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A SHELL MODEL OF THE HOLLOW SHAFT FOR AN ANALYSIS OF THE STRESSED STATE IN FITTED ZONE

Summary. The method of analysis of the contact pressure concentration, that occurred near the boundary of cylindrical bush fitted with interference on hollow shaft, is presented.

Keywords: contact pressure, interference, hollow shaft

INTRODUCTION

Using hollow shafts in modern machine building is conditioned by their constructive and technical-economic advantages in contrast with solid shafts. Thus the methods of analysis of hollow shafts, founded on a bar model, in many cases is not efficient enough, and unacceptable, since it does not take into account the essential features of the state of stress of thin walls in the zones of joints with interference of the shaft and rings of bearings, hubs of gear wheels, couplings, flywheels and others.

OBJECT AND METHOD RESEARCH

That's why an estimation of distribution of loading in connection with hollow shaft and cylindrical bushing planted on it with diametrical interference δ is considered (Fig. 1). The simultaneous condition of displacements of contacting surfaces has a kind

$$u_1 + u_2 + u_3 = 0{,}5\delta \qquad (1)$$

where:

u_1 – displacement of the shaft;

u_2 – displacement of internal surface of the bushing;

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u_3 – displacement because of contact deformation of micro-roughness surfaces of the shaft and bushing.

The function of the displacement $u_1(x)$ on the length of the joint is possible to describe by differential equation of the moment theory of the cylindrical shells [1]

$$\frac{d^4u_1(x)}{dx^4} + 4\beta^4 u_1(x) = \frac{p(x)}{D} \tag{2}$$

where:

$p(x)$ – contact pressure;

$D = E_1h^3 / 12(1 - \mu_1^2)$ – cylindrical rigidity of the wall of the shaft;

$\beta = [3(1 - \mu_1^2) / R^2h^2]^{0,5}$ – factor of the dying down of the displacements;

R, h – radius of middle surface and thickness of the wall of the shaft;

E_1, μ_1 – module of elasticity of elongation and factor of Poisson of the material of the shaft.

The shaft and bushing are accepted long, that is such, that their left and right edges are removed from start coordinates 0 more then on value R (Fig. 1a).

Obviously, outside the bushing the contact pressure is equal

$$p(x)\big|_{x<0} = 0 \tag{3}$$

For a description of the function $u_2(x)$ the well- known decision of Lyame is used

$$u_2(x) = c_2p(x), \;\; c_2 = \frac{d}{2E_2}\left(\frac{1+k_2^2}{1-k_2^2} + \mu_2\right)^2, \tag{4}$$

where:

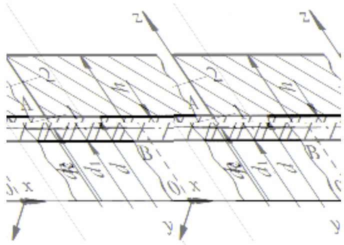
$k_2 = d / d_2$ – geometric feature of the bushing;

E_2, μ_2 – module of elasticity of elongation and factor of Poisson of the material of the bushing.

Third summand in a condition (1) $u_3(x)$ is accepted connected with contact pressure by nonlinear dependence

$$u_3(x) = c\sqrt{p(x)} \tag{5}$$

a



b

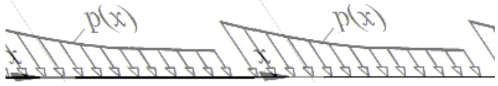


Fig. 1. The geometrical characteristics of connection with interference (a) and diagram of distribution of contact pressure (b)

In this equation a factor of a pliability c depends on elastic properties of materials, both the roughness of surfaces of the shaft and bushing. The experimental studies for steel and cast-iron details show different ways their mechanical processing is installed $c = (0,4...6,3) \cdot 10^{-3}\ \mathrm{mm}^2 \cdot \mathrm{N}^{-0,5}$ [2]. Thus the smaller values result in a higher accuracy degree of surface cleanliness in the processing.

Experimentally it is established, that the contact pressure has the maximal value at the edge of joint, with removing from which it sharply decreases and stabilizes. Equation is accepted for numerical image of such functions in the form

$$p(x) = P_0 + P_1 e^{-\beta x} \tag{6}$$

Thus pressure at edges of the bushing and far from it will be accordingly equal

$$p(0) = P_0 + P_1; \;\; p(\infty) = P_0 \tag{7}$$

The non-uniform equation (2) for loading (6) at $x \geq 0$ is expressed by the sum of the general and private decisions, and both summands must not increase with a growth of x.

Decision has the form

$$u_{1r}(x) = e^{-\beta x}(A_1 \sin \beta x + A_2 \cos \beta x) + b(P_0 + 0,8 P_1 e^{-\beta x}), \;\; x \geq 0 \tag{8}$$

where:

$b = R^2 / E_1 h$.

For negative values of the abscissa $x < 0$ the right part of equation (6) is a zero so for the left part the following accounting scheme is accepted (Fig. 1, b), that the not growing decision will take place

$$u_{1l}(x) = e^{\beta x}(B_1 \sin \beta x + B_2 \cos \beta x) \tag{9}$$

In formulas (8) and (9) indexes r and l designate the right and left parts of the shaft in section at the edge of joint.

The functions (8) and (9) are bound between itself with four border conditions

$$u_{1l}^{(k)}(x) = u_{1r}^{(k)} \text{ under } x = 0 \tag{10}$$

where:

factor $k = \overline{0{,}3}$ marks the functions (k = 0) and their derivative (up the third inclusive).

The realization of conditions (10) allows to find the meanings of factors

$$\begin{aligned} A_1 &= 0{,}5bP_1; \qquad A_2 = -0{,}5b(P_0 + P_1); \\ B_1 &= -0{,}1bP_1; \qquad B_2 = 0{,}5b(P_0 + 0{,}6P_1). \end{aligned} \tag{11}$$

Condition (1) with account of decisions (4) - (6), (8), (11) gives equation for the right part of the shaft

$$\begin{aligned} &b\left\{ 0{,}5e^{-\beta x}[P_1 \sin \beta x - (P_0 + P_1)\cos \beta x] + (P_0 + 0{,}8P_1 e^{-\beta x})\right\} + \\ &+ c_2(P_0 + P_1 e^{-\beta x}) + c\sqrt{P_0 + P_1 e^{-\beta x}} = 0{,}5\delta, \end{aligned} \tag{12}$$

which is nonlinear in the ratio to the sought parameters P_0 and P_1.

The equation (12) for extreme meanings of the argument allows to get two equations:

$$\text{under } x = \infty \quad b_0 P_0 + c\sqrt{P_0} - 0{,}5\delta = 0; \tag{13}$$

$$\text{under } x = 0 \quad b_1(P_0 + P_1) + c\sqrt{P_0 + P_1} + 0{,}2bP_0 - 0{,}5\delta = 0. \tag{14}$$

Here it is marked

$$b_0 = b + c_2; \; b_1 = 0{,}3b + c_2. \tag{15}$$

The expressions (13) and (14) are considered as the quadratic equations. The appropriate decisions look like

$$(P_0)_{1,2} = \left(\frac{-c \pm \sqrt{c^2 + 2b_0\delta}}{2b_0} \right)^2; \tag{16}$$

$$(P_0 + P_1)_{1,2} = \left(\frac{-c \pm \sqrt{c^2 + 2b_1\delta - 0{,}8bb_1P_0}}{2b_1} \right)^2. \tag{17}$$

The numerical analysis of the radicand expressions have shown that value c^2 in contrast with the constructive realization of the rest parameter is small enough. By accepting $c = 0$, that is without consideration the contact deformations of surfaces, is received

$$P_0 = \frac{0{,}5\delta}{b_0}. \tag{18}$$

At the account of deformations of roughness the contact pressure obviously decrease. This implies, that in the formulas (16) and (17) before a square root it is necessary to take a mark +.

For an estimation of strength in the most intense section at $x = 0$ all the loadings working here are found, considering the shaft as the axis-symmetrical loaded cylindrical shell.

The meridian bending moment, with provision of functions (9), (11) look like

$$M_x(x)\big|_{x=0} = 0{,}2b\beta^2 DP_1. \tag{19}$$

Axial power is absent, that is $N_x = 0.$

District bending moment is proportional to the meridian moment

$$M_y(x)\big|_{x=0} = \mu_1 M(x). \tag{20}$$

District force is

$$N_y\big|_{x=0} = 0{,}5R(P_0 + 0{,}6P_1). \tag{21}$$

Transverse force is

$$Q_x\big|_{x=0} = 2b\beta^3 D(0{,}5P_0 + 0{,}4P_1). \tag{22}$$

The maximum stresses from action of the bending moment appear in the most remote from the middle surface points *A* and *B* (see Fig. 1):

$$\sigma_x = \pm\frac{6M_x}{h^2} = \pm\frac{RP_1}{10h}\sqrt{\frac{3}{1-\mu^2}}\,, \tag{23}$$

$$\sigma_y^M = \mu_1\sigma_x\,. \tag{24}$$

In the formula (23) the mark minus corresponds to stress σ_x in the point *A*, and mark plus — in the point *B*.

The stresses from district force N_y are distributed on the thickness of the wall evenly and are equal

$$\sigma_y^N = \frac{N_y}{h} = \frac{0{,}5R(P_0 + 0{,}6P_1)}{h}. \tag{25}$$

The stresses from transverse force Q_x are small, and in the first approximation of the decision may be neglected. The radial stresses on an outside surface of the shaft are defined by the formula (7), and on an internal surface up to zero decrease linearly. Since the pressure *p(x)* is compressing, sign of parameters P_0 and P_1 in formula for forces and stresses follows to turn negative.

As an example there is considered a calculation of the joints with interference of rings of bearings and toothed gear with the hollow shaft of the drive in a passenger car generator (Fig. 2).

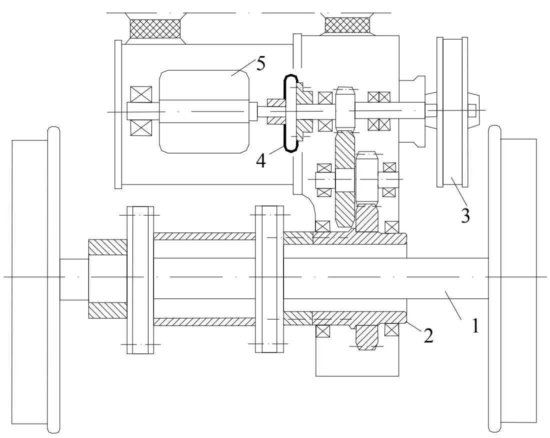


Fig. 2. Scheme of the generator drive in a passenger car: 1 – wheel axis; 2 - entrance the hollow shaft of gear ; 3 – conducting pulley belt drive; 4 – flexible coupling ; 5 – flywheel

It is established, that at an increase of the space factor of the shaft $\alpha = d_0 / d > 0{,}7$ there sharply increase the components of the stresses σ_x and σ_y. At criteria estimation of the shaft strength in accordance to the third and fourth theories of strength [3] the equivalent stresses σ_E change in 1,6-2 times at deviations of actual interference from the nominal within tolerance of the fit (Fig. 3).

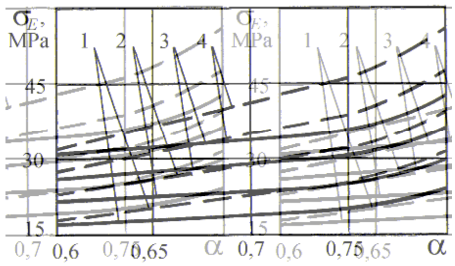


Fig. 3. Dependence σ_E on the space factor α for joint with interference of the hollow shaft d = 200 mm with the bearing (curves 1; 2; 3; 4 corresponds to interference's δ = 0.08; 0.10; 0.12; 0.14 mm, continuous lines – for point *A*, shaped – *B*)

It is established, that the larger values of σ_E meet to a point *B* (on an internal surface of the hollow shaft), that in a combination to possible constructive concentrators of stresses (the joints, keyways), technological defects (the foundry sinks, traces of the mechanical processing) and the surrounding ambiences influence (the corrosion) can result in fatigue cracks [4].

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