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# MODELING OF THE FRICTION PROCESS IN THE CONTACT OF THE TOOTH GEAR

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**Summary.** The algorithm of calculation of speed of sliding between the surfaces of friction of gear-wheels of hyperboloidal transmission is presented. Speed of sliding in the contact of friction is one of the factors influencing the processes of friction in contact and wear of working surfaces of points. A correct determination of its value will allow to forecast operating descriptions of gearing.

**Keywords:** gearing, sliding friction, friction of woobling, hyperboloidal gearing, sliding speed

## INTRODUCTION

The contacting points of wheels form a kinematic pair [1, 2] with linear or point character of the working surfaces touch. Both woobling and sliding appears as a result. It results in the wear of working surfaces and, consequently – the collapse of sections of the different technical systems of the used gearings. Therefore, the designers face a task of diminishing friction sliding and friction of woobling costs. One of the main parameters, influencing the processes of friction, is sliding speed in contact. Determination of speed sliding is an important task for the construction of theoretical presentation of the processes of friction and wear.

## OBJECTS AND PROBLEMS

We will consider hyperboloid gearing. The large enough level of sliding has this type of gearing as compared to other types of gearings, and, consequently, is to a greater degree exposed to the processes of friction and wear.

As main motion relative speed of sliding, direction of which coincides with direction of points of cylindrical wheel, is accepted (Fig. 1).

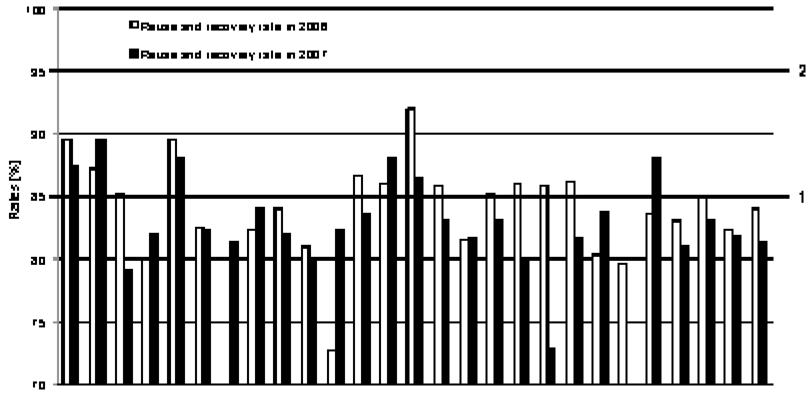


Fig. 1. Relative speed of sliding

The third class of kinematics charts, which is characterized by the fact that motion of surface of contacting points is instantaneous spiral motion which is the result of two rotations round crossings axes, is applied in this case. In this research relative motion of cylindrical wheel in relation to quasi-hyperboloidical can be presented as woobling with sliding of cylinder on a quasi-hyperboloid (Fig. 1). Axes of directtooth straight-toothed cylindrical wheel 1 and quasi-hyperboloidical wheel 2 cross in space under an angle.

The resulting in the process of hooking type of sliding is determined from Fig. 1 according to the following equation:

$$[V^{(12)}]^2 = [V_x^{(12)}]^2 + [V_y^{(12)}]^2 + [V_z^{(12)}]^2. \tag{1}$$

Projections of relative speed:

$$\begin{aligned} V_x^{(12)} &= -(f_1 \sin\varphi_1 + f_2 \cos\varphi_1)(1 - u_{21}\cos\gamma) + [a_\omega u_{21}\cos\gamma(f_2' - f_1'\cos\varphi_1) - \\ &\quad -(1 - u_{21}\cos\gamma)(f_2 f_2' + f_1' f_1)] / (f_2'\cos\varphi_1 + f'_1 \sin\varphi_1), \\ V_y^{(12)} &= (f_1\cos\varphi_1 - f_2\sin\varphi_1)(1 - u_{21}\cos\gamma) - a_\omega u_{21}\cos\gamma, \\ V_z^{(12)} &= (f_1\cos\varphi_1 - f_2\sin\varphi_1 + a_\omega)u_{21}\sin\gamma. \end{aligned} \tag{2}$$

Applying equation (1), the speed of sliding is determined according to the types of wheels. Because at $\varphi_1 = const$ equation (1) determines relative speed in contact point lines, at relative speed in a type of points of wheel, and will epress the principle of relative speed change on the length of tooth of cylindrical wheel.

Expressing the right part of equation (1) as zero, we get the equation for determination of parameters, characterizing points on the surfaces of cylindrical wheel, in which sliding speed is equal to the zero. It means that it is necessary to change the geometrical parameters of cylindrical and quasi-hyperboloidical wheels, so that the possible motion sliding speed was the minimum one.

For the reception of true values of projections of relative speed it is necessary to increase right parts of equalizations (2) on the angulator of cylindrical wheel $\vec{\omega}_1$.

We will consider the speed of movement of contacting quasi-hyperboloidical surfaces in direction, perpendicular to the lines of contact. We will enter the followings denotations:
$\vec{V}^{(1)}$ – vector of point's speed at motion on the basic surface of tooth of cylindrical wheel;
$\vec{V}^{(2)}$ – vector of point's speed of basic surface of quasi-hyperboloidical wheel;
$\vec{V}^{(12)}$ – vector of speed of the relative sliding.

Between the resulted speeds of contact points there is the following connection:

$$\vec{V}^{(2)} - \vec{V}^{(1)} = \vec{V}^{(12)} \Rightarrow \vec{V}^{(2)} = \vec{V}^{(1)} + \vec{V}^{(12)}. \tag{3}$$

Rate of movement of point of contact on the surface of points of cylindrical wheel it is possible to get in the mobile system of co-ordinates $x_1 y_1 z_1$:

$$\vec{V}^{(1)} = \vec{r}_1^{\lambda} d\lambda / dt + \vec{r}_1^{\mu} d\mu / dt, \tag{4}$$

where: $\vec{r}_1^{\lambda}$, $\vec{r}_1^{\mu}$ are partials of vector $\vec{q}$ to on $\lambda$ u $\mu$, accordingly.

Let a unit vector to be set in the system of co-ordinates $x_1 y_1 z_1$ $\vec{q}$. We will define the speed of movement of point of contact in the direction perpendicular to this vector. Multiplying scalar equality (4) by a vector $\vec{q}$ we will get:

$$(\vec{r}_1^{\lambda} \vec{q}) d\lambda / dt + (\vec{r}_1^{\mu} \vec{q}) d\mu / dt = 0. \tag{5}$$

We will add to this equality correlation, got at differentiation of equalization of the machine-tool hooking:

$$F^{\lambda} d\lambda / dt + F^{\mu} d\mu / dt + F^{\varphi_1} d\varphi_1 / dt = 0. \tag{6}$$

In correlation (5) and (6) there are three unknown values, $d\varphi_1/dt$. Set one of the unknown, putting, for example, $d\varphi_1/dt = 1$. The other unknown will be defined by the system of equations (5) and (6). The decision of these equations looks like:

$$\begin{aligned} d\lambda / dt &= (\vec{r}_1^{\mu} \vec{q}) F^{\varphi_1} / [(\vec{r}_1^{\lambda} \vec{q}) F^{\mu} - (\vec{r}_1^{\mu} \vec{q}) F^{\lambda}, \\ d\mu / dt &= -(\vec{r}_1^{\lambda} \vec{q}) F^{\varphi_1} / [(\vec{r}_1^{\lambda} \vec{q}) F^{\mu} - (\vec{r}_1^{\mu} \vec{q}) F^{\lambda}]. \end{aligned} \tag{7}$$

From expressions (4) (7) ensues:

$$\vec{V}^{(1)} = F^{\varphi_1} / [(\vec{r}_1^{\lambda} \vec{q}) F^{\mu} - (\vec{r}_1^{\mu} \vec{q}) F^{\lambda}] [\vec{r}_1^{\lambda} (\vec{r}_1^{\lambda} (\vec{r}_1^{\mu} \vec{q}) - \vec{r}_1^{\mu} (\vec{r}_1^{\lambda} \vec{q})]. \tag{8}$$

Let's transform the expressions of the second factor of equality (8) to the next kind:

$$[\vec{r}_1^{\lambda} (\vec{r}_1^{\mu} \vec{q}) - \vec{r}_1^{\mu} (\vec{r}_1^{\lambda} \vec{q})] = \left| \vec{N} \right| [\vec{q} \times \vec{l}_1],$$

where: - module vector is perpendicular to the surface of tooth of cylindrical wheel;
$\bar{l}_1$ - unit vector is perpendicular to this surface.
Taking into account this equation we will get:

$$\vec{V}^{(1)} = F^{\varphi}[\vec{q} \times \vec{l}_1]\left|\vec{N}\right| / [(\vec{r}_1^{\,v}\vec{q})F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v}],$$

(9)

$$\vec{V}^{(2)} = F^{\varphi}[\vec{q} \times \vec{l}_1]\left|\vec{N}\right| / [(\vec{r}_1^{\,v}\vec{q})F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v}] + \vec{V}^{(12)}.$$

At research of the specific sliding of the cut points and other indexes of capacity of transmissions there are tasks of determination of rate of movement of points of contact in the direction of the set vector. We will define this speed. Let a unit vector, perpendicular, to be set as $\vec{q}$. It is required to define $\overline{V}^{(12)} \neq 0$ in the direction of the set vector. In the examined case a vector, included in correlations (9), is equal to:

$$\vec{q} = [\vec{a} \times \vec{l}_1] = [\vec{a} \times (\vec{r}_1^{\,v} \times \vec{r}_1^{\,u})]1 / \left|\vec{N}\right|. \tag{10}$$

Exposing double vectorial work, we have:

$$q = [\vec{r}_1^{\,v}(\vec{a}\vec{r}_1^{\,u}) - \vec{r}_1^{\,u}(\vec{a}\vec{r}_1^{\,v})] / \left|\vec{N}\right|.$$

Taking into account this equality we get:

$$\left|\vec{N}\right|[\vec{q} \times \vec{l}_1] = -[\vec{r}_1^{\,v}(\vec{a}\vec{r}_1^{\,v})G_1 + \vec{r}_1^{\,u}(\vec{a}\vec{r}_1^{\,u})E_1 / \left|\vec{N}\right|, \qquad ,$$
$$(\vec{r}_1^{\,v}q)F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v} = [(\vec{a}\vec{r}_1^{\,u})E_1F^{u} + (\vec{a}\vec{r}_1^{\,v})G_1F^{v}] / \left|\vec{N}\right|,$$

where: $E_1$, $G_1$ are coefficients of the first quadratic form of contacting points and surfaces of cylindrical wheel.

Whereupon the above correlations (9) can be brought to the kind:

$$\vec{V}^{(1)} = -[(\vec{a}\vec{r}_1^{\,v})G_1\vec{r}_1^{\,v} + (\vec{a}\vec{r}_1^{\,u})E_1\vec{r}_1^{\,u}]F^{\varphi} / [(\vec{a}\vec{r}_1^{\,u})E_1F^{u} + (\vec{a}\vec{r}_1^{\,v})G_1F^{v}]; \tag{11}$$

$$\vec{V}^{(2)} = -[(\vec{a}\vec{r}_1^{\,v})G_1\vec{r}_1^{\,v} + (\vec{a}\vec{r}_1^{\,u})E_1\vec{r}_1^{\,u}] \cdot F^{\varphi} / [(\vec{a}\vec{r}_1^{\,u})E_1F^{u} + (\vec{a}\vec{r}_1^{\,v})G_1F^{v}] + \vec{V}^{(12)}.$$

We will define the total rate of movement of points of contact in the direction perpendicular to vector $\vec{q}$. For this purpose we will take advantage of correlations (9). In obedience to these correlations the vector of total speed of moving of points of contact in the direction perpendicular to unit vector is equal to:

$$\vec{u} = \vec{V}^{(1)} + \vec{V}^{(2)} = 2F^{\varphi}[\vec{q} \times \vec{l}_1]\left|N\right| / [(\vec{r}_1^{\,v}\vec{q})F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v} + \vec{V}^{(12)}. \tag{12}$$

For finding the true value of speed let's project vector $\vec{u}$ on the unit vector, perpendicular to vector $\vec{q}$:

$$\vec{q} = [\vec{q} \times \vec{l}_1]. \tag{13}$$

Increasing the scale of both parts of equation (12) on vector (13), after transformations we have:

$$\vec{u}_{\bar{q}} = 2F^{\varphi} + \vec{V}^{(12)}[\vec{q} \times \vec{l}_1](\vec{r}_1^{\,v}\vec{q})F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v} / [(\vec{r}_1^{\,v}\vec{q})F^{u} - (\vec{r}_1^{\,u}\vec{q})F^{v}]. \tag{14}$$

Dependence (14) determines the total rate of movement of points of contact in arbitrary direction, determined by unit vector $\vec{q}$. Furthermore, we will define the total speed of points of contact in the direction of vector $\vec{a}$ under the corner $\psi$ to the vector tangent to the contact line of basic surfaces of kinematics pair. We will present vector $\vec{a}$ as:

$$\vec{a} = (\vec{r}_1^{\,\lambda}\frac{d\lambda}{d\mu} + \vec{r}_1^{\,\mu}) / \sqrt{E_1(\frac{d\lambda}{d\mu})^2 + G_1}. \tag{15}$$

The corner between vectors $\vec{a}$ and $\vec{\tau}_1$ can be defined from the correlation:

$$tg\psi = \frac{\left|[\vec{\tau} \times \vec{a}]\right|}{(\vec{\tau}_1,\ \vec{a})}. \tag{16}$$

Transforming the right part of this equality with the use of correlation (15) and having in mind that:

$$\vec{\tau}_1 = \vec{r}_1^{\,\lambda}F^{\mu} - \vec{r}_1^{\,\mu}F^{\lambda},$$

we: will get

$$tg\psi = \left(\vec{N}\left((F_1^{\lambda}\frac{d\lambda}{d\mu} + F^{\mu}\right) / (E_1F^{\mu}\frac{d\lambda}{d\mu} - G_1F^{\lambda}\right).$$

Deciding the last equation relative, we get:

$$d\lambda / d\mu = -(G_1F^{\lambda}tg\psi + \left(\vec{N}\left(F^{\mu}\right) / (\left(\vec{N}\left(F^{\lambda} - E_1F^{\mu}tg\psi\right).\right.\right.\right. \tag{17}$$

Correlations (17) can be used for the arbitrary value of corner $\psi$. So, for example at $\psi = 0$ we get the corresponding direction of vector $\vec{\tau}_1$. In this case vector $\vec{a}$ is directed on vector $\vec{\tau}_1$. Putting (17) in equation (15), after transformations, we have:

$$\vec{a} = a\vec{r}_1^{\,\lambda} + b\vec{r}_1^{\,\mu}, \tag{18}$$

where: coefficients $a$ $u$ $b$ are determined as equations:

$$\begin{aligned} a &= -(G_1F^{\lambda}tg\psi + \left(\vec{N}\left(F^{\mu}\right)a_{\omega},\right.\right. \\ b &= (\left(\vec{N}\left(F^{\lambda} - E_1F^{\mu}tg\psi\right)a_{\omega}\right.\right. \end{aligned}, \tag{19}$$

$$a_{\omega} = [(E_1G_1F^{\lambda}tg\psi' + \left|\vec{N}\right|F^{\mu})^2 + (G_1F^{\lambda}\left|\vec{N}\right| - E_1G_1F^{\mu}tg\psi')^2]^{1/2}. \tag{20}$$

From correlations (13) at replacement of vector $\vec{q}$ and vector $\vec{a}$ after transformations of dependence (15), we get

$$u_{\vec{a}} = \frac{2F^{\omega\lambda} + [bG_1(\vec{r}_1^{\,\lambda}\vec{V}^{(12)} - aE_1(\vec{r}_1^{\,\mu}\vec{V}^{(12)})] \cdot [(\vec{r}_1^{\,\lambda}q)F^{\mu} - (\vec{r}_1^{\,\lambda}q)F^{\lambda}] / E_1G_1}{[(\vec{r}_1^{\,\lambda}\vec{q})F^{\mu} - (\vec{r}_1^{\,\mu}\vec{q})F^{\lambda}] / \left|\vec{N}\right|}. \tag{21}$$

Supposing that in correlations (19) and (21) $\psi = 0$, we have the following formula for determination of total rate of movement of points of contact in direction, perpendicular to vector $\vec{\tau}_1$.

$$u_{\Sigma} = \frac{2F^{\varphi d} + [G_1 F^{d}(\vec{r}_1^{\,d}\vec{V}^{(12)}) + E_1 F^{\mu}(\vec{r}_1^{\,\mu}\vec{V}^{(12)})] \cdot \dfrac{[(\vec{r}_1^{\,d}\vec{q})F^{\mu} - (\vec{r}_1^{\,\mu}\vec{q})F^{d}}{A_1 E_1 G_1}}{[(\vec{r}_1^{\,d}\vec{q})F^{\mu} - (\vec{r}_1^{\,\mu}\vec{q})F^{d}] / |N|}, \tag{22}$$

where: the module of vector;
$E_1$, $G_1$ - coefficients of quadratic forms of basic surface, equal;

$$E_1 = f_1'^2 + f_2'^2;\ f_1 = 0;\ G_1 = 0.$$

$F^{\varphi d}$, $F^{d}$, $F^{\mu}$ - partials, found from equalization of the machine-tool hooking

$$\begin{aligned} F^{\varphi d} &= -\mu u_{21} \sin\gamma(-f_2' \sin\varphi_1 + f_1' \cos\varphi_1) - \\ &\quad - a_w u_{21} \cos\gamma(f_2' \cos\varphi_1 + f_1' \sin\varphi_1), \\ F^{d} &= -(1 - u_{21}\cos\gamma)[f_2'^2 + f_2 f_2'' + f_1''(f_1 - r_1) + f_1'^2] - \\ &\quad - \mu u_{21} \sin\gamma(f_2'' \cos\varphi_1 + f_1'' \sin\varphi_1 - a_w u_{21} \cos\gamma(f_2'' \sin\varphi_1 - f_1'' \cos\varphi_1), \\ F^{\mu} &= -u_{21} \sin\gamma(f_2' \cos\varphi_1 + f_1' \sin\varphi_1) \end{aligned} \tag{23}$$

$\vec{\tau}^0 = \vec{q} = \dfrac{\vec{r}_1^{\,d} F^{\mu} - \vec{r}_1^{\,\mu} F^{d}}{a_w}$ is a unit vector of tangent line to the contact line.

Putting vector $\vec{q}$ in (22) and taking (23) into account, we get:

$$\begin{aligned} u_{\Sigma} &= \frac{2F^{\varphi d}E_1 + F^{\mu}(\vec{r}_1^{\,d}\vec{V}^{(12)}) - F^{\mu}E_1(\vec{r}_1^{\,\mu}\vec{V}^{(12)})}{\sqrt{E_1[E_1(F^{\mu})^2 + (F^{d})^2]}} = \\ &= \frac{2F^{\varphi d}(f_1'^2 + f_2'^2) - F^{d}(\vec{r}_1^{\,d}\vec{V}^{(12)} - (f_1'^2 + f_2'^2)F^{\mu}(\vec{r}_1^{\,\mu}\vec{V}^{(12)})}{\sqrt{(f_1'^2 + f_2'^2)[(f_1'^2 + f_2'^2)(F^{\mu})^2 + (F^{d})^2]}} \end{aligned}, \tag{24}$$

we: will get:

$$\begin{aligned} (\vec{r}_1^{\,d}\vec{V}^{(12)}) &= [-f_1^d f_2 + f_2^d(f_1 - r_1)](1 - u_{21}\cos\gamma) - \mu u_{21}\sin\gamma(f_1^d \cos\varphi_1 - , \\ &\quad - f_2^d \sin\varphi_1) - a_w u_{21} \cos\gamma(f_1^d \sin\varphi_1 + f_2^d \cos\varphi_1); \\ (\vec{r}_1^{\,\mu}\vec{V}^{(12)}) &= [(f_1 - r_1)\cos\varphi_1 - f_2 \sin\varphi_1 + a_w] u_{21} \sin\gamma \end{aligned} \tag{25}$$

at

$$2F^{\varphi d}(f_1'^2 + f_2'^2) - F^{d}(\vec{r}_1^{\,d}\vec{V}^{(12)}) - (f_1'^2 + f_2'^2)F^{\mu}(\vec{r}_1^{\,\mu}\vec{V}^{(12)}) = 0. \tag{26}$$

The total rate of movement of points of contact $u_{\Sigma} = 0$.

## CONCLUSIONS

Correlations (26) can be applied for the points of contact, for which $u_z = 0$, i.e. those points in which there occur the most unfavorable conditions for work of cylinder-giperboloidical transmission.

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## MODELOWANIE PROCESÓW TARCIA W KONTAKCIE ZĘBÓW SKRZYNI ZĘBATEJ

**Streszczenie.** Przestawiono algorytm obliczenia szybkości styku między powierzchniami tarczą kół zębatych hiperbolicznej przekładni szybkość połączenia kontaktowego tarcza okazuje się, że jest jednym z ważnych czynników określających wpływ na procesy tarczą i zużycia powierzchni zębów. Prawidłowe określenie tych zjawisk pozwoli na prognozowanie eksploatacyjnych charakterystyk przekładni.

**Słowa kluczowe:** przekładnia zębata, tarcze połączenia, tarcze toczne, hiperbaliczna przekładnia zębata