation (9) turns to dependence $\lambda_1(\Gamma_1)=\lambda_2(\Gamma_2)$, finishes the theorem proof.

Consequence 1. The system solution $\begin{cases}\lambda_1(\Gamma_1)=\lambda_2(\Gamma_2),\\ p\Gamma_1[\beta_1(\Gamma_1)-\beta_C]+q\Gamma_2[\beta_2(\Gamma_2)-\beta_C]=0\end{cases}$ defines values Γ_1, Γ_2 allowing to provide the maximum output of a concentrate with set ash content at separate enrichment of two coals.

The theorem 2. At separate enrichment of various n coals for the purpose of reception set average ash content β_C, the maximum cumulative concentrate output will be received if all boundary ash contents will be equal $\lambda_1(\Gamma_1)=\lambda_2(\Gamma_2)=...=\lambda_n(\Gamma_n)$.

The proof of this theorem is spent similarly previous. Let designate $p_1, p_2, ..., p_n$ shares of each coal in a mix participation, $p_1 + p_2 + ... + p_n = 1$. An output of a concentrate Γ and its average ash content are equal β_c $\Gamma = \sum_{i=1}^{n} p_i \Gamma_i$, $\frac{1}{\sum p_i \Gamma_i} \sum p_1 \Gamma_i \beta_i(\Gamma_i) = \beta_C$.

Consequence 2. To define optimum values $\Gamma_1, \Gamma_2, ..., \Gamma_n$ it is necessary to solve system in common $\begin{cases} \lambda_1(\Gamma_1) = \lambda_2(\Gamma_2) = ... = \lambda_n(\Gamma_n) \\ \sum p_i \Gamma_i (\beta_i - \beta_C) = 0 \end{cases}$. The found values will provide a maximum of an concentrate output at set average ash content.

CONCLUSION

1. The analytical solution method of the ordinary coals mix characteristics finding is offered, allowing to carry out calculations with the set accuracy and to use it as a component of the solution more general problems on the computer.
2. The expression describing relation between an output and boundary ash content is found.
3. The problem of a finding optimum ash content of the mix divisions is solved at demanded average ash content of the concentrate on the basis of the offered method and practical use of the Reinhardt theorem. It allows to solve a problem of ordinary coals optimum batch composite definition.

REFERENCES

1. Garus V. K, Grachyov O. V, Pozhidayev V. F, Polulyah A.D., 2003: Formalization of Dividing Processes Results in Coal Enrichment: the Monography. Publishing Open Company "NVF" the Stack ", Lugansk: 176.
2. Meller E.F., 1935: Theory of coals enrichment research. Kharkov: 115.
3. Fomenko T.G., Butovetsky B.C., Pogartseva E.M., 1978: Coals enrichment research. Nedra, Moscow: 263.
4. Tikhonov O. N., 1984: Laws effective separation of minerals in mineral processing. Nedra, Moscow: 207.
5. Sokolov V. G., 1962: Coals' concentratibility curves. Gosgortekhizdat, Moscow:191.
6. Kotkin A. M. , Gerashchenko K. D.,1972: Determination of rates of withdrawal and enrichment as a commercial product // Enrichment and briquetting of coal, № 6: P.76-79.
7. Ulshin V.A., Pozhydayev V. F., Grachyov O. V., 2009: Synthesis of batching problem and optimization of the concentrating factory scheme. // V.Dal East Ukrainian National University Journal, №1 (131): 243-249
8. Grachyov O. V., Pozhydayev V. F., Ulshin V.A., 2005: System of process management for the preparation of the coal batch in order to optimize the separation on an arbitrary criterion. // Minerals enrichment, scientific works, Dnipropetrovsk: 158-165.

9. Grachyov O. V., 1999: Practical analysis of one class of antimodal distributions in order to find their analytical description // V.Dal East Ukrainian National University Journal, №3 (18): 73-76.
10. Pozhydayev V. F., Grachyov O. V., 2002: Finding the distribution of particle sizes as a random variable for the different fractions. // Lugansk branch of IAI works, № 1 (4): 72-75.
11. Pozhydayev V. F., Grachyov O. V., 2002 Type of the weight distribution function of density and ash content of coal fractions // Scientists to the enterprises and institutions in the region, scientific works, Lugansk: 35.
12. Grachyov O. V., Pozhydayev V. F., Ulshin V.A., 2006: Developing an algorithm of recovering the parameters of the particle size distribution function on the minimum amount of testing. // Lugansk branch of IAI works, № 2 (13): 107-110.
13. Grachyov O. V., 2007: Algorithm for calculating the parameters of the distribution of the coal mixture. // Minerals enrichment, scientific works, Dnipropetrovsk: 39-41.
14. Pozhydayev V. F., Grachyov O. V., 2000: Terms of mathematical description of the coal ash distribution by fractions. // V.Dal East Ukrainian National University Journal, №9 (31): 99-105.
15. Ulshin V.A., Pozhydayev V. F., Grachyov O. V., 2009: Synthesis of batching problem and optimization of the concentrating factory scheme. // V.Dal East Ukrainian National University Journal, №1 (131): 243-249
16. Grachyov O. V., Pozhydayev V. F., 2009: Products division quality management system with the calculation of expected enrichment rates. // Minerals enrichment, scientific works, Dnipropetrovsk: 197-203.
17. Grachyov O.V., 2003: Efficiency of stability increase of coal concentrates quality. // V.Dal East Ukrainian National

АНАЛИТИЧЕСКОЕ РЕШЕНИЕ ЗАДАЧИ ОПРЕДЕЛЕНИЯ ХАРАКТЕРИСТИК СМЕСИ НЕСКОЛЬКИХ УГЛЕЙ

Пожидаев В.Ф., Грачев О.В.

Аннотация. В данной статье рассмотрена возможность аналитического решения задачи определения характеристик смеси нескольких углей.

Ключевые слова: обогащение смеси углей, кривые обогатимости, суммарный выход концентрата, средняя зольность, демаркационные слои, характеристики смеси.