# THE TWO-TIME GREEN'S FUNCTIONS IN THE METHOD OF BRIEF DESCRIPTION

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**Summary.** On the basis of method N. Bogolyubov about brief description of the non-equilibrium states asymptotic of Green's function is investigated in the hydrodynamic approaching for the degenerated systems of Boze-particles with weak co-operation. Lowfrequency asymptotic of Green's normal functions in approaching of ideal and viscid liquid is calculated.

Key words: green's functions, lowrequency asymptotics, quantum kinetics.

### INTRODUCTION

Lowfrequency asymptotic of Green's electrodynamic function was in-process [1] investigated with bringing in of vehicle of phenomenological theory. Other, more successive going near the calculation of kinematics asymptotic of Green's function Boze-systems with the spontaneously broken symmetry, was illustrated in the article [2].

In the article [2] lowfrequency asymptotic of Green's functions was investigated in terms of the linearized interval of collisions of quasi-particles. In given works asymptotic of Green's functions is studied in a hydrodynamic limit, when frequency is small as compared to reverse time of relacsation of  $\tau_1^{-1}$ , and a wavevector is small as compared to reverse length of free run of particles  $\Gamma^1$ . This task is related to consideration of equalizations of hydrodynamics of degenerated Boze-systems [3]. Here we will save denotations, in-use in works [2-4].

#### **OBJECTS AND PROBLEMS**

# 1 Connection between lowfrequency asymptotic of Green's functions and equalizations of hydrodynamics.

Following work [2], will write the kinematics parameters of the examined system.

 $F_{FP}(x,t)$  - is a function of distributing of quasi-particles [4],  $\eta_F^2(x,t)$  - is a closeness of Boze-runback [4],  $V_{FK}(x,t)$  - is speed of superfluid components [4] (the index of «F» means that the proper sizes are certain for the system, being in the variable external field of F(x,t)).

On the basis of method N.Bogolyubov about brief description of the non-equilibrium states [5] will suppose that for time of t, large as compared to time of relaxation of  $\tau_1$  for the kinematics parameters of the system  $\xi_{Fa}(x,t)$  correlation will be just:

$$\xi_{F\alpha}(x,t) \xrightarrow{l \to n} \xi_{F\alpha}(x,\psi_{F\beta}(\widetilde{x},t);F(t),...) \equiv \xi_{F\alpha}(x,\psi_{F\beta}(\widetilde{x},t),\tag{1}$$

where  $\xi_{F\alpha}(x,\psi_{F\beta}(\widetilde{x},t))$  - is functional from  $\psi_{F\beta}(x,t)$  and  $F(x,t),\dot{F}(x,t),...$ 

Here:  $\psi_{F\beta}(x,t) = \{Y_{F0}(x,t), \eta_{F0}(x,t), U_{Fk}(x,t), \gamma_{Fk}(x,t)\}$  - are hydrodynamic parameters:  $Y_{F0}(x,t)$  - it is a reverse temperature,  $\eta_{F0}(x,t)$  - is a parameter, related to the closeness of Boze-condensate,  $U_{Fk}(x,t)$  - it is speed normal components and at  $\gamma_{Fk}(x,t) = V_{Fk}(x,t) - U_{Fk}(x,t)$ .

Taking (1.1) into account, will present the sizes of  $\eta_{\xi a}^{\xi}(x,t)$  determined a formula [2], in a kind:

$$\eta_{\xi a}^{\xi}(x-x',t-t') = \int d^3x'' R_{\xi a;\psi\beta}(x-x''S(_{\psi\beta}^{(\xi)}(x''-x',-t') + g_{\xi a}^{(\xi)}(x-x',t-t'))$$
 (2)

where: C - numerical functions;  $R_{\xi\alpha;\psi_{\beta}}(x), S_{\psi\beta}^{(\xi)}(x,t)$  and  $g_{\xi\alpha}^{(\xi)}(x,t)$  set correlations

$$\begin{split} R_{\xi\alpha;\psi_{\beta}}(x-x'') &= \left[\frac{\delta_{\xi\alpha}(x,\psi(\widetilde{x},t))}{\delta\psi_{\beta}(x',t)}\right]_{0}, \ S_{\psi\beta}^{(\xi)}(x-x',t-t') = \left[\frac{\delta\psi_{F\beta}(x,t)}{\delta F(x',t')}\right]_{0}, \\ g_{\psi\alpha}^{(\xi)}(x-x',t-t') &= \left[\frac{\delta\xi_{F\alpha}(x,\psi_{F}(\widetilde{x},t);t)}{\delta F(x',t)}\right]_{0}, \ \psi_{\beta}(x,t) = \psi_{F\beta}(x,t)\Big|_{F} = 0, \\ \xi_{\alpha}(x,\psi(\widetilde{x},t)) &= \xi_{F\alpha}(x,\psi_{F}(\widetilde{x},t);t)\Big|_{F} = 0 \end{split}$$

(here: in future  $[A]_0$  the value of size means an equilibrium A).

We will notice that after the calculation of variation derivates in (1.2), it is necessary to put  $[U_{Fk}(x,t)]_0 = [\gamma_{Fk}(x,t)]_0 = 0$ .

Passing in (1.2) to Fur'e-components, will get:

$$\eta_{\xi\alpha}^{(\xi)}(k,\omega) = R_{\xi\alpha;\psi\beta}(k)S_{\psi\beta}^{(\xi)}(k,\omega) + g_{\xi\alpha}^{(\xi)}(k,\omega), \qquad (3)$$

We will find equalization for the calculation of sizes  $S_{\psi\beta}^{(\xi)}(k,\omega)$  and  $g_{\xi\alpha}^{(\xi)}(k,\omega)$ . Functions  $R_{\xi\alpha,\psi\beta}(k)$  assumed known (they are determined the results of joint decision of kinetic and hydrodynamic equalizations of superfluid Boze-liquid [3],[4]).

Putting expression (1.3) in equalization [2], find:

$$-i\omega R_{\xi\alpha;\psi\beta}(k)S_{\psi\beta}^{(\xi)}(k,\omega) - N_{\xi\alpha;\xi\beta}(k)R_{\xi\beta;\psi\alpha}(k)S_{\psi\alpha}^{(\xi)}(k,\omega) =$$

$$= q_{\xi\alpha}^{(\xi)}(k,\omega) + N_{\xi\alpha;\xi\beta}(k)g_{\xi\beta}^{(\xi)}(k,\omega) + i\omega g_{\xi\beta}^{(\xi)}(k,\omega)$$
(4)

Here  $N_{\xi\alpha;\xi\beta}(k)$  - Fur'e is components of sizes  $N_{\xi\alpha;\xi\beta}(x)$ , determined together with sources  $q_{\xi\alpha}^{(\xi)}(k,\omega)$  in [2].

We will transform equalization (1.4) to more simple kind. To that end we will rewrite kinetic equalization:

$$\frac{\partial \xi_{\alpha}(x,t)}{\partial t} = L_{\xi\alpha}(x,t) \tag{5}$$

in form conditioned correlation (1.1):

$$-\int d^3x' \int d^3x'' \frac{\delta_{\xi\alpha}(x,\psi(\widetilde{x},t))}{\delta\psi_{\beta}(x',t)} \frac{\delta\psi_{\beta}(x',t)}{\delta\zeta_{\gamma}(x'',t)} \frac{\partial\zeta_{\gamma_n}(x'',t)}{\partial x_n''} = L_{\xi\alpha}(x,t).$$
 (6)

(Integrals of collisions  $L_{\xi\alpha}(x,t)$  determined formulas [2].) Thus we utillized equalizations of hydrodynamics [3]:

$$\frac{\partial \zeta_{\alpha}(x,t)}{\partial t} = \frac{\partial \zeta_{\alpha n}(x,t)}{\partial x_n},\tag{7}$$

$$\begin{split} & \zeta_{\alpha}(x,t) = \left\{ \varepsilon(x,t), \sigma(x,t), \pi_k(x,t), V_m(x,t) \right\}, \ \varepsilon(x,t) \ \text{- density of energy,} \ \sigma(x,t) \ \text{- density of weight,} \ V_m(x,t) \ \text{- speed superfluid components,} \ \pi_k(x,t) \ \text{- impulse density,} \\ & \zeta_{cm}(x,t) = \left\{ q_k(x,t), \pi_k(x,t), t_{ik}(x,t), z(x,t) \delta_{ik} \right\}, \ q_k(x,t) \ \text{- density of a stream of energy,} \\ & t_{ik}(x,t) \ \text{- density of a stream of an impulse,} \ z(x,t) = h(x,t) + \frac{1}{2} V(x,t)^2, \\ & h(x,t) = h_F(x,t) \Big|_E = 0 \ (\text{look [2]}). \end{split}$$

Varying equalization (1.6) on parameters  $\psi_{\alpha}(x,t)$  and utillizing a formula (1.2) will get in terms of Fur'e-component:

$$-ik_{n}R_{\xi_{\alpha};\psi_{\lambda}}(k)\Lambda_{\psi_{\lambda};\zeta_{\gamma}}^{-1}T_{\zeta_{\gamma_{n}};\psi_{\delta}}(k) = N_{\xi\alpha;\xi\lambda}(k)R_{\xi\lambda;\psi\delta}(k)$$

$$\Lambda_{\psi_{\lambda};\zeta_{\gamma}}^{-1} = \left[\frac{\partial\psi_{\lambda}}{\partial\zeta_{\gamma}}\right], T_{\zeta_{\gamma_{n}};\psi_{\delta}}(k) = \int d^{3}x e^{-ikx}T_{\zeta_{\gamma_{n}};\psi_{\delta}}(k)$$

$$T_{\zeta_{\gamma_{n}};\psi_{\delta}}(x-x') = \left[\frac{\partial\zeta_{\gamma_{n}}(x,t)}{\partial\zeta_{\psi_{\delta}}(x',t)}\right]_{0}.$$
(8)

Taking into account a formula (1.8), will rewrite equalization (1.4) in a kind

$$\left[-\omega\Lambda_{\zeta_{\alpha};\psi_{\beta}}^{-1} + K_{n}T_{\zeta\alpha_{n};\psi_{\beta}}(k)\right]S_{\psi_{\beta}}^{(\xi)}(k,\omega) = Q_{\zeta_{\alpha}}^{(\xi)}(k,\omega),$$

$$Q_{\zeta\alpha}^{(\xi)}(k,\omega) = -i\sum_{\zeta_{\alpha};\zeta_{\beta}}q_{\zeta_{\beta}}^{(\xi)}(k,\omega) - i\sum_{\zeta_{\alpha};\xi_{\beta}}N_{\xi_{\beta};\xi_{\gamma}}(k)g_{\xi\gamma}^{(\xi)}(k,\omega)$$

$$\sum_{\zeta_{\alpha};\xi_{\beta}} = \left[\frac{\partial\zeta_{\alpha}}{\partial\xi_{\beta}}\right]_{0} \Lambda_{\zeta_{\alpha};\psi_{\gamma}} = \left[\frac{\partial\zeta_{\alpha}}{\partial\psi_{\gamma}}\right]_{0}$$
(9)

Correlations were thus utillized:

$$\sum_{\zeta_{\alpha};\xi_{\beta}} R_{\xi_{\beta};\psi_{\gamma}}(k) = \Lambda_{\zeta_{\alpha};\psi_{\gamma}} \qquad \sum_{\zeta_{\alpha};\xi_{\beta}} g_{\xi\beta}^{(\xi)}(k,\omega) = 0.$$
 (10)

Easily to find independent equalization for determination of sizes  $g_{\xi\beta}^{(\xi)}(k,\omega)$ . Indeed on foundation (1.4), (1.8), (1.9) have:

$$-iR_{\xi_{\alpha};\psi_{\delta}}(k)\Lambda_{\psi_{\delta};\zeta_{\gamma}}^{(-1)}\left\{\sum_{\zeta_{\gamma};\xi_{\beta}}Q_{\xi_{\beta}}^{(\xi)}(k,\omega)+\sum_{\zeta_{\gamma};\xi_{\beta}}N_{\xi_{\beta};\xi_{\lambda}}(k)g_{\xi_{\lambda}}^{(\xi)}(k,\omega)\right\}=$$

$$=-iq_{\xi_{\alpha}}^{(\xi)}(k,\omega)-iN_{\xi_{\alpha};\xi_{\beta}}(k)g_{\xi_{\beta}}^{(\xi)}(k,\omega)+\omega g_{\xi_{\alpha}}^{(\xi)}(k,\omega)$$

$$\sum_{\zeta_{\alpha};\xi_{\beta}}g_{\xi_{\beta}}^{(\xi)}(k,\omega)=0.$$
(11)

Equalizations (1.9), (1.11) it is possible to decide in the theory of indignations on small wavevectors k and to the small parameter of effective co-operation  $\lambda$  [6], [7].

We will write hydrodynamic asymptotic of Green's functions in terms of sizes  $S_{w_{-}}^{(\xi)}(k,\omega)$ . Putting correlations (1.3) in equalization (1,4.5), will get:

$$G_{\xi'\xi}^{(t)}(k,\omega) = \left[ Sp \, \sigma_{\xi_{\alpha}}(k) \widetilde{\xi}'_{2}(k) + Sp \rho_{eq} \widetilde{\xi}'_{\xi_{\alpha}}(k) \right] R_{\xi_{\alpha};\psi_{\beta}}(k) S_{\psi_{\beta}}^{(\xi)}(k,\omega) +$$

$$+ g_{\xi_{\alpha}}^{(\xi)}(k,\omega) \left[ Sp \, \sigma_{\xi_{\alpha}}(k) \widetilde{\xi}'_{2}(k) + Sp \rho_{eq} \widetilde{\xi}'_{\xi_{\alpha}}(k) \right] + Sp \overline{\rho}^{(\xi)}(k,\omega) \widetilde{\xi}'_{2}$$

$$(12)$$

Sizes  $\sigma_{\xi_a}(k)$ ,  $\overline{\rho}^{(\xi)}(k,\omega)$ ,  $\widetilde{\xi}'_{\xi_a}(k)$ ,  $\widetilde{\xi}'_2$  determined correlations [2].

A formula (1.12) establishes a connection between asymptotic of Green's functions and hydrodynamic parameters of superfluid Boze-liquid.

#### 2. Approaching of ideal liquid

From definition (1.12) and the equations (1.9) follows that the polar part of hydrodynamic asymtotics Green's functions  $G^{(t)}_{\xi'\xi}(k,\omega)$  is connected with sizes  $S^{(\xi)}_{\psi_\alpha}(k,\omega)$ . In this section we will give the general analysis of sizes  $S^{(\xi)}_{\psi_\alpha}(k,\omega)$ , proceeding from the equation (1.9), being limited to approach of an ideal liquid. As representations  $S^{(\xi)}_{U_n}(k,\omega) = \frac{K_n}{K} S^{(\xi)}_1(k,\omega)$ ,  $S^{(\xi)}_{\gamma_n}(k,\omega) = \frac{K_n}{K} S^{(\xi)}_2(k,\omega)$ , the equation (1.9) is possible to formulate for four scalar functions are fair  $S^{(\xi)}_{\gamma_0}(k,\omega)$ ,  $S^{(\xi)}_1(k,\omega)$ ,  $S^{(\xi)}_2(k,\omega)$ .

$$\left[-\omega\Lambda_{\zeta_{\alpha};\psi_{\beta}} + K_{n}T_{\zeta\alpha_{n};\gamma_{0}}(k)\right]S_{\gamma_{0}}^{(\xi)}(k,\omega) + \left[-\omega\Lambda_{\zeta_{\alpha};U_{n}}\frac{K_{n}}{K} + \frac{K_{L}K_{n}}{K}T_{\zeta_{\alpha_{L}};U_{n}}(k)\right]S_{1}^{(\xi)}(k,\omega) + \left[-\omega\Lambda_{\zeta_{\alpha};\gamma_{n}}\frac{K_{n}}{K} + \frac{K_{L}K_{n}}{K}T_{\zeta_{\alpha_{L}};\gamma_{n}}(k)\right]S_{2}^{(\xi)}(k,\omega) + \left[-\omega\Lambda_{\zeta_{\alpha};\eta_{0}} + T_{\zeta_{\alpha};\eta_{0}}(k)\right]S_{\eta_{0}}^{(\xi)}(k,\omega) = Q_{\zeta_{\alpha}}^{(\xi)}(k,\omega)$$
(13)

Sizes  $T_{\zeta_{2nw_{R}}}(k)$  it is possible to find, drawing on the results of work [3] on the theory of indignations on small wavevectors k:  $T_{\zeta_{2n;\psi_{\beta}}}(k) = T_{\zeta_{2n;\psi_{\lambda}}}^{[0,]} + T_{\zeta_{2n;\psi_{\lambda}}}^{[1,]} + \dots.$ 

$$T_{\zeta_{2n;W,\beta}}(k) = T_{\zeta_{2n;W,\beta}}^{[0,]} + T_{\zeta_{2n;W,\beta}}^{[1,]} + \dots$$
 (14)

Calculating  $T^{[0,]}_{\zeta_{2n;w_2}}$ ,  $\Lambda_{\zeta_{2;w_2}}$  and taking into account  $Q^{(\xi)(0,)}_{\pi_n}=Q^{(\xi)(0,)}_{V_n}=0$ , will get on foundation (2.1):

$$-\omega \frac{\partial}{\partial \mathcal{E}} S_{Y_0}^{(\xi)}(k,\omega) + k(p + \frac{\partial}{\mathcal{E}}) S_1^{(\xi)}(k,\omega) + \delta_{\xi} \mu S_2^{(\xi)}(k,\omega) - 2\omega \sqrt{n_0 \frac{\mathcal{E}}{\partial n_0}} S_{n_0}^{(\xi)}(k,\omega) = Q_{\varepsilon}^{(\xi)(0,\cdot)},$$

$$K \frac{\partial P}{\partial Y_0} S_{Y_0}^{(\xi)}(k,\omega) - \omega \sigma S_1^{(\xi)}(k,\omega) + \omega \sigma_s S_2^{(\xi)}(k,\omega) + 2k \sqrt{n_0} \frac{\partial P}{\partial n_0} S_{n_0}^{(\xi)}(k,\omega) = 0$$

$$-\omega \frac{\partial \sigma}{\partial Y_0} S_{Y_0}^{(\xi)}(k,\omega) + k\sigma S_1^{(\xi)}(k,\omega) + k\sigma_s S_2^{(\xi)}(k,\omega) - 2\omega \sqrt{n_0} \frac{\partial \sigma}{\partial n_0} S_{n_0}^{(\xi)}(k,\omega) = Q_{\sigma}^{(\xi)(0,\cdot)}$$

$$k \frac{\partial \mu}{\partial Y_0} S_{Y_0}^{(\xi)}(k,\omega) - \omega S_1^{(\xi)}(k,\omega) - \omega S_2^{(\xi)}(k,\omega) + 2k \frac{\partial \mu}{\partial n_0} \sqrt{n_0} S_{n_0}^{(\xi)}(k,\omega) = 0,$$

$$(15)$$

(here:  $[\eta_0]_0 = \sqrt{n_0}$ , p - pressure in system Boze-particles,  $\mu$  - the chemical potent [3],  $A^{[n,]}$  means order n on a wave vector k and any on parametre of effective interaction  $\lambda$  of size A in the hydrodynamic theory,  $B^{(m,)}$  order m on k and any on sizes B sizes in the kinetic theory).

Determinant of this system  $\Delta(k, \omega)$  equal

$$\Delta(k,\omega) = 2\sqrt{n_0\sigma_n \frac{\partial(\dot{\varepsilon},\sigma)}{\partial(n_0, Y_0)}} \sigma(k,\omega) , \qquad (16)$$

$$\sigma(k,\omega) = \omega^4 - k^2 \omega^2 \left[ \left( \frac{\partial P}{\partial \sigma} \right) \frac{s}{\sigma} + \frac{s^2}{Y_0 \sigma C_v} \frac{\sigma^3}{\sigma_n} \right] + k^4 \left[ \frac{\sigma^3}{\sigma_n} \frac{s^2}{\sigma Y_0 C_v} \left( \frac{\partial P}{\partial \sigma Y_0} \right) \right] ,$$

where:  $S = Y0(\dot{\varepsilon} + p - \sigma\mu)$  it is entropy of the system [3],  $C_v = -\frac{1}{Y_0^2} \left( \frac{\partial \dot{\varepsilon}}{\partial Y_0} \right)_{\hat{\rho}}$  it is a heat capacity at a permanent volume,  $\sigma_s$  and  $\sigma_n$  are closenesses, accordingly, the superfluid and normal component,  $\frac{\partial (\dot{\mathcal{E}},\sigma)}{\partial (n_0,Y_0)}$  it is a determinant of Jacobi.

Expression for  $\sigma(k,\omega)$  it is possible to present in a form

$$\sigma(k,\omega) = (\omega^2 - \tilde{N}_0^2 K^2)(\omega^2 - \tilde{N}_1^2 K^2)$$
(17)

Here  $C_0$ ,  $C_1$  are speeds of the first and second sounds [8]:

$$C_{0,1}^{2} = \frac{1}{2} \left[ \left( \frac{\partial P}{\partial \sigma} \right)_{\frac{s}{\sigma}} + \frac{S^{2}}{Y_{0}} \frac{\sigma^{3}}{\sigma_{n}} \frac{1}{\sigma C_{\nu}} \right] \pm \sqrt{\frac{1}{4} \left[ \left( \frac{\partial P}{\partial \sigma} \right)_{\frac{s}{\sigma}} + \frac{S^{2}}{Y_{0}} \frac{\sigma^{3}}{\sigma_{n}} \frac{1}{\sigma C_{\nu}} \right] - \frac{\sigma^{3}}{\sigma_{n}} \left( \frac{\partial P}{\partial \sigma Y_{0}} \right) \frac{S^{2}}{\sigma Y_{0} C_{\nu}}}$$
(18)

Sizes  $S_{Y_n}^{(\xi)}(k,\omega)$ ,  $S_1^{(\xi)}(k,\omega)$ ,  $S_2^{(\xi)}(k,\omega)$ ,  $S_{n_0}^{(\xi)}(k,\omega)$ , thus, determined formulas:

$$\begin{split} S_{Y_0}^{(\xi)}(k,\omega) &= \frac{2\sqrt{n_0}}{\Delta(k,\omega)} \Bigg\{ \omega^3 \sigma_n (Q_{\varepsilon}^{(\xi)(0,)} \frac{\partial \sigma}{\partial n_0} - Q_{\sigma}^{(\xi)(0,)} \frac{\partial \dot{\varepsilon}}{\partial n_0}) + \omega k^2 \times \\ &\times \Bigg[ Q_{\sigma}^{(\xi)(0,)}(\sigma_3(\sigma\mu - p - \dot{\varepsilon}) \frac{\partial \mu}{\partial n_0} + (p + \dot{\varepsilon} - \sigma_s \mu) \frac{\partial P}{\partial n_0} - \sigma_n Q_{\sigma}^{(\xi)(0,)} \frac{\partial P}{\partial n_0} \Bigg] \Bigg\} \\ S_1^{(\xi)}(k,\omega) &= \frac{2\sqrt{n_0}}{\Delta(k,\omega)} \Bigg\{ k^3 \sigma^3 \frac{\partial (P,\mu)}{\partial (Y_0,n_0)} (\mu Q_{\sigma}^{(\xi)(0,)} - Q_{\varepsilon}^{(\xi)(0,)}) + k\omega^2 \times \\ &\times \Bigg[ Q_{\varepsilon}^{(\xi)(0,)} \Bigg( \frac{\partial (P,\sigma)}{\partial (Y_0,n_0)} - \sigma_s \frac{\partial (P,\mu)}{\partial (Y_0,n_0)} \Bigg) + Q_{\sigma}^{(\xi)(0,)} \Bigg( \frac{\partial (\dot{\varepsilon},\sigma)}{\partial (Y_0,n_0)} - \sigma_s \frac{\partial (\dot{\varepsilon},\mu)}{\partial (Y_0,n_0)} \Bigg) \Bigg] \Bigg\} \\ S_1^{(\xi)}(k,\omega) &= \frac{2\sqrt{n_0}}{\Delta(k,\omega)} \Bigg\{ k^3 \sigma^3 \frac{\partial (P,\mu)}{\partial (Y_0,n_0)} (\mu Q_{\sigma}^{(\xi)(0,)} - Q_{\varepsilon}^{(\xi)(0,)}) + k\omega^2 \times \\ &\times \Bigg[ Q_{\varepsilon}^{(\xi)(0,)} \Bigg( \frac{\partial (P,\sigma)}{\partial (Y_0,n_0)} - \sigma_s \frac{\partial (P,\mu)}{\partial (Y_0,n_0)} \Bigg) + Q_{\sigma}^{(\xi)(0,)} \Bigg( \frac{\partial (\dot{\varepsilon},\sigma)}{\partial (Y_0,n_0)} - \sigma_s \frac{\partial (\dot{\varepsilon},\mu)}{\partial (Y_0,n_0)} \Bigg) \Bigg] \Bigg\} \\ S_2^{(\xi)}(k,\omega) &= \frac{2\sqrt{n_0}}{\Delta(k,\omega)} \Bigg\{ k^3 \frac{\partial (P,\mu)}{\partial (Y_0,n_0)} \Bigg[ \sigma Q_{\varepsilon}^{(\xi)(0,)} - (p + \dot{\varepsilon}) Q_{\sigma}^{(\xi)(0,)} \Bigg] + k\omega^2 \times \\ &\times \Bigg[ \Bigg( \frac{\partial (\sigma,P)}{\partial (Y_0,n_0)} - \sigma \frac{\partial (\sigma,\mu)}{\partial (Y_0,n_0)} \Bigg) Q_{\varepsilon}^{(\xi)(0,)} + \Bigg( \frac{\partial (P,\dot{\varepsilon})}{\partial (Y_0,n_0)} - \sigma \frac{\partial (\mu,\dot{\varepsilon})}{\partial (Y_0,n_0)} \Bigg) Q_{\sigma}^{(\xi)(0,)} \Bigg] \Bigg\} \\ S_{\eta_0}^{(\xi)}(k,\omega) &= \frac{1}{\Delta(k,\omega)} \Bigg\{ \omega^3 \sigma_n (Q_{\sigma}^{(\xi)(0,)} \frac{\partial \dot{\varepsilon}}{\partial Y_0} - Q_{\varepsilon}^{(\xi)(0,)} \frac{\partial \sigma}{\partial Y_0} + \omega k^2 \times \\ &\times \Bigg[ \sigma_n \frac{\partial P}{\partial Y_0} Q_{\varepsilon}^{(\xi)(0,)} + Q_{\sigma}^{(\xi)(0,)} (\frac{\partial P}{\partial Y_0} (\sigma_s \mu - p - \dot{\varepsilon}) + \sigma_s (p + \dot{\varepsilon} - \sigma_s \mu) \frac{\partial \mu}{\partial Y_0} \Bigg] \Bigg\} \end{aligned}$$

The structure of correlations (2.7) specifies on that hydrodynamic asymptotic of Green's functions  $G_{\xi'\xi}^{(+)}(k,\omega)$  in approaching of ideal liquid has two poles, proper different elementary excitations  $\omega = C_0 K$  and  $\omega = C_1 K$ .

#### 3. Approaching of viscid liquid

In the previous section of research of poles of Green's function  $G_{\xi'\xi}^{(+)}(k,\omega)$  led in approaching of ideal superfluid Boze-liquids, therefore dissipative effects were not taken into account in a formula (2.4). We will rotin here, as viscidity and heat conductivity influence on forming of poles of hydrodynamic asymptotic of Green's functions  $G_{\xi'\xi}^{(+)}(k,\omega)$ . To that end we will rewrite equalization (2.1) in the interesting us approaching:

$$\left(-\omega\frac{\partial\dot{\varepsilon}}{\partial Y_{0}} + ik^{2}\frac{\theta}{Y_{0}^{2}}\right)S_{Y_{0}}^{(\xi)}(k,\omega) + k(p+\dot{\varepsilon})S_{1}^{(\xi)}(k,\omega) + \\
+k\sigma_{s}\mu S_{2}^{(\xi)}(k,\omega) - 2\omega\frac{\partial\dot{\varepsilon}}{\partial n_{0}}\sqrt{n_{0}}S_{\eta_{0}}^{(\xi)}(k,\omega) = Q_{\varepsilon}^{(\xi)(0,0)} \\
k\frac{\partial P}{\partial Y_{0}}S_{Y_{0}}^{(\xi)}(k,\omega) + \left[-\omega\sigma - ik^{2}\left(\frac{4}{3}\eta + \zeta_{2}\right)\right]S_{1}^{(\xi)}(k,\omega) + (-\omega\sigma_{s} - ik^{2}\zeta_{1}) \times \\
\times S_{2}^{(\xi)}(k,\omega) + 2k\sqrt{n_{0}}\frac{\partial P}{\partial n_{0}}S_{\eta_{0}}^{(\xi)}(k,\omega) = \frac{K_{n}}{K}Q_{\pi_{n}}^{(\xi)(1,0)} \\
-\omega\frac{\partial\sigma}{\partial Y_{0}}S_{Y_{0}}^{(\xi)}(k,\omega) + k\sigma S_{1}^{(\xi)}(k,\omega) + k\sigma_{s}S_{2}^{(\xi)}(k,\omega) - \\
-2\omega\sqrt{n_{0}}\frac{\partial\sigma}{\partial n_{0}}S_{\eta_{0}}^{(\xi)}(k,\omega) = Q_{\sigma}^{(\xi)(0,0)} \\
k\frac{\partial\mu}{\partial Y_{0}}S_{Y_{0}}^{(\xi)}(k,\omega) + (-\omega\sigma - ik^{2}\zeta_{4})S_{1}^{(\xi)}(k,\omega) + (-\omega-ik^{2}\sigma_{s}\zeta_{3}) \times \\
\times S_{2}^{(\xi)}(k,\omega) + 2k\sqrt{n_{0}}\frac{\partial\mu}{\partial n_{0}}S_{\eta_{0}}^{(\xi)}(k,\omega) = \frac{K_{n}}{K}Q_{\nu_{n}}^{(\xi)(1,0)}$$
(20)

Here: we do not conduct the calculation of sizes  $O_{\zeta_{2n;\psi_{\beta}}}^{[1,]}$ , which easily to do by job performances [3]. We will notice only, that in the system (3.1)  $\theta$   $\eta$   $\zeta_{1},\zeta_{2},\zeta_{3},\zeta_{4}$  mean dissipative coefficients (see [3]), determinant  $\Delta(k,\omega)$  has the systems of equalizations (3.1) kind:

$$\Delta(k,\omega) = 2\sqrt{n_0} \,\sigma_n \,\frac{\partial(\dot{\varepsilon},\sigma)}{\partial(n_0,Y_0)} \,\delta(k,\omega)\,,$$

$$\delta(k,\omega) = \omega^{4} - k^{2} \left\{ \omega^{2} \left[ \left( \frac{\partial P}{\partial \sigma} \right)_{\frac{s}{\sigma}} + \frac{\sigma_{s}}{\sigma_{n}} \frac{s^{2}}{\sigma Y_{0} C_{v}} \right] - i\omega^{3} \times \right.$$

$$\times k^{4} \left\{ \frac{\sigma_{s}}{\sigma_{n}} \frac{s^{2}}{\sigma Y_{0} C_{v}} \left( \frac{\partial P}{\partial \sigma} \right)_{Y_{0}} - i\omega \left[ \frac{\sigma_{s}}{\sigma_{n}} \left( \frac{4}{3} \eta + \zeta_{2} + \sigma \zeta_{1} \right) \left( \frac{\partial \mu}{\partial \sigma} \right)_{\dot{\varepsilon}} + \right.$$

$$\left. + \frac{\sigma_{s}}{\sigma_{n}} (\sigma \zeta_{3} - \zeta_{1}) \left( \frac{\partial P}{\partial \sigma} \right)_{\dot{\varepsilon}} + \frac{\sigma_{s}}{\sigma_{n}} (\zeta_{3} (p + \dot{\varepsilon}) - \zeta_{1} \mu) \left( \frac{\partial P}{\partial \dot{\varepsilon}} \right)_{\sigma} + \right.$$

$$\left. + \frac{\sigma_{s}}{\sigma_{n}} \left( \mu \left( \frac{4}{3} \eta + \zeta_{3} \right) - (p + \dot{\varepsilon}) \zeta_{1} \right) \left( \frac{\partial \mu}{\partial \dot{\varepsilon}} \right)_{\sigma} + \frac{\theta}{C_{v}} \left( \frac{\partial P}{\partial \sigma} \right)_{Y_{0}} \right] \right\}$$

$$\left. + \frac{\sigma_{s}}{\sigma_{n}} \left( \mu \left( \frac{4}{3} \eta + \zeta_{3} \right) - (p + \dot{\varepsilon}) \zeta_{1} \right) \left( \frac{\partial \mu}{\partial \dot{\varepsilon}} \right)_{\sigma} + \frac{\theta}{C_{v}} \left( \frac{\partial P}{\partial \sigma} \right)_{Y_{0}} \right] \right\}$$

Research of equalization  $\Delta(k, \omega) = 0$  results in two decisions:

$$\kappa = \frac{\omega}{c_0} + i\gamma_1 \quad \kappa = \frac{\omega}{c_1} + i\gamma_1 \tag{22}$$

where: decrements  $\gamma_0$  and  $\gamma_1$ , accordingly, first and second sounds given formulas:

$$\gamma_0 = \frac{\omega^2}{2c_0^3} \frac{\Delta^2}{\Delta^2 - 1} \left( \Lambda - \frac{1}{c_0^2} \psi \right), \ \gamma_1 = \frac{\omega^2}{2c_1^3} \frac{\Delta^2}{\Delta^2 - 1} \left( \Lambda - \frac{1}{c_1^2} \psi \right)$$

$$\Delta = \frac{c_1}{c_0}$$
(23)

Sizes  $\Lambda$  and  $\psi$  look like:

$$\Lambda = \frac{1}{\sigma_{n}} \left( \frac{4}{3} \eta + \zeta_{2} + \sigma \sigma_{s} - 2 \sigma_{s} \zeta_{1} \right) + \frac{\theta}{c_{v}}$$

$$\psi = \frac{\sigma_{s}}{\sigma_{n}} (\sigma \zeta_{3} - \zeta_{1}) \left( \frac{\partial P}{\partial \sigma} \right)_{\dot{\varepsilon}} + \frac{\sigma_{s}}{\sigma_{n}} \left( \frac{4}{3} \eta + \zeta_{2} - \sigma \zeta_{1} \right) \left( \frac{\partial \mu}{\partial \sigma} \right)_{\dot{\varepsilon}} + \frac{\sigma_{s}}{\sigma_{n}} \times$$

$$\times \left[ \zeta_{3} (p + \dot{\varepsilon}) - \zeta_{1} \mu \left( \frac{\partial P}{\partial \dot{\varepsilon}} \right)_{\sigma} + \frac{\sigma_{s}}{\sigma_{n}} \left[ \mu \left( \frac{4}{3} \eta + \zeta_{2} \right) - (p + \dot{\varepsilon}) \zeta_{1} \right] \left( \frac{\partial \mu}{\partial \dot{\varepsilon}} \right)_{\sigma} + \frac{\theta}{C_{v}} \left( \frac{\partial P}{\partial \sigma} \right)_{Y_{0}} \tag{24}$$

Correlations (3.3) specify on that of Green's function  $G^{(+)}_{\xi'\xi}(k,\omega)$  in area of small  $\omega$  and  $\kappa$  poles has, proper poorly-fading to the sound-waves which spread in a superfluid Boze-liquids. Obvious expressions for sizes  $S^{(\xi)}_{Y_0}(k,\omega)$ ,  $S^{(\xi)}_1(k,\omega)$ ,  $S^{(\xi)}_1(k,\omega)$ , in a kind their bulkyness, we do not lead here.

Influence of dissipative processes on distribution of sound-waves in the superfluid systems was probed also in works [9-11].

# **CONCLUSIONS**

It is considered in to become appendix of variation communication theory with the calculation of low-frequency asymptotic of Green's two-time functions, which describe linear processes in the systems of many particles. However, possibilities of variation theory, in our view, will allow to execute the proper constructions for the n-time (n = 3,4,...) Green's functions, taking into account nonlinear co-operations of different collective influences, poorly investigational in a microscopic theory.

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# ДВУХВРЕМЕННЫЕ ФУНКЦИИ ГРИНА В МЕТОЛЕ СОКРАЩЕННОГО ОПИСАНИЯ

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**Аннотация.** На основе метода Н.Н. Боголюбова о сокращенном описании неравновесных состояний исследована асимптотика функции Грина в гидродинамическом приближении для вырожденных систем бозе-частиц со слабым взаимодействием. Вычислены низкочастотные асимптотики нормальных функций Грина в приближении идеальной и вязкой жидкости.

Ключевые слова: асимптотика, бозе-система, функция Грина, диссипативные процессы.