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QUASI-SELF-ADJOINT MAXIMAL ACCRETIVE EXTENSIONS OF NONNEGATIVE SYMMETRIC OPERATORS

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Summary. We give a new parametrization of all quasi-self-adjoint maximal accretive extensions and maximal sectorial extensions (with vertex at $z=0$ and the acute semi-angle α) for a densely defined closed nonnegative symmetric operator. An application to point interactions model on the real line is considered.

Key words: nonnegative symmetric operators, quasi-self-adjoint, accretive, sectorial extensions, transversal operators, point interactions.

INTRODUCTION

Let H be a separable Hilbert space with the inner product $(\cdot,\cdot)$. A linear operator T in H is called accretive if $\operatorname{Re}(Tf, f) \geq 0$ for all $f \in D(T)$. An accretive operator T is called *maximal* accretive (m-accretive) if one of the equivalent conditions is satisfied [14]:

- the operator T has no accretive extensions in H;
- the resolvent set $\rho(T)$ is nonempty;
- the operator T is densely defined and closed, and T^* is accretive operator.

The resolvent set $\rho(T)$ of m-accretive operator contains the open left half-plane and

$$\left\|\left(T - zI_{\mathsf{H}}\right)^{-1}\right\| \leq \frac{1}{|\operatorname{Re} z|}, \ \operatorname{Re} z < 0 .$$

It is well known [14] that if T is m-accretive operator, then the one-parameter semi-group:

$$T(t) = \exp(-tT),\ t \ge 0\,,$$

is contractive. Conversely, if the family $\{T(t)\}_{t\ge 0}$ is a strongly continuous semi-group of bounded operators in a Hilbert space H, with $T(0) = I_{\mathsf{H}}$ (C_0-semi-group) and $T(t)$ is a contraction for each t, then the generator T of $T(t)$:

$$Tu := \lim_{t\to +0} \frac{(I_{\mathsf{H}} - T(t))u}{t},\ u \in D(T)\,,$$

is an m-accretive operator in H.

Let $\alpha \in [0;\pi/2)$ and let:

$$\mathcal{S}(\alpha) := \{z \in \mathbf{C}\mathbf{:}\ |\arg z| \le \alpha\}\,,$$

be a sector on the complex plane $\mathbf{C}$ with the vertex at the origin and the semi-angle α.

A linear operator T in a H is called sectorial with vertex at $z = 0$ and the semi-angle α [14] if its numerical range:

$$W(T) = \{(Tu,u) \in \mathbf{C}\mathbf{:}\ u \in D(T),\ \|u\| = 1\}\,,$$

is contained in $\mathcal{S}(\alpha)$. If T is m-accretive and sectorial with vertex at $z = 0$ and the semi-angle α, then it is called m-sectorial with vertex at the origin and semi-angle α. We call shortly these operators m-α-sectorial. The resolvent set of m-α-sectorial operator T contains the set $\mathbf{C}\backslash\mathcal{S}(\alpha)$ and:

$$\left\|(T - zI_{\mathsf{H}})^{-1}\right\| \le \frac{1}{\operatorname{dist}(z,\mathcal{S}(\alpha))},\ z \in \mathbf{C}\setminus\mathcal{S}(\alpha)\,.$$

It is well-known [14] that a C_0-semi-group $T(t) = \exp(-tT),\ t \ge 0$ has *contractive* and *holomorphic* continuation into the sector $\mathcal{S}(\pi/2 - \alpha)$ if and only if the generator T is m-α-sectorial operator.

Let S be a linear, closed, densely defined non-negative symmetric operator on H, i.e. $(Sf,f) \ge 0$ for all $f \in D(S)$ and S^* be its adjoint. A linear operator T possessing property:

$$S \subset T \subset S^* \tag{1}$$

is called quasi-self-adjoint (proper, intermediate) extension ns of S. In particular, all self-adjoint extensions ($T = T^*$) satisfy (1).

We are interested in a description of all quasi-self-adjoint m-accretive and m-sectorial extensions of S. This problem is a part of more general Phillips problem [19] of a parametrization of all m-accretive extensions of a given densely defined accretive operator. It was established by Phillips that any closed densely defined accretive operator has an m-accretive extension. In order to obtain a description of all m-accretive extension Phillips proposed to use the approach connected with geometry of spaces with indefinite inner product. His approach has been used in [12] for ordinary differential operators and in [18] for an abstract positive definite symmetric operator with finite defect numbers. The fractional-linear transformation reduces the Phillips problem to the dual problem of a parametrization of all contractive extensions of a given non-densely defined contraction. Such parametrization has been obtained in [9]. The literature on contractive extensions of a given contraction is too extensive and we refer in this matter on [7] and references therein. A special case is an existence and description of all nonnegative self-adjoint extensions. This case has been considered by J. von Neumann, K. Friedrichs, M. Krein and later by M. Birman in their well known papers [15], [8]. Applications to partial differential operators in terms of boundary conditions and other approaches are discussed in [13]. Recently in [6] a new approach has been proposed.

The problem of existence and description of quasi-self-adjoint m-accretive extensions of a nonnegative symmetric operator via fractional-linear transformation has been solved in [5] and via abstract boundary conditions in [17], [16], [3], [10], [11]. We note that the reader can find references related to the described problem in the survey [6]. Here we announce new results based on the method developed in [6].

Following the Krein's notations [15] we denote by $S[\cdot,\cdot]$ the closure of the form (Su,v) and by $D[S]$ its domain for non-negative symmetric operator S. The same notations we preserve for the case of a nonnegative linear relation. As it is well known [14], [15] the Friedrichs non-negative self-adjoint extension S_F of S is defined as a non-negative self-adjoint extension associated with the form $S[\cdot,\cdot]$. Clearly, $D(S_F) = D[S]\cap D(S^*)$, $S_F = S^*\big|D(S_F)$. We would like to mention basic results of the theory of nonnegative self-adjoint extensions that have been established by M. Krein [15]. He showed that a non-negative symmetric operator S admits, so called, minimal non-negative self-adjoint extension. This extension we call the Krein-von Neumann extension S_K. The operator S_K can be defined as follows: $S_K = \left((S^{-1})_F\right)^{-1}$, where S^{-1} denotes in this context the inverse nonnegative linear relation to the graph S. Thus, for every non-negative self-adjoint extension $\tilde{S}$ of S the inequality holds in sense of the associated quadratic forms: $S_K \le \tilde{S} \le S_F$. It was shown in [2] that the relations below take place:

$$D[S_K] = \left\{u \in \mathsf{H}:\ \sup_{f\in D(S)} \frac{|(u,Sf)|^2}{(Sf,f)} < \infty\right\},\quad \sup_{f\in D(S)} \frac{|(u,Sf)|^2}{(Sf,f)} = S_K[u],\ u\in D[S_K].$$

In addition $D\left[\tilde{S}\right] = D[S]\dot{+}\mathsf{N}_z \cap D\left[\tilde{S}\right]$, where $\mathsf{N}_z = \mathrm{Ker}\left(S^* - zI\right)$ are the defect subspaces of S.

MAIN RESULTS

Consider the domain $D(S^*)$ of an operator S^* as a Hilbert space H_+ with the inner product $(f,g)_+ = (f,g) + (S^*f, S^*g)$. The (+)-orthogonal decomposition holds $H_+ = D(S)\oplus\mathsf{N}_i\oplus\mathsf{N}_{-i}$. Let N_F be (+)-orthogonal complement of $D(S)$ in $D(S_F)$ and let M_F be be (+)-orthogonal complement of $D(S_F)$ in H_+. So, we have $H_+ = D(S)\oplus\mathsf{N}_F\oplus\mathsf{M}_F$. It is easy to see that $\mathsf{N}_F = (S_F + iI)^{-1}\mathsf{N}_i = (S_F - iI)^{-1}\mathsf{N}_{-i}$

and $\mathfrak{N}_F = (I+V_F)\mathfrak{N}_i$, $\mathfrak{M}_F = (I-V_F)\mathfrak{N}_i$, where V_F is an isometry from $\mathfrak{N}_i$ onto $\mathfrak{N}_{-i}$ which determines a self-adjoint extension S_F by the von Neumann formula $D(S_F) = D(S) \dot{+} (I+V_F)\mathfrak{N}_i$. The following relations hold: $\mathfrak{N}_F = \left\{ f \in D(S_F) : S_F f \in H_+,\ S^* S_F f = -f \right\}$,

$$\mathfrak{M}_F = \left\{ f \in D(S^*) : S^* f \in D(S_F),\ S_F S^* f = -f \right\},\ \mathfrak{M}_F = S_F \mathfrak{N}_F,\ \mathfrak{N}_F = S^* \mathfrak{M}_F .$$

Recall that two self-adjoint extensions $\tilde S_1$ and $\tilde S_2$ of a symmetric operator S are disjoint (relatively prime) if $D(\tilde S_1) \cap D(\tilde S_2) = D(S)$ and transversal if $D(\tilde S_1) + D(\tilde S_2) = D(S^*)$. The necessary and sufficient condition of transversality of the Friedrichs and Krein - von Neumann extensions is the following: $D(S^*) \subset D[S_K]$. This condition is equivalent to $\mathfrak{N}_z \subset R(S_F^{1/2})$ for some (and then for all) $z \in \rho(S_F)$. It follows from the M.G. Krein result [15] that the operator S has unique non-negative self-adjoint extension iff $R(S_F^{1/2}) \cap \mathfrak{N}_F = \{0\}$. Suppose that

$$\mathfrak{N}_0 = R(S_F^{1/2}) \cap \mathfrak{N}_F \neq \{0\}, \tag{2}$$

and define a non-negative sesquilinear form

$$\omega_0[e,g] = \left(S_F^{1/2}e, S_F^{1/2}g\right) + \left(\hat{S}_F^{-1/2}e, \hat{S}_F^{-1/2}g\right) = \left(\hat{S}_F^{-1/2}e, \hat{S}_F^{-1/2}g\right)_+,\ e,g \in \mathfrak{N}_0. \tag{3}$$

The form ω_0 is closed in the Hilbert space H_+ and $\omega_0[e] \geq 2\|e\|^2$ for all $e \in \mathfrak{N}_0$. Let $\mathbf{W}_0$ be a (+)-nonnegative self-adjoint linear relation in $\mathfrak{N}_F$ associated with ω_0 given by (3). In view of $\omega_0[f] > 0$ for all $f \neq 0 \in \mathfrak{N}_0$, the inverse l.r. $\mathbf{W}_0^{-1}$ is densely defined in $\mathfrak{N}_F$ and therefore is the graph of a (+)-self-adjoint nonnegative operator. We denote this operator by W_0^{-1}. The next theorem gives a description of all non-negative self-adjoint extensions of S and their associated closed forms in terms of W_0^{-1} and some auxiliary operators in $\mathfrak{N}_F$.

Theorem 1. *[6]* Let condition (2) be fulfilled. Then the formulas

$$D(\tilde S) = D(S) \oplus (I + S_F \tilde U) D(\tilde U),$$

$$\tilde S(f_0 + e + S_F \tilde U e) = S_F(f_0 + e) - \tilde U e,\ f_0 \in D(S),\ e \in D(\tilde U),$$

$$D[\tilde S] = D[S] \dot{+} S_F R(\tilde U^{1/2}),$$

$$\tilde S[f + S_F h] = \|S_F^{1/2} f - \hat{S}_F^{-1/2} h\|^2 + \tilde U^{-1}[h] - \omega_0[h],\ f \in D(S),\ h \in R(\tilde U^{1/2}),$$

give a one-to-one correspondence between all nonnegative self-adjoint extensions $\tilde S$ of S, their associated closed forms and all (+)-self-adjoint operators $\tilde U$ in $\mathfrak{N}_F$

satisfying the condition $0 \le \tilde{U} \le W_0^{-1}$. An extension $\tilde{S}$ coincides with S_K iff $\tilde{U} = W_0^{-1}$. The extensions S_F and S_K are disjoint if and only if $\mathfrak{N}_0$ is a dense in $\mathfrak{N}_F$ and transversal if and only if $\mathfrak{N}_0 = \mathfrak{N}_F$. Here $\hat{S}_F^{-1/2} := \left(S_F^{1/2} \big| \overline{R\left(S_F^{1/2}\right)}\right)^{-1}$ stands for Moore-Penrose pseudo-inverse.

The theorem below plays an essential role in the investigation of quasi-self-adjoint m-accretive extensions.

Theorem 2. [4]. Let S be a nonnegative symmetric operator and let $\tilde{S}$ be a maximal accretive extension of S. The following conditions are equivalent:

1) $\tilde{S} \subset S^*$,

2) $D\left(\tilde{S}\right) \subset D[S_K]$ *and* $\mathrm{Re}\left(\tilde{S} f, f\right) \ge S_K[f]$ *for all* $f \in D\left(\tilde{S}\right)$,

3) $|(Sg, f)|^2 \le (Sg, g)\,\mathrm{Re}\left(\tilde{S} f, f\right)$ *for all* $f \in D\left(\tilde{S}\right)$, $g \in D(S)$.

The extension $\tilde{S}$ *is quasi-self-adjoint and m-*α*- sectrorial if and only if* $D(S) \subset D(S_K^{1/2})$ *and the sesquilinear form* $\left(\tilde{S} f, h\right) - \left(S_K^{1/2}, S_K^{1/2} h\right)$, $f, h \in D\left(\tilde{S}\right)$ *is sectorial with the same semi-angle* α *and the vertex at the origin.*

Next theorem gives the parametrization of all quasi-self-adjoint maximal accretive and maximal sectorial extensions of S.

Theorem 3. The formulas

$$\begin{gathered} D\left(\tilde{S}\right) = D(S) \oplus \left(I + S_F \tilde{U}\right) D\left(\tilde{U}\right), \\ \tilde{S}\left(\varphi + h + S_F \tilde{U} h\right) = S_F(\varphi + h) - \tilde{U} h, \ \varphi \in D(S), \ h \in D\left(\tilde{U}\right) \end{gathered} \tag{4}$$

give a one-to-one correspondence between all quasi-self-adjoint and maximal accretive extensions $\tilde{S}$ *of* S *and all (+)-maximal accretive operators* $\tilde{U}$ *in* $\mathfrak{N}_F$ *satisfying the condition*

$$R\left(\tilde{U}\right) \subset \mathfrak{N}_0 \text{ and } \mathrm{Re}\left(\left(\tilde{U} P_{\tilde{U}}\right)^{-1} e, e\right)_+ \ge \omega_0[e] \text{ for all } e \in R\left(\tilde{U}\right), \tag{5}$$

where: $P_{\tilde{U}}$ *is the (+)-orthogonal projection in* $\mathfrak{N}_F$ *onto* $\overline{R\left(\tilde{U}\right)}$.

The extension $\tilde{S}$ given by (4) is m- α -sectorial if and only if:

$\tilde{U}$ *in* $\mathfrak{N}_F$ *is (+)-maximal accretive,* $R\left(\tilde{U}\right) \subset \mathfrak{N}_0$*, the form*

$$\left(\left(\tilde{U} P_{\tilde{U}}\right)^{-1} e, h\right)_+ - \omega_0[e, h], \ e, h \in D\left(\tilde{U}\right) \tag{6}$$

is sectorial with the semi-angle α *and the vertex at the origin.*

The extension $\tilde{S}$ given by (4) is relatively prime with S_F if and only if the operator $\tilde{U}$ is invertible and transversal to S_F iff $\tilde{U}^{-1}$ is bounded.

Proposition 1. Suppose that the operator W_0^{-1} is (+)-bounded in $\mathfrak{N}_F$. Then

1) the formula:

$$\hat{U} = \frac{1}{2}W_0^{-1} + \frac{1}{2}W_0^{-1/2}\hat{Z}W_0^{-1/2}; \tag{7}$$

gives one-to-one correspondence between (+)-maximal accretive operators $\hat{U}$ *in* $\mathfrak{N}_F$, *satisfying condition (5) and (+)-contractive operators* $\hat{Z}$ *in* $\overline{\mathfrak{N}_0}$,

2) the formula (7) gives one-to-one correspondence between operators $\hat{U}$ *in* $\mathfrak{N}_F$, *satisfying condition (6) and operators* $\hat{Z}$ *in* $\overline{\mathfrak{N}_0}$, *such that* $\| \hat{Z}\sin\alpha \pm i\cos\alpha I \| \le 1$.

Let M_0 be the linear fractional transformation of W_0^{-1}:

$$M_0 = \left(W_0^{-1} - I\right)\left(W_0^{-1} + I\right)^{-1}. \tag{8}$$

Then M_0 is a (+)-contraction in $\mathfrak{N}_F$.

Theorem 4. There is the one-to-one correspondence between quasi-self-adjoint maximal accretive extensions $\tilde{S}$ of a nonnegative symmetric operator S and (+)-contractions $\hat{Y}$ in $\overline{\mathfrak{N}_0}$. This correspondence is given by the formulas:

$$D\left(\tilde{S}\right) = D(S) \oplus \left(I + S_F\tilde{U}\right)D\left(\tilde{U}\right),$$

$$\tilde{U} = \left(I - \hat{U}\right)\left(I + \hat{U}\right)^{-1},$$

$$\hat{U} = I - \frac{1}{2}\left(I + M_0\right)^{1/2}\left(I + \hat{Y}\right)\left(I + M_0\right)^{1/2},$$

where; M_0 *is given by (8). The extension* $\tilde{S}$ *is quasi-self-adjoint and m-*α*-sectorial if and only if the operator* $\hat{Y}$ *satisfies the condition* $\| \hat{Y}\sin\alpha \pm i\cos\alpha I \| \le 1$.

1-D SHRODINGER OPERATOR WITH δ' INTERACTIONS

Next we give applications of the above mentioned results to point-interaction in $\mathbf{R}$ [1]. Let $y_1, y_2, \ldots, y_m \in \mathbf{R}$. Consider operator S defined as follows:

$$D(S) = \left\{\varphi(x) \in H_2^2(\mathbf{R}) :\ \varphi'\left(y_j\right) = 0,\ j = 1, \ldots, m\right\},\quad S\varphi = -\frac{d^2\varphi}{dx^2}, \tag{9}$$

where: $H_2^2(\mathbf{R})$ is the Sobolev space. As it is well known [1] the operator S is symmetric and non-negative in $L^2(\mathbf{R}, dx)$ with defect numbers $\langle m, m\rangle$ and its Friedrichs extension S_F is given by:

$$D(S_F) = H_2^2(\mathbf{R}),\quad S_F = -\frac{d^2}{dx^2}.$$

Let $\mathcal{F}\, f=\tilde{f}(p)=s-\lim_{R\to\infty}(2\pi)^{-1/2}\int_{|x|\le R} f(x)\exp(-ixp)dx$ be the Fourier transform. In the p-representation we obtain the nonnegative symmetric operator A and its Friedrichs extension A_F:

$$D(A)=\left\{h(p)\in L^2(\mathbf{R},dp),\ \int_{\mathbf{R}} \mathbf{h}(p)\,p\exp(ipy_j)\,dp=0,\ \ j=1,\dots,m\right\},$$

$$D(A_F)=H_2(\mathbf{R}):=L^2\left(\mathbf{R},\left(p^4+1\right)dp\right),$$

$$Ah=p^2h(p),\ h(p)\in D(A),\ A_F f=p^2 f(p),\ \ f(p)\in D(A_F).$$

Let $e_j(p)=p\exp(-ipy_j)(1+p^4)^{-1},\ \ j=1,\dots,m$. Clearly,

$$\mathfrak{N}_F=span\{e_1(p),\dots,e_m(p)\},\ \ \mathfrak{M}_F=A\mathfrak{N}_F=span\{p^2e_1(p),\dots,p^2e_m(p)\}.$$

The adjoint operator A^* is given by the following relations:

$$D(A^*)=D(A)\dot{+}\mathfrak{N}_F\dot{+}\mathfrak{M}_F=H_2(\mathbf{R})\dot{+}\mathfrak{M}_F,$$

$$A^*\left(f(p)+\sum_{j=1}^m \lambda_j p^2 e_j(p)\right)=p^2 f(p)-\sum_{j=1}^m \lambda_j e_j(p),\ \ f(p)\in H_2(\mathbf{R}).$$

Let $H_+=D(A^*)$. Since $D\left(A_F^{1/2}\right)=H_1(\mathbf{R}):=L^2\left(\mathbf{R},\left(p^2+1\right)dp\right)$, $A_F^{1/2} f=pf(p)$, we obtain that $A_F^{-1/2}e_j(p)\in H_1(\mathbf{R})$, $j=1,\dots,m$. It follows that $\mathfrak{N}_0=\mathfrak{N}_F$ and A_F is transversal to A_K. By the direct calculation we get the following equalities:

$$g_{kj}=\left(e_k(p),e_j(p)\right)_+=\frac{\pi}{\sqrt{2}}\exp\left(-\frac{|y_k-y_j|}{\sqrt{2}}\right)\left(\cos\frac{|y_k-y_j|}{\sqrt{2}}-\sin\frac{|y_k-y_j|}{\sqrt{2}}\right),$$

$$\omega_{kj}=\left(A_F^{1/2}e_k(p),A_F^{1/2}e_j(p)\right)+\left(A_F^{-1/2}e_k(p),A_F^{-1/2}e_j(p)\right)=$$

$$=\frac{\pi}{\sqrt{2}}\exp\left(-\frac{|y_k-y_j|}{\sqrt{2}}\right)\left(\cos\frac{|y_k-y_j|}{\sqrt{2}}+\sin\frac{|y_k-y_j|}{\sqrt{2}}\right).$$

Let $W_0=\|\omega_{kj}\|_{k,j=1}^m,\ G=\|g_{kj}\|_{k,j=1}^m$. Providing calculation with the inverse Fourier transform $\mathcal{F}^{-1}$ we obtain

$$\mathcal{F}^{-1}e_j(p)=g_j(x)=i\sqrt{\frac{\pi}{2}}\exp\left(-\frac{|x-y_j|}{\sqrt{2}}\right)\sin\frac{|x-y_j|}{\sqrt{2}},$$

$$\mathcal{F}^{-1}A_F e_j(p)=h_j(x)=i\sqrt{\frac{\pi}{2}}\exp\left(-\frac{|x-y_j|}{\sqrt{2}}\right)\cos\frac{|x-y_j|}{\sqrt{2}}.$$

We have $S=\mathcal{F}^{-1}A\mathcal{F}$, $S_F=\mathcal{F}^{-1}A_F\mathcal{F}$, $S_K=\mathcal{F}^{-1}A_K\mathcal{F}$. From Theorem 1 we get the following description of all m-accretive extensions of S.

Theorem 5. If the operator S is given by (9), then the formulas:

$$D\left(\tilde{S}\right)=\left\{ f_0(x)+\sum_{j=1}^{m}\lambda_j g_j(x)+\sum_{k,j=1}^{m}u_{kj}\lambda_k h_j(x)\right\},$$

$$f_0(x)\in D(S),\ (\lambda_1,\ldots,\lambda_m)\in \mathbf{C}^m,$$

$$\tilde{S}\left(f_0(x)+\sum_{j=1}^{m}\lambda_j g_j(x)+\sum_{k,j=1}^{m}u_{kj}\lambda_k h_j(x)\right)=-\frac{d^2}{dx^2}f_0(x)+\sum_{j=1}^{m}\lambda_j h_j(x)-\sum_{k,j=1}^{m}u_{kj}\lambda_k g_j(x),$$

establish a one-to-one correspondence between the set of all $m\times m$ matrices $U=\| u_{kj}\|_{k,j=1}^{m}$ satisfying the condition:

$$UG+GU^*\geq 2UW_0U^*:$$

and the set of all m-accretive extensions of S. The operator $\tilde{S}$ is m-α-sectorial if and only if:

$$\begin{cases}\operatorname{tg}\alpha\cdot\left(UG+GU^*\right)+i\left(UG-GU^*\right)\geq 2\operatorname{tg}\alpha\cdot UW_0U^*,\\ \operatorname{tg}\alpha\cdot\left(UG+GU^*\right)-i\left(UG-GU^*\right)\geq 2\operatorname{tg}\alpha\cdot UW_0U^*.\end{cases}$$

In particular, if $m=1$ then

$$D\left(\tilde{S}\right)=\left\{ f_0(x)+\lambda\exp\left(-\frac{|x-y|}{\sqrt{2}}\right)\left(\sin\frac{|x-y|}{\sqrt{2}}+u\cos\frac{|x-y|}{\sqrt{2}}\right)\right\},$$

$$f_0(x)\in D(S),\ \lambda,u\in \mathbf{C},\ y\in \mathbf{R},$$

$$\tilde{S}\left(f_0(x)+\lambda\exp\left(-\frac{|x-y|}{\sqrt{2}}\right)\left(\sin\frac{|x-y|}{\sqrt{2}}+u\cos\frac{|x-y|}{\sqrt{2}}\right)\right)=$$

$$=-\frac{d^2}{dx^2}f_0(x)+\lambda\exp\left(-\frac{|x-y|}{\sqrt{2}}\right)\left(\cos\frac{|x-y|}{\sqrt{2}}-u\sin\frac{|x-y|}{\sqrt{2}}\right),$$

where: u satisfies the conditions: 1) $\left(\operatorname{Re}u-\frac{1}{2}\right)^2+\left(\operatorname{Im}u\right)^2\leq\frac{1}{4}$ for m-accretive extensions; 2) $\left(\operatorname{Re}u-\frac{1}{2}\right)^2+\left(\operatorname{Im}u\pm\frac{\operatorname{ctg}\alpha}{2}\right)^2\leq\frac{1}{4\sin^2\alpha}$ for m - α -sectorial extensions.

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КВАЗИСАМОСОПРЯЖЕННЫЕ МАКСИМАЛЬНЫЕ АККРЕТИВНЫЕ РАСШИРЕНИЯ НЕОТРИЦАТЕЛЬНЫХ СИММЕТРИЧЕСКИХ ОПЕРАТОРОВ

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Аннотация. Мы даем новую параметризацию всех квазисамосопряженных максимальных аккретивных расширений и максимальных секториальных расширений (с центром в начале координат и острым полууглом α) для плотно определенного замкнутого неотрицательного симметрического оператора. Рассматриваем применение к точечным взаимодействиям на вещественной прямой.

Ключевые слова. Неотрицательный симметрический оператор, квазисамосопряженный, аккретивный, секториальный, расширение, трансверсальные операторы, точечные взаимодействия.