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DETERMINATION OF MEKCHANICAL SYSTEMS RELIABILITY FUNCTION AT CONSTANT INCREASE OF REFUSALS INTENSITY

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Summary. The conduct of unduplicated mechanical system at gradual accumulation of damages has been described. The function of readiness, that determines the initial and stationary value of coefficient of readiness has been considered.

Key words: calculation of states, intensity of refusals, state of the system, coefficient of readiness

INTRODUCTION

For many subsystems of sophisticated equipment from the point of view of reliability, the gradual increase of intensity of refusals in the absence of reservations is a typical situation. In such cases the formation of the streams of refusals cannot be described as Marc casual process of transitions of systems into different possible (capable or incapable of working) states. For analytical description of the conduct of systems in operation the introduction of the additional fictitious state has been suggested [1]. Thus, not Marc processes are replaced by Marc ones, but the mathematical description of the conduct of systems is more difficult due to the increase of their possible states.

The establishment of function of readiness of the mechanical system is the purpose of this research, the level of reliability of which goes down with the increase of operation.

RESULTS AND DISCUSSION

The graph of the states and transitions of the systems in which in the course of time the operation on the refusal is decreasing, and the intensity of refusals of aging systems is accordingly increasing, is shown in Fig.1.

On the basis of the built graph of transitions the differential equations of dynamic balance for probabilities of the states have been worked out:

$$\begin{cases} \frac{d}{dt}P_0(t) \quad \lambda_0 P_0(t) + \mu_1 P_1(t); \\ \frac{d}{dt}P_1(t) \quad \lambda_2 P_2(t) + \mu_1 P_1(t); \\ \frac{d}{dt}P_2(t) \quad \lambda_0 P_0(t) + \mu_2 P_2(t). \end{cases} \tag{1}$$

where $P_0(t)$ - is probability of the system in the capable of working state;
$P_2(t)$ - is probability of the system in the transient state;
$P_1(t)$ - is probability of the system in the disabled state.

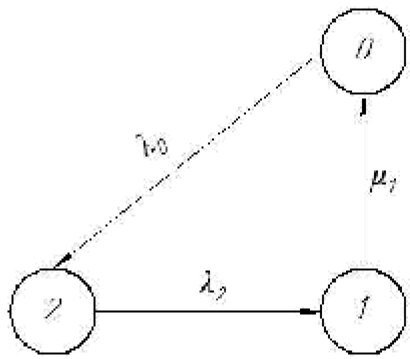

Fig.1. Graph of states and transitions of the system the intensity of refusals of which is growing

"0" - is the capable of working state;
"1" - is the disabled state (renewal);
"2" - is intermediate (fictitious state);
$\lambda_0, \lambda_2, \mu_1$ - is the proper intensity of transitions.
The rationed condition for the systems which necessarily are in one of the possible states is:

$$P_0(t) + P_1(t) + P_2(t) = 1. \tag{2}$$

From the first equation of the system (1) in the Laplas transformations we have:

$$S\varphi_0(S) = -\lambda_0\varphi_0(S) + C, \tag{3}$$

where: C - is a permanent value which is determined from the initial conditions.

The initial condition is rightfully adopted as the situation when the system starts working in good condition. Thus, for time $t = 0$ $P_0(t) = 1$. Then, inserting the initial condition in equation (3) and executing the Laplas transformation for the second equation of the system (1), taking into account the rationed condition (2) we will write down:

$$\begin{cases} (S+\lambda_0)\phi_0(S) \quad \mu_1\phi_1(S) \quad 1, \\ (S+\mu_1)\phi_1(S) \quad \lambda_2\phi_2(S) \quad 0, \\ S\phi_0(S) + S\phi_1(S) + S\phi_2(S) \quad 1. \end{cases} \tag{4}$$

The determinant of the system is $\begin{vmatrix} S+\lambda_0 & \mu_1 & 0 \\ 0 & S+\mu_1 & \lambda_2 \\ S & S & S \end{vmatrix} \quad \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$,

where the column of free members is on the right.

The solution of system (4) in relation to unknown $\varphi_0\,(S)$ can be carried out according to the Kramer's rule. Then we have:

$$\varphi_0(S) \quad \frac{S^2+(\mu_1+\lambda_2)S+\mu_1\lambda_2}{S[S^2+(\mu_1+\lambda_0+\lambda_2)S+\lambda_0\mu_1+\lambda_2\lambda_0]}. \tag{5}$$

We will represent a denominator in the expansion of equation in terms of roots:

$$S[S^2+(\mu_1+\lambda_0+\lambda_2)S+\lambda_0\mu_1+\lambda_2\lambda_0] \quad 0.$$

Obviously, the two roots $S_{1,\,2}$ are the solution of a quadratic, and the third $S_3 = 0$. As a discriminant of the quadratic $D < 0$, the roots S_1 and S_2 are valid

$$S_1 \quad \frac{\mu_1+\lambda_0+\lambda_2}{2}+\sqrt{\left(\frac{\mu_1+\lambda_0+\lambda_2}{2}\right)^2 \quad \lambda_0\mu_1 \quad \lambda_2\lambda_0}, \tag{6}$$

$$S_2 \quad \frac{\mu_1+\lambda_0+\lambda_2}{2} \quad \sqrt{\left(\frac{\mu_1+\lambda_0+\lambda_2}{2}\right)^2 \quad \lambda_0\mu_1 \quad \lambda_2\lambda_0} \tag{7}$$

Function (5) expanded in terms of roots can be written down as a sum

$$\varphi_0(S) \quad \frac{A}{S \quad S_3}+\frac{B}{S \quad S_1}+\frac{C}{S \quad S_2}, \tag{8}$$

where A, B, C are some unknown coefficients which satisfy the equality (5).

Hence, after the transformations and simplifications we have:

$$\varphi_0(S) \quad \frac{(A+B+C)S^2 \quad \left[A(S_1+S_2)+BS_2+CS_1\right]S+AS_1S_2}{S(S \quad S_1)(S \quad S_2)}. \tag{9}$$

The value of coefficients A, B and C can be found on the basis of comparison of expressions (5) and (9). Equalizing the numerators we will write down:

$$S^2(\mu_1+\lambda_2)S+\mu_1\lambda_2 \quad (A+B+C)S^2 \quad \left[A(S_1+S_2)+BS_2+CS_1\right]S+AS_1S_2.$$

So, as the left part equals the right one, then the coefficients are equal too at identical degrees unknown. Thus, it is possible to write down a new system of equations:

$$\begin{cases} 1 \quad A+B+C; \\ \mu_1+\lambda_2 \quad A(S_1+S_2) \quad BS_2 \quad CS_1, \\ \mu_1+\lambda_2 \quad AS_1S_2. \end{cases} \tag{10}$$

Thus, for the determination of unknown coefficients A, B and C we have three equations. We solve the obtained system according to the Kramer's rule.

The determinant of the system (10) is represented by the D matrix:

$$Д \quad \begin{vmatrix} 1 & 1 & +1 \\ (S_1+S_2) & S_2 & S_1 \\ S_1S_2 & 0 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 \\ \mu_1+\lambda_2 \\ \mu_1\lambda_2 \end{vmatrix}, \tag{11}$$

$$Д \quad S_1 \cdot S_2^2 \quad S_1^2 S_2.$$

By analogy, the matrices are solved for determination of coefficients *A*,*B* and *C*:

$$Д_A \quad \mu_1\lambda_2 (S_2 \quad S_1), \tag{12}$$

$$Д_B \quad \mu_1\lambda_2 S_2 (1+S_1^2), \tag{13}$$

$$Д_C \quad \mu_1\lambda_2 S_1 + (\mu_1+\lambda_2) S_1 S_2 + S_1 S_2^2. \tag{14}$$

The values of coefficients *A*, *B* and *C* are found from the relations:

$$A \quad \frac{D_A}{D}; \quad B \quad \frac{D_B}{D}; \quad C \quad \frac{D_C}{D}.$$

Inserting the obtained values of matrices from equations (11-14) we will write down:

$$A \quad \frac{\mu_1\lambda_2 (S_2 \quad S_1)}{S_1 S_2^2 \quad S_1^2 S_2};$$

$$B \quad \frac{\mu_1 \quad \lambda_2 (1 \quad S_1^2)}{S_1 (S_2 \quad S_1)};$$

$$C \quad \frac{\mu_1\lambda_2 (\mu_1+\lambda_2) S_2 + S_2^2}{S_2^2 \quad S_1 S_2}.$$

After the substitution of values of coefficients *A*, *B* and *C* in equalization (8) we have:

$$\varphi_0(S) \quad \frac{\dfrac{\mu_1\lambda_2 (S_2 \quad S_1)}{S_1 S_2^2 \quad S_1^2 S_2}}{S \quad S_3} \quad \frac{\dfrac{\mu_1\lambda_2 (S_2 \quad S_1)}{S_1 S_2^2 \quad S_1^2 \cdot S_2}}{S \quad S_1} + \frac{\dfrac{\mu_1\lambda_2 (\mu_1+\lambda_2) S_2 + S_2^2}{S_2^2 \quad S_1 S_2}}{S \quad S_2}.$$

For the represented equation it is possible to apply the reverse transformation of Laplas. Then, for the capable of working state, the probability of finding the system in it is written down in a kind of:

$$P_0(t) \quad \frac{\mu_1\lambda_2 (S_2 \quad S_1)}{S_1 S_2^2 \quad S_1^2 S_2} \exp[\quad S_3 \cdot t] \quad \frac{\mu_1 \quad \lambda_2 (1 \quad S_1^2)}{S_1 (S_2 \quad S_1)} \exp[\quad S \cdot t] +$$

$$+ \frac{\mu_1\lambda_2 (\mu_1+\lambda_2) S_2 + S_2^2}{S_2^2 \quad S_1 S_2} \exp[\quad S \cdot t]. \tag{15}$$

Taking into account that root $S_3 = 0$ and that the probability of finding the system in the capable of working state $P_0(t)$ in the physical essence answers the readiness of the system to work, i.e. $P_0(t) = K_{RO}(t)$, we can represent the equation (15) as:

$$K_{го}(t) \quad \frac{\mu_1\lambda_2(S_2 \quad S_1)}{S_1S_2^2 \quad S_1^2S_2} \quad \frac{\mu_1 \quad \lambda_2(1 \quad S_1^2)}{S_1(S_2 \quad S_1)}\exp[S_1\cdot t]+$$
$$+\frac{\mu_1\lambda_2(\mu_1+\lambda_2)S_2+S_2^2}{S_2^2 \quad S_1S_2}\exp[\quad S_2\cdot t]. \tag{16}$$

The values of roots S_1 and S_2 expressed through the intensity of transitions, λ_2 and μ_1 are represented in equations (6 and 7).

Thus, the certain function of readiness of the systems which reduce the level of reliability at the increase of intensity of refusals (aging system) has been determined. The maximum value of the coefficient of readiness is the beginning of exploitation at $t = 0$. Then, on the basis that equation (16), the coefficient of readiness after the simplifications and transformations is equal to:

$$K_{го} \quad \frac{\mu_1\lambda_2}{S_1}\left(\frac{S_2 \quad S_1}{S_2^2 \quad S_1S_2} \quad \frac{1 \quad S_1^2}{S_2 \quad S_1}+\frac{S_1}{S_2^2 \quad S_1S_2}\right)+\frac{\mu_1+\lambda_2+S_2}{S_2 \quad S_1}. \tag{17}$$

The exposure of conduct of the coefficient of readiness is the second extreme case when $t\to\infty$. Analyzing equation (16), in this case we get the endless and opposite values of the second and third constituent. Then the value of the coefficient of readiness is limited to the first member:

$$K_{го}(t\to\infty)_{stat} \quad \frac{\mu_1\lambda_2(S_2 \quad S_1)}{S_1S_2^2 \quad S_1^2S_2}.$$

The obtained equation characterizes the stationary value of the coefficient of readiness which the system makes for asymptotically at the unlimited increase of the time of its exploitation.

From the comparison of values of coefficients of readiness at the beginning of work and asymptotic close to the stationary value it is clear, that the first value exceeds the second one. That means that the function of the coefficient of readiness is falling down, and its value gradually diminishes from the beginning of exploitation of the system to the stationary value. It is represented graphically in Fig. 2.

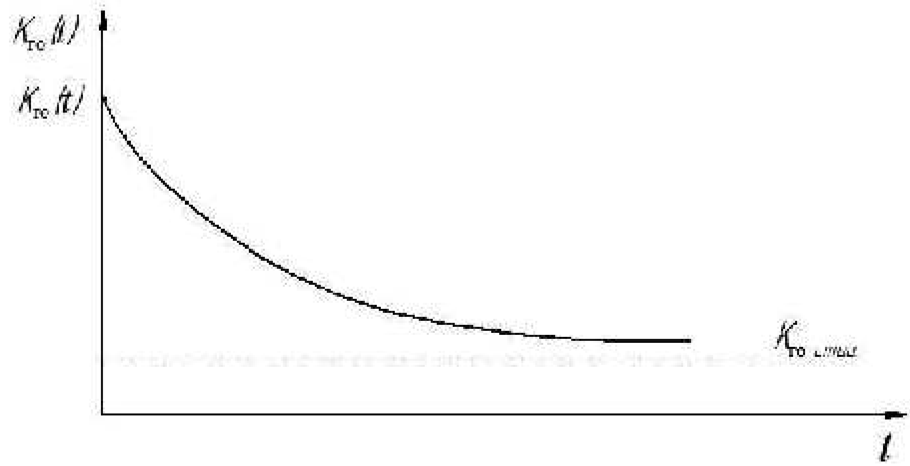

Fig. 2. - Change of coefficient of readiness of unduplicated aging system

CONCLUSIONS

Readiness of the unduplicated mechanical systems at accumulation of damages is described by a nonlinear function $K_{zd}(t)$. The maximum value of the coefficient of readiness $K_{zd}(t)_{max}$ answers the beginning of work t = 0x0 with the gradual asymptotic making for the stationary value $K_{zd}(t \to \infty)_{st}$

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WYZNACZENIE FUNKCJI PRZYSZŁOŚCIOWYCH SYSTEMÓW INTESYWNEGO ROZWOJU JAKICH JEST CORAZ WIĘCEJ

Streszczenie. Analityczny zapis mechanicznych systemów. Optymalna funkcja gotowości, która wyznacza początkowe i stacjonarne, znaczenia współczynnika gtowości.

Słowa kluczowe: stan systemu, współczynnyki gotowości pracy.