Polish Academy of Sciences
University of Engineering and Economics in Rzeszów
Lviv Polytechnic National University
University of Life Sciences in Lublin
Faculty of Production Engineering

ECONTECHMOD

AN INTERNATIONAL QUARTERLY JOURNAL ON ECONOMICS IN TECHNOLOGY, NEW TECHNOLOGIES AND MODELLING PROCESSES

Vol. 5, No 4

LUBLIN–RZESZÓW 2016

All the articles are available on the webpage:
http://www.pan-ol.lublin.pl/wydawnictwa/Econtechmod.html

ISSN 2084-5715

All the scientific articles received positive evaluations by independent reviewers

Linguistic consultant: Mikhail *Suknov*
Typeset: *Tetiana Serhiienko, Adam Niezbecki*
Cover design: *Hanna Krasowska-Kołodziej*

Editorial

Editorial Office Address
Polish Academy of Sciences Branch in Lublin
Pałac Czartoryskich, Plac Litewski 2, 20-080 Lublin, Poland
e-mail: eugeniusz.krasowski@up.lublin.pl

Edition 200+16 vol.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 3-10

ABOUT ONE CLASS OF THE PROBLEMS OF OPTIMAL STOCHASTIC CONTROL OF HYBRID DYNAMICAL SYSTEMS

A. Tevyashev, O. Matviyenko

Kharkiv National University of Radio Electronics; e-mail: olga_mat@ukr.net

Received September 06.2016: accepted September 29.2016

Abstract. A new class of the problems of optimal stochastic control of hybrid dynamical systems different from well-known ones by the introduction of additional extreme and probabilistic constraints on the phase variables is studied in the present work. The mathematical formulation and approximate method of solution of the examined class of the problems are presented in this work. The effectiveness of the use of this class of the problems is illustrated on the example of one of the largest water main of Ukraine.

Key words: optimal stochastic control, hybrid dynamical systems, probabilistic constraints on the phase variables, water main.

INTRODUCTION

Pipeline Energy Systems (PES) play an important role in the energetics, industry, communal household sector and include oil and gas pipelines, water supply systems [1].

A special feature of PES is the presence in them some specific technological elements: underground gas storages, tank batteries, clean water reservoirs (CWR), receiving tanks. The dynamic properties of the processes of changes of the levels of target products in reservoirs or overpressure in the underground gas storages differ significantly from the dynamic properties of the transport processes of the target products on the sections of the pipeline.

If the dynamic processes in the technological elements of the system can be divided into two disjoint classes - fast and slow flowing and the dynamic properties of the system are determined by the dynamic properties of the interaction of the end system of the technological elements with slowly flowing processes, then such a system will be called a hybrid dynamical system. This class of the systems includes pipeline energy system, involving main pipelines of the target products (water, oil and oil products, condensation and so on) with tank batteries both at its inputs and outputs and within the system, multi crossing line sections and multi shop pumping stations (PS).

As an object of control hybrid dynamical systems belong to the class of multi-dimensional, multi connected, nonlinear dynamical systems.

In this work hybrid dynamical systems are considered as a stochastic object functioning in a stochastic environment. The stochastic properties of the object of control are manifested in the fact that the parameters of technological equipment are unknown authentically a priori, but replaced by the estimates obtained from experimental samples of the finite length, which are random variables.

The stochastic properties of the environment appear as main disturbing factors – processes of supply and consumption of the target products, which are random processes, dependent on three main groups of the factors - meteorological, chronological, organizational [2].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The whole technological process of transportation and distribution of the target products in PES is based on the use of the energy developed by pumping stations, which, in their turn, are major consumers of the electricity. In the cost price of transportation and distribution of the target product in PES the electricity cost reaches more than 90% [3-5]. The continuous increase of the electricity tariff taking place now in Ukraine has led to the urgent need in the development and implementation of new energy and resource saving technologies of the control in PES [2]. The analysis of the actual modes of operation of PES [6-9] has shown that in modern PES there are huge reserves of saving of material and energy costs, but their implementation requires the development of new mathematical models and methods of control of the technological process of production, preparation, transportation and distribution of the target products of PES. One of the main way of the saving of material and energy resources is the transition from the traditional to the optimal control with the use of an adjustable valve for pumping units (PU) [10-14]. The practical implementation of three-band tariffs for the electricity opens up new opportunities of the saving of financial costs for the electricity due to more efficient use of storage capacity of the tank batteries of the main pipelines [15-20].

In this work we introduce a new class of the problems of optimal stochastic control of hybrid dynamical systems with three-band tariff for the electricity for the practical implementation of these two possibilities.

OBJECTIVES

The purpose of the study is the selection of a new class of the problems of optimal stochastic control of hybrid dynamical systems.

To achieve this goal the following problems are being solved:

1. The mathematical formulation of a new class of the problems of optimal stochastic control of hybrid dynamical systems with a three-band tariff for the electricity.

2. The evaluation of the potential of resource and energy saving while the implementation of the examined problem in the systems of operational dispatch management of the modes of operation of the water main.

MAIN RESULTS OF THE RESEARCH

For the mathematical formulation of this class of the problems the following must be known: the structure of the system presented as an oriented graph the inputs and outputs of which are reservoirs; the parameters of multiline pipelines, multi shop pumping stations and reservoirs. The interval of adjustment [0, *T*] (one day), which is divided into 24 sub-intervals corresponding to each hour of the period of control $k = 0, \ldots, 23$. At each k time sub-interval the predictions of inflow of the target products into the reservoirs at the inputs of the system, the incidental selections and the selections from the reservoirs at the output of the system in the form of conditional mathematical expectations

$$\overline{q}_{i0}(l) = \underset{\omega}{M}(q_{il}(\omega))$$

and their dispersions

$$\sigma^2_{q_{i0}}(l) = \underset{\omega}{D}(q_{il}(\omega))$$

calculated at the time interval $k = 0$ proactively $l = 1, 2, \ldots, 23$; the measured values of the levels of the target products in each z reservoir $H_{zk}(\tilde{\omega})$; the actual quantity of the switched PU are known.

The objective function of the problem of optimal stochastic control of the modes of operation of hybrid dynamical systems is presented in the form of the mathematical expectation of the integrated cost of the electricity consumed by all operating PU at the interval of control [0, T]:

$$\underset{\omega}{M}\sum_{k=0}^{23}\sum_{i=1}^{n}\sum_{j=1}^{m_i} N_{ijk}(q_{ik}(\omega))\cdot r_k \to \min_{u(k)\in\Omega}, \qquad (1)$$

where: $N_{ijk}(q_{ik}(\omega))$ – the power expended by j PU of i PS at the k time interval; r_k – the value of three-band tariff for the electricity at k time interval; n – the number of PS; m_i – the number of PU operating on i PS.

The area of constraints Ω is determined by the stochastic model of quasi-stationary modes of transportation and distribution of the target products in the pipeline energy systems [2, 21]:

$$\underset{\omega}{M}\Big(H^{\alpha}_{r\text{н}}(\omega) - H^{\alpha}_{r\text{к}}(\omega) + \sum_{i\in L} b_{1ri}h_{NAik}\ (q_{ik}(\omega)) + \sum_{i\in R} b_{1ri}h_{RZik}\ (q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega))\Big) = 0,$$
$$(r = \nu, \ldots, \nu + \eta_2 - 1; \quad k = 0, \ldots, 23), \qquad (2)$$

$$\underset{\omega}{M}\Big(h^{c}_{r}(q_{rk}(\omega)) - H_{1k}(\omega) + \sum_{i\in L} b_{1ri}h_{NAi}\ (q_{ik}(\omega)) + \sum_{i\in R} b_{1ri}h_{RZi}\ (q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega) + h^{g}_{i})\Big) = 0,$$
$$(r = \nu + \eta_2, \ldots, \nu + \eta_2 + \xi_1 - 1), \qquad (3)$$

$$\underset{\omega}{M}\Big(H^{\alpha}_{r\text{н}}(\omega) - H^{\alpha}_{r\text{к}}(\omega) + h^{g}_{r} - H_{zk}(\omega) + H_{1k}(\omega) + \sum_{i\in L} b_{1ri}h_{NAi}\ (q_{ik}(\omega)) + \sum_{i\in R} b_{1ri}h_{RZi}\ (q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega) + h^{g}_{i})\Big) = 0,$$
$$(r = \nu + \eta_2 + \xi_1, \ldots, e; \quad z = 1, \ldots, Z), \qquad (4)$$

$$\underset{\omega}{M}\left(\sum_{r=\nu}^{\nu+\eta_2-1} b_{1ri}q_{rk}(\omega) + \sum_{r=\nu+\eta_2}^{e} b_{1ri}q_{rk}(\omega) - q_{ik}(\omega)\right) = 0,$$
$$(i = 1, \ldots, \nu - 1), \qquad (5)$$

The temperature of the target product at any point x of the pipeline section is found as follows:

$$T_x = T_{cp} + (T_{\text{н}} - T_{cp})e^{-\theta x}, \qquad (6)$$

where: x – the distance from the beginning of the section to the point of this section with the coordinate x; T_{cp} – the average at a certain time interval temperature of the environment where the section of the pipeline is situated; $T_{\text{н}}$ – the temperature of the target product at the beginning of the section; θ – heat transfer parameter [2].

If in j nod there is some mixing m of the incoming streams of the target product with different temperature $T_i(\omega), \quad (i = 1, 2, \ldots, m)$, the temperature of the target

product $T_j(\omega)$, which outputs out of this nod is determined as follows:

$$\underset{\omega}{\mathrm{M}}\left(\sum_{i=1}^{m} T_{ik}(\omega) q_{ik}(\omega) - T_{jk}(\omega) \sum_{i=1}^{m} q_{ik}(\omega) \right) = 0, \quad (j = 1, \ldots, v), \tag{7}$$

$$q_{ik}(\omega) > 0, \quad i \in L. \tag{8}$$

$$H_{i\text{н}}^{\alpha}(\omega) - H_{i\text{к}}^{\alpha}(\omega) = sgn\, q_{ik}(\omega) S_i(\omega) q_{ik}^2(\omega), \quad i \in M, \quad \alpha \geq 1, \tag{9}$$

$$h_{NAik}(q_{ik}(\omega)) = a_{0i}(\omega) + a_{1i}(\omega) q_{ik}(\omega) + a_{2i}(\omega) q_{ik}^2(\omega), \quad i \in L, \tag{10}$$

$$\eta_{NAik}(q_{ik}(\omega)) = d_{0i}(\omega) + d_{1i}(\omega) q_{ik}(\omega) + d_{2i}(\omega) q_{ik}{}^2(\omega), \quad i \in L, \tag{11}$$

$$N_{NAik}(q_{ik}(\omega)) = \frac{9{,}81 \cdot h_{NAik}(q_{ik}(\omega)) \cdot q_{ik}(\omega)}{0{,}9 \cdot \eta_{NAik}(q_{ik}(\omega))}, \quad i \in L, \tag{12}$$

$$h_{RZik}(\mathrm{q}_{ik}(\omega)) = \frac{\mathrm{q}_{ik}(\omega) \mathrm{C}_i(\omega)}{\mathrm{E}_{ik}^2}, \quad \mathrm{i} \in \mathrm{R}, \tag{13}$$

and dynamical models of the reservoirs

$$H_{zk}(\omega) = H_{zk-1}(\omega) + c_{zk}(q_{zvhk}(\omega) - q_{zvihk}(\omega)), \quad (z = 1, \ldots, Z), \tag{14}$$

with probabilistic constraints on the phase variables:

$$P\left(H_{zk}(\omega) \leq H_z^{\max}\right) \geq \alpha,$$
$$P\left(H_{zk}(\omega) \geq H_z^{\min}\right) \geq \alpha, \quad \alpha \approx 0{,}97, \tag{15}$$

and extreme values of the constraints on the phase variables for the settled time interval $k = 6$ and $k = 23$:

$$\underset{\omega}{M}\{H_{z6}(\omega)\} \rightarrow \underset{q_{zvhk} \in \Omega}{extr}, \quad (k = 0, \ldots, 6), \tag{16}$$

$$\underset{\omega}{M}\{H_{z23}(\omega)\} \rightarrow \underset{q_{zvhk} \in \Omega}{extr}, \quad (k = 0, \ldots, 23), \tag{17}$$

where: $u(k)$ – vector of control which determines the amount of operating PU, the position of adjustable valves (AV); $H_{zk}(\omega)$ – level of the target product in z reservoir at a given k time interval, $H_z^{\min}$, $H_z^{\max}$ – minimum and maximum allowable level of the target product in each z reservoir.

Random variables characterize: $q_{ik}(\omega)$ – consumption of the target product on i section of the pipeline at k time interval; $H_{i\text{н}}^{\alpha}(\omega) - H_{i\text{к}}^{\alpha}(\omega)$ – pressure drop on i section of the pipeline; $h_{NAik}(q_{ik}(\omega))$ – pressure of i PU; $S_i(\omega) = f(l_i, d_i, T_{cpi}, k_i)$ – hydraulic resistance of i section of the pipeline ($i \in M$), k_i – coefficient of heat transfer from the target product to the environment; $h_{RZik}(\mathrm{q}_{ik}(\omega))$ – evaluation of the pressure drop on i AV; $\eta_{NAik}(q_{ik}(\omega))$ – evaluation of the coefficient of the efficiency of i PU; $a_{0i}(\omega), a_{1i}(\omega), a_{2i}(\omega), d_{0i}(\omega), d_{1i}(\omega), d_{2i}(\omega)$ – evaluation of the parameters of PU ($i \in L$); $C_i(\omega)$ – evaluation of the parameters of AV ($\mathrm{i} \in \mathrm{R}$); E_{ik} – opening degree of AV (E (0,1]); l_i, d_i, h_i^g – length, diameter and geodesic mark of i section of the pipeline ($i \in M$), b_{1ri} – cyclomatic matrix element; $q_{zvh}(\omega)$, $q_{zvih}(\omega)$ – consumption of the target product at input and output of the reservoir; $\underset{\omega}{M}\{\cdot\}$ – mathematical expectation of the random variable $\{.\}$.

For the solvability of the problem (1–17), the system of equations (2–17) is supplemented with the boundary conditions in the form of the predictions of the consumption of all consumers of the system $q_{ik}(l)$ (incidental and end) at all inputs, the predictions of the temperature of the target product supplied into the inputs of the main pipeline, calculated as conditional mathematical expectations at the time interval k = 0, proactively l = 1,2, ... , 23 as well as the initial conditions when k = 0 is in the form of estimates of the mathematical expectations of the levels of the target product in each z reservoir – H_{z0}.

Extreme values of the constraints on the phase variables are determined by the specifics of the object of control and the three-band tariff for the electricity. For the main pipelines in the restriction (16) *extr* is replaced by *min*, in the restriction (17) *extr* is replaced by *max* [3, 4]. For the sewerage systems in the restriction (16) *extr* is replaced by *max*, in the restriction (17) *extr* is replaced by *min* [5]. In the work [3] an approximate method of solving the problem in question by the transition from a stochastic problem (1 – 17) to its deterministic equivalent is presented here.

The deterministic equivalent of the problem of optimal stochastic control of the modes of operation of hybrid dynamical systems at the time interval [0, T] takes the form:

$$\sum_{k=0}^{23} \sum_{i=1}^{n} \sum_{j=1}^{m_i} \bar{N}_{ijk}(\bar{q}_{ik}) \cdot r_k \rightarrow \min_{u(k) \in \Omega}, \tag{18}$$

$$\bar{H}_{r\text{н}}^{\alpha} - \bar{H}_{r\text{к}}^{\alpha} + \sum_{i \in L} b_{1ri} \bar{h}_{NAik}(\bar{q}_{ik}) + \sum_{i \in R} b_{1ri} \bar{h}_{RZik}(\bar{q}_{ik}) + \sum_{i \in M_1} b_{1ri} (\bar{H}_{i\text{н}}^{\alpha} - \bar{H}_{i\text{к}}^{\alpha}) = 0, \quad (r = v, \ldots, v + \eta_2 - 1; \quad k = 0, \ldots, 23), \tag{19}$$

$$\bar{h}_r^c(\bar{q}_{rk}) - \bar{H}_{1k} + \sum_{i\in L} b_{1ri}\bar{h}_{NAi}(\bar{q}_{ik}) + \\ + \sum_{i\in R} b_{1ri}\bar{h}_{RZi}(\bar{q}_{ik}) + \sum_{i\in M_1} b_{1ri}(\bar{H}_{i\text{н}}^{\alpha} - \bar{H}_{i\text{к}}^{\alpha} + h_i^g)\Big) = 0, \\ (r = v+\eta_2,\ldots,v+\eta_2+\xi_1-1), \quad (20)$$

$$\bar{H}_{r\text{н}}^{\alpha} - \bar{H}_{r\text{к}}^{\alpha} + h_r^g - \bar{H}_{zk} + \bar{H}_{1k} + \\ + \sum_{i\in L} b_{1ri}\bar{h}_{NAi}(\bar{q}_{ik}) + \sum_{i\in R} b_{1ri}\bar{h}_{RZi}(\bar{q}_{ik}) + \\ + \sum_{i\in M_1} b_{1ri}(\bar{H}_{i\text{н}}^{\alpha} - \bar{H}_{i\text{к}}^{\alpha} + h_i^g)\Big) = 0, \\ (r = v+\eta_2+\xi_1,\ldots,e; \quad z = 1,\ldots,Z), \quad (21)$$

$$\bar{q}_{ik} = \sum_{r=v}^{v+\eta_2-1} b_{1ri}\bar{q}_{rk} + \sum_{r=v+\eta_2}^{e} b_{1ri}\bar{q}_{rk}, \\ (i = 1,\ldots,v-1). \quad (22)$$

$$\bar{q}_{ik} > 0, \quad i \in L. \quad (23)$$

$$\bar{H}_{i\text{н}}^{\alpha} - \bar{H}_{i\text{к}}^{\alpha} = sgn\,\bar{q}_{ik}\bar{S}_i\bar{q}_{ik}^2, \\ i \in M, \quad \alpha \geq 1 \quad (24)$$

$$\bar{h}_{NAik}(\bar{q}_{ik}) = \bar{a}_{0i} + \bar{a}_{1i}\bar{q}_{ik} + \bar{a}_{2i}\bar{q}_{ik}^2, \quad i \in L, \quad (25)$$

$$\bar{\eta}_{NAik}(\bar{q}_{ik}) = \bar{d}_{0i} + \bar{d}_{1i}\bar{q}_{ik} + \bar{d}_{2i}\bar{q}_{ik}{}^2, \quad i \in L, \quad (26)$$

$$\bar{N}_{NAik}(\bar{q}_{ik}) = \frac{9{,}81\cdot\bar{h}_{NAik}(\bar{q}_{ik})\cdot\bar{q}_{ik}}{0{,}9\cdot\bar{\eta}_{NAik}(\bar{q}_{ik})}, \\ i \in L, \quad (27)$$

$$\bar{h}_{RZik}(\bar{q}_{ik}) = \frac{\bar{q}_{ik}\bar{C}_i}{E_{ik}^2}, \quad i \in R, \quad (28)$$

$$\bar{H}_{zk} = \bar{H}_{zk-1} + c_{zk}(\bar{q}_{zvhk} - \bar{q}_{zvihk}), \\ (z = 1,\ldots,Z). \quad (29)$$

$$\bar{H}_{zk} \leq H_z^{\max}, \quad \bar{H}_{zk} \geq H_z^{\min}, \quad (30)$$

$$H_{z6} \to \underset{q_{zvhk}\in\Omega}{extr}, \ (k = 0,\ldots,6), \quad (31)$$

$$H_{z23} \to \underset{q_{zvhk}\in\Omega}{extr}, \ (k = 0,\ldots,23). \quad (32)$$

The problem (18–32) belongs to the class of the problems of nonlinear dynamic programming with nonlinear objective function (18), non-linear constraints in the form of qualities (19–21), extreme constraints (31–32), coupling equations (22–29) and the unilateral restrictions variables (23), (30).

The solution of the problem (18–32) is carried out by a modified method of branches and borders [3].

THE EVALUATION OF THE POTENTIAL OF RESOURCE AND ENERGY SAVING OF THE WATER MAIN

Practical use of the examined class of the problems in the systems of the operational dispatch management allows to embody not only energy and resource saving technologies, but also to get unbiased, efficient and consistent evaluations of the potential of resource saving while the modernization of technological equipment of WM, which is the basis for the investment projects. Without loss of generality, we will assess the potential of energy and resource saving for WM at the beginning while the transition from the traditional system of control of the modes of operation of WM to the system of optimal stochastic control with the following possible variants of modernization of technological equipment. The evaluation of the potential of resource and energy saving was carried out for one of the largest Ukrainian WM the scheme of which is shown in Fig. 1.

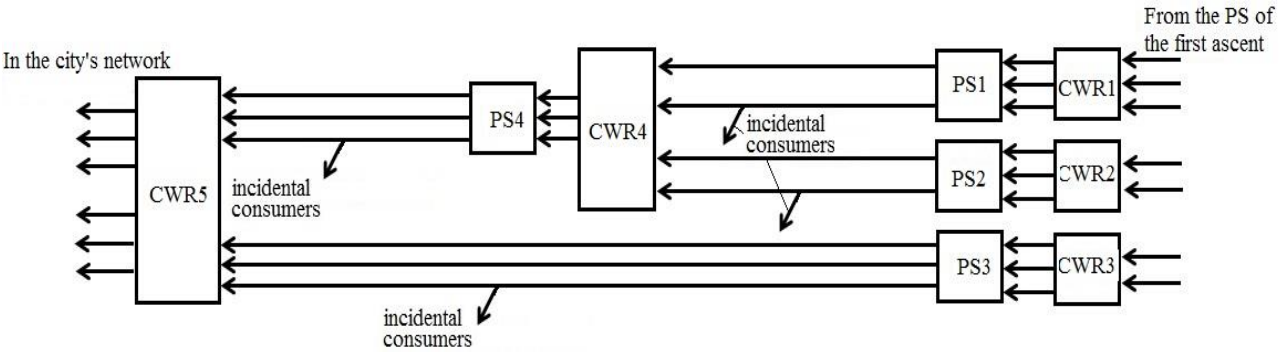


Fig. 1. The structure of the water main

The structuring of the examined class of the problems applied to WM is given in the work [3]. The use of the system of optimal stochastic control has initially allowed to evaluate the potential of resource saving while the transition from the existing system of operational dispatch management of the modes of operation of WM to the system of optimal stochastic control with three-band tariff for the electricity. Further the evaluation of the potential of energy and resource saving, associated with the modernization of technological equipment, was carried out for the system of optimal stochastic control.

The energy cost is determined according to three-band tariff (see Table 1).

Table 1. The electricity rate according to the hours

Hours	Coefficient	Cost of 1 кW/h, UAH
6.00–8.00	1,02	1,6
8.00–10.00	1,8	2,82
10.00–18.00	1,02	1,6
18.00–22.00	1,8	2,82
22.00–23.00	1,02	1,6
23.00–6.00	0,35	0,39

The examined WM includes PS of the second lift (PS1, PS2, PS3) and PS4 of the third lift. There are clean water reservoirs (CWR) at the input of each PS and at the output of WM.

The input data for the problem of optimal stochastic control of the modes of operation of WM at the time interval $[0, T]$ (7 days) are as follows:

• static data including the structure of WM: lengths; diameters; geodetic marks of the sections of the pipeline; the estimates of the parameters of the mathematical models of PU for each PS; the estimates of hydraulic resistances of AV on each PS; physical dimensions of each of CWR;

• dynamic data, including the prediction of the daily water consumption from CWR5; the prediction of the water consumption by incidental consumers.

PS of the second lift PS1, PS2 and PS3 are equipped with the same type, connected in parallel PU with the same characteristics, PS4 of the third lift is equipped with the same type PU with different characteristics. At time zero k=0 the mathematical expectation of the water levels in CWR4 is $H_{1,0} = 2,6$ m; in CWR5 $H_{2,0} = 3,9$ m. The allowable ranges of change in water levels in CWR4 is [2-4.9], in CWR5 is [1.45-4.9]. The actual parameters of WM the technological equipment are given in Table 2.

Table 2. The actual parameters of the process equipment of the MW

PS	PU type	q, m^3/h	h, m	N, kW	n_1, min^{-1}.	d, mm	amount of PU
PS1	20NDS	3420	71	960	1000	765	4
PS2	24NDS	6500	79	1600	750	1040	5
PS3	22NDS	4799	90	1000	1250	825	4
PS4	24NDS	6500	79	1600	750	990–1040	6

Table 3 shows the parameters of the modes of operation of WM for the actual operating mode and the operating mode at the optimal stochastic control (S – costs for the electricity).

Table 3. The estimates of the parameters of the modes of operation of WM at the time interval 7 days

Day	Actual modes		Optimal modes	
	N, kW	S, UAH	N, kW	S, UAH
1	261567	390840	288533	362761
2	265399	396530	300658	386446
3	258970	393765	293739	370907
4	251876	391068	289142	363551
5	251998	391339	281864	352021
6	269445	398191	298675	385346
7	270893	400120	279714	361951
Amount	1830152	2761856	2032327	2582988

From Table 3 it can be seen that the transition from the existing system of control of the modes of operation of WM to the system of optimal stochastic control allowed to reduce the actual costs for the electricity by 6.5%.

Table 4 shows 6 variants of modernization of the technological equipment of WM for the existing structure of WM and 2 variants for WM with the altered structure. The point of the structural changes of WM was in switching of the outputs of PS3 to the inputs of CWR4 in accordance with the scheme shown in Fig. 2.

Fig. 3, Fig. 4 show the estimates of the mathematical expectation of the power and energy costs and the potential of resource and energy saving at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization.

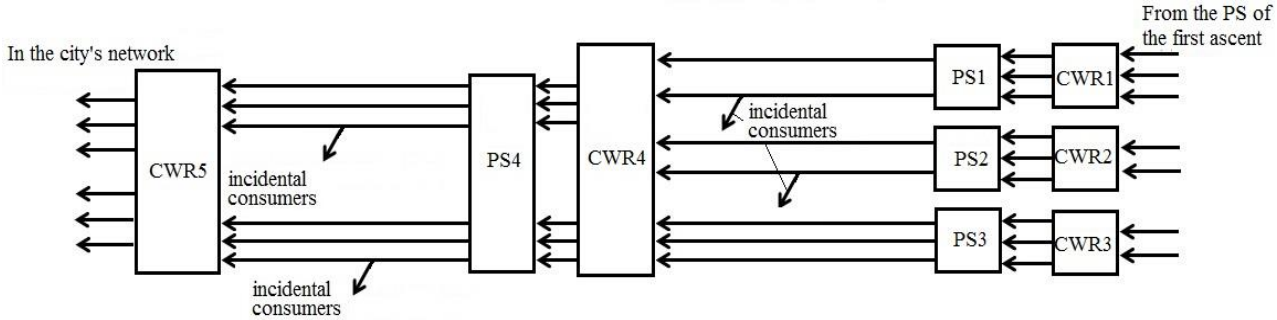


Fig. 2. The scheme a modified water main structure

Table 4. The variants of modernization of the technological equipment of WM

Variant	The point of modernization	Actual parameters of the operating equipment	Changed parameters of the operating equipment
1	Replacing the engine to the PS1	$n1$=960 min^{-1}	$n1$*=750 min^{-1}
2	Cutting wheels of PU on PS1	d=765 mm	d*=612 mm
3	Cutting wheels of PU on PS2	d=990 mm	d*=792 mm
4	Replacing the engine to the PS1, cutting wheels of PU on PS2	$n1$=960 min^{-1} d=990 mm	$n1$*=750 min^{-1} d*=792 mm
5	Cutting wheels of PU on PS1, cutting wheels of PU on PS2	d=765 mm d=990 mm	d*=612 mm d*=792 mm
6	Cutting wheels of PU on PS1 cutting wheels of PU on PS2, one adjustable drive on PS3, two adjustable drives on PS4.	d=765 mm d=990 mm $n1$=1000 min^{-1} $n1$=750 min^{-1}	d*=612 mm d*=792 mm $n1$*=700 min^{-1} $n1$*=650 min^{-1}
7	Modification of the structure of WM, cutting wheels of PU on PS1, cutting wheels of PU on PS2, cutting wheels of PU on PS3, cutting wheels of PU on PS4,	 d=765 mm d=990 mm d=825 mm d=990 mm	 d*=612 mm d*=792 mm d*=660 mm d*=792 mm
8	Modification of the structure of WM, cutting wheels of PU on PS1, cutting wheels of PU on PS2, cutting wheels of PU on PS3, two adjustable drives on PS4,	 d=765 mm d=990 mm d=825 mm $n1$=750 min^{-1}	 d*=612 mm d*=792 mm d*=660 mm $n1$*=650 min^{-1}

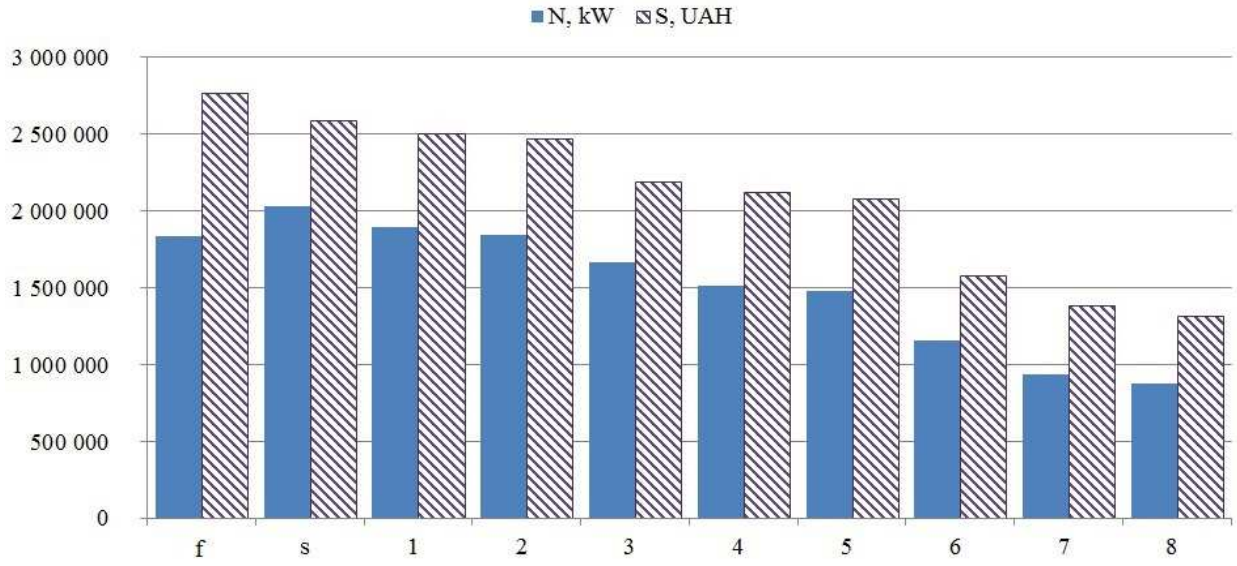


Fig. 3. The estimates of the mathematical expectation of the power and energy costs at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization WM

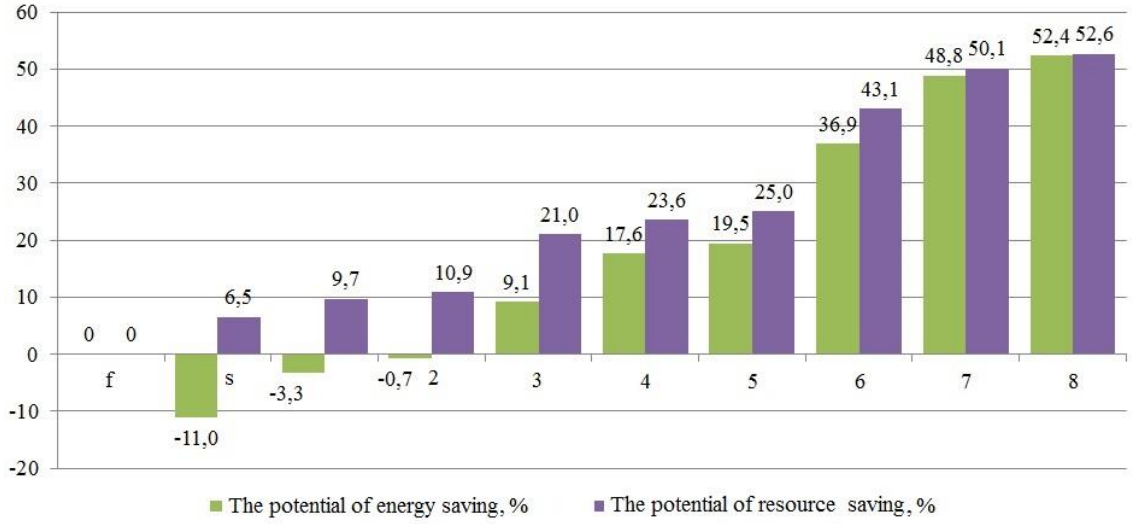


Fig. 4. The estimates of the mathematical expectation of the potential of resource and energy saving at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization WM

The analysis of the obtained results showed that all variants of modernization have significant potential of resource and energy saving, but for its implementation require various material costs. Among the least expensive variants of modernization connected with cutting wheels on PU, variant 5 with the potential of energy saving 19.5% and resource saving 25% must be emphasized. More promising, with the potential of energy saving 52.4% and resource saving 52.6% is variant 8 providing cutting wheels of PU and installation of two adjustable valves on PS4. It should be noted that all examined variants of the modernization of WM are economically attractive, and the payback period does not exceed one year.

CONCLUSIONS

In the present work:

1. A new class of the problems of optimal stochastic control of hybrid dynamical systems different from existing ones by the introduction of additional extreme and probabilistic constraints on the phase variables, the use of which allows to implement practically the energy saving technologies in the PES is proposed;

2. It is shown that the use of this class of the problems allowed to obtain unbiased, efficient and consistent estimates of the potential of resource and energy saving for one of the largest water main of Ukraine.

The obtained results are the basis for the development of science-based investment projects on the modernization of WM.

REFERENCES

1. **Novitsky N., Sukharev M., Tevyashev A., 2015.** Energy Pipeline System: Methodological and Applied Problems of Mathematical Modeling. - Novosibirsk: Nauka, 476. (in Russian).

2. **Evdokimov A., Teviashev A., 1980.** Operational management of load flow in engineering networks. - Kharkiv: Vyscha shkola, 144. (in Russian).

3. **Karelin V., Minayev A., 1986.** Pumps and pumping stations. M.: Stroyizdat, 320. (in Russian).

4. **Chupin R., Melehov E., 2011.** The Theory and Practice of Modeling and Optimization of Water and Wastewater. – Irkutsk: Publisher Irkutsk State Technical University, 323. (in Russian).

5. **Lobachev P., 1983.** Pumps and pumping stations. - Moscow: Stroyizdat, 191. (in Russian).

6. **Teviashev A., Matvienko O., 2014.** About one Approach to Solve the Problem of Management of the Development and Operation of Centralized Water–Supply Systems. Econtechmod. An International Quarterly Journal, Vol. 3, № 3, 61–76.

7. **Teviashev A., Matvienko O., 2015.** Mathematical Model and Method of Optimal Stochastic Control of the Modes of Operation of the Water Main. East–European Journal of Enterprise Technologies, 6/4(78), 45-53. (in Russian).

8. **Tevyashev A., Matviyenko O., 2015.** About One Problem of Optimal Stochastic Control of the Modes of Operation of Water Mains. Econtechmod. An International Quarterly Journal, Vol. 4, № 3, 3–12**.**

9. **Tevyashev A., Nikitenko G., Matviyenko O., 2015.** Optimal Stochastic Control of the Modes of Operation of the Sewage Pumping Station. Econtechmod. An International Quarterly Journal. 4, Issue 3, 47–55**.**

10. **Sotnikov D., 2014.** Development of Methodology for Improving the Energy Efficiency of Pumping Stations. Journal of Vilnius Gediminas Technical University, 1, 105–106.

11. **Petrov D., 2005.** Adjustable Drive to Pump Applications. Power Electronics, 4, 18–22.

12. **Pulido–Calvo I., Gutiérrez–Estrada J., 2011.** Selection and operation of pumping stations of water distribution systems. Environmental Research Journal, Nova Science Publishers, 5 (3), 1–20.

13. **Leznov B., 2006.** Energy saving and adjustable drive for pump and blower installations. – Moscow: Energoatomizdat, 358. (in Russian).

14. **Reinbold C., Hart V., 2011.** The search for energy savings: optimization of existing & new pumping stations. Florida Water Resources Journal, 44–52.

15. **Lipták B., 2009.** Pumping station optimization. Control promoting excellence in process automation, 12–19.

16. **Burgschweiger J., Gnadig B., Steinbach M., 2009.** Optimization Models for Operative Planning in Drinking Water Networks. Optimization and Engineering. 10, №1, 43–73.

17. **Gulyi A., Kobyzska A., 2012.** Pumping Equipment Effectiveness Increase by Means of Ejector Application as Preliminary Stage for High-Speed Pump Units. Motrol. Commission of Motorization and Energetics in Agriculture Polish Academy of Sciences. University of Engineering and Economics in Rzeszow. Lublin – Rzeszow. 14, №1, 158–164.

18. **Ruuskanen A., 2007.** Optimization of Energy Consumption in Wastewater Pumping. Lappeenranta University of Technology Department of Energy- and Environmental Technology. 99.

19. **Saliev E., 2013.** Reliability of the Functioning of the Water Supply and Sewerage System. Motrol. Commission of Motorization and Energetics in Agriculture Polish Academy of Sciences. University of Engineering and Economics in Rzeszow. Lublin – Rzeszow. 15, №5, 53–61.

20. **Steinbach M., 2001.** General Information Constraints in Stochastic Programs. Konrad-Zuse-Zentrum fur Informations technik. Berlin, ZIB-Report, 5.

21. **Bertsekas D., Shreve S., 1985**. Stochastic Optimal Control. The Discrete Time Case. Moscow: Nauka, 280. (in Russian).

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 11-19

ON THE CONSTRUCTION OF TWO-SIDED APPROXIMATIONS TO POSITIVE SOLUTIONS OF SOME ELLIPTIC PROBLEM

S. Kolosova, V. Lukhanin

Kharkiv National University of Radio Electronics; e-mail: lanakol@ukr.net

Received September 07.2016: accepted September 30.2016

Abstract: In this paper we have investigated the existence, uniqueness and possibility of constructing of two-sided approximations to the positive solution of a heat conduction problem with two sources.

The investigation is based on methods in operator equations theory in half-ordered spaces. In this case we have considered a nonlinear operator equation that corresponds to the initial boundary value problem in a cone of non-negative continous functions. The properties of the corresponding operator define conditions which provide the existence and uniqueness of the solution. The conditions link the parameters of the problem implicitly meaning that they don't provide the range of allowed values but need to be verified for each specific parameters value set separately.

During the investigation we have provided the scheme of a two-sided iteration process which must satisfy the conditions in order to converge to the positive solution from both sides.

The computational experiment have been conducted in two domains – unit disk and unit half disk. We have applied both two-sided approximations method and Green's quasifunction method for the problem solving. The obtained results are presented as a surface and level lines plots and also as a table. The results in corresponding domains obtained by different methods have been compared with each other.

Key words: two-sided approximations, operator equation, positive solution, concave operator, conical interval, Green's function, Green's quasifunction.

INTRODUCTION

Modern science is highly interested in processes that take place in nonlinear environments. Mathematical models of these processes typically are represented by nonlinear boundary value problems of mathematical physics of the following form

$$-\Delta u = f(\lambda, u) \quad \forall x \in \Omega \subset R^n, \qquad (1)$$

$$u > 0, \quad u|_{\partial\Omega} = 0, \qquad (2)$$

where: λ is a numerical parameter.

Many profound problems are reduced to equation (1). For example:

1) various problems in the theory of elasticity, where the parameter represents the load;

2) temperature distribution during conduction of electrical current through a body (the parameter is a value of electrical current);

3) auto-oscillation problems (the parameter is the unknown period) etc.

More specifically, if

$$f(u) = e^{-u},$$

then problem (1), (2) is a mathematical model of a flow in conductive environment inside an impenetrable cylinder [1]; when

$$f(u) = \lambda e^{u},$$

equation (1) is a stationery equation of the thermal theory of spontaneous ignition of chemically active gas mixture inside a vessel [2-5], in this case problem (1), (2) is called the Liouville-Gelfand problem; if

$$f(u) = u^{p}, \ p > 0,$$

then we have a mathematical model of gas density distribution in a star (equation (1) in that case is called the Lane–Emden equation) [6]; the problem of model selection of population migration in genetics leads to problem (1), (2) with

$$f(u) = \lambda(1+u)^{q} \quad [7];$$

problem (1), (2) with

$$f(u) = \lambda + u^{p}, \ f(u) = \lambda u^{q} + u^{p},$$
$$f(u) = \lambda\left(e^{u} + \ + e^{\gamma u}\right)$$

are considered in [8, 9] and with

$$f(u) = au^{-q} + bu^{p}, \ a > 0, \ b > 0, \ q > 0, \ p > 0$$

in [10].

Problem (1), (2) is equivalent to the integral equation in $C(\Omega)$

$$u(x) = \int_{\Omega} G(x,s) f(\lambda u(s)) ds, \qquad (3)$$

where: $G(x,s)$ is a Green's function for the operator $-\Delta u$ of the first boundary value problem in the domain

$$\Omega,\ x=(x_1,\ldots,x_n),\ s=(s_1,\ldots,s_n).$$

Now we rewrite equation (3) as follows:

$$u=Tu,$$

where: $Tu=\int_{\Omega}G(x,s)f(\lambda u(s))ds$ is an operator with domain $D(T)=K$, K is a cone of non-negative functions in the space $C(\Omega)$.

It's naturally to expect that the existence and uniqueness of the positive solution of equation (1), and hence problem (1), (2), significantly depends on properties of the operator T and the form of $f(\lambda,u)$. The cases of monotone and antitone operator Tu are considered in [11-13].

Since the construction of Green's functions can be quite complicated even for two-dimensional problems there are only few cases for which a constructive solution can be obtained. In complex domains Green's quasifunction method can be used [14]. The method is based on the construction of a boundary equation. R-functions theory plays a significant role in solving this task [14, 15, 16].

In this work we investigate the following problem [8]

$$\begin{gathered}-\Delta u=\lambda u^q+u^p \quad \forall x\in\Omega,\\ u>0,\quad u|_{\partial\Omega}=0,\end{gathered} \tag{4}$$

where: $0<q<1<p,\quad \lambda>0$.

The equation of the problem (4) is a stationary heat conduction equation with two sources of different power and describes heat distribution over a plate (domain Ω) that doesn't change in time. It happens when stationary sources of heat act for a long time and transitional processes caused by them have been finished. The terms λu^q and u^p represent the power of heat sources.

EXISTENCE OF POSITIVE SOLUTIONS

Problem (4) is equivalent to the integral equation in $C(\Omega)$

$$u(x)=\int_{\Omega}G(x,s)[\lambda u^q(s)+u^p(s)]ds. \tag{5}$$

We need following definitions in the sequel [17-19].

Definition 1. A convex closed set K in Banach space E is called a *cone* if this set contains, together with each element u, ($u\neq\theta$), all the elements of the form tu for $t\geq 0$ and does not contain the element $-u$, where θ is the zero element of E.

Definition 2. The cone K is called *normal* if there exists an $N(K)$ such that:

$$\|u\|\leq N(K)\|v\| \text{ for } 0\leq u\leq v,\ u,v\in K.$$

More precisely, the cone K is called *normal* if there exists a $\delta>0$ such that the inequality:

$$\|f_1+f_2\|\geq\delta$$

is satisfied for all

$$f_1,f_2\in K,\ \|f_1\|=\|f_2\|=1.$$

The cone of non-negative functions is normal in the space C.

Definition 3. The collection of elements $u\in K$ for which $v_0\leq u\leq w_0$ is called the *conical interval* $<v_0,w_0>$.

Definition 4. An operator T is *monotone* if $Tv\leq Tw$ follows from $v\leq w$, $v,w\in K$.

Definition 5. An operator T is *positive* if $TK\subset K$.

Definition 6. Let E and F be Banach spaces. An operator, acting from E into F, is called *completely continuous* if it maps every bounded set of the space E onto a (relatively) compact set of the space F.

Definition 7. Let $f(\lambda,x,u(x))$ be a non-negative and concave function (i.e.

$$f(\lambda,x,tu(x))-tf(\lambda,x,u(x))>0 \tag{6}$$

for all $t\in(0,1)$, $u>0$ and $x\in\Omega$). Then an operator

$$Tu=\int_{\Omega}G(x,s)f(\lambda,s,u(s))ds$$

is called u_0-concave on K if

$$\alpha u_0(x)\leq Tu\leq\beta u_0(x)\ \ \forall u\geq 0,\ \alpha,\beta>0, \tag{7}$$

where: $u_0\in K$ is a fixed non-zero element.

Suppose K is a cone of non-negative functions in $C(\Omega)$. Let

$$u=Tu \tag{8}$$

be an operator equation defined over K, where:

$$Tu=\int_{\Omega}G(x,s)[\lambda u^q(s)+u^p(s)]ds.$$

Since the cone K is normal and the function $f(u)=\lambda u^q+u^p$ is continuous in u, it follows that the operator T is completely continuous if it maps $C(\Omega)$ on itself [17, 18].

First, we state

Lemma 1. The operator T has following properties:

1) T is monotone.

2) There is a conical interval $< v_0, w_0 >$ such that

$$T < v_0, w_0 > \subset < v_0, w_0 >.$$

3) T is u_0-concave, where:

$$u_0 = \int_\Omega G(x,s)ds.$$

Proof. 1) The proof is trivial.

2) Let us build $< v_0, w_0 >$.

It is advised in [17] to put $v_0 = 0$ if $f(\lambda, x, u)$ is monotonically increasing in *u*. Following this advice we get

$$v_1 = Tv_0 = \int_\Omega G(x,s)[\lambda v_0^q + v_0^p]ds = 0.$$

Therefore, the interval's left endpoint stays still if we apply the successive approximation scheme:

$$u_{n+1}(x) = \int_\Omega G(x,s)[\lambda u_n^q(s) + u_n^p(s)]ds, \quad n = 1,2,\ldots. \tag{9}$$

It means that we obtain approximations from above only instead of two-sided ones.

Now we introduce the following concept. Let v_0 be

$$v_0(x) = \varepsilon\omega(x),$$

where: $\omega(x) > 0$ in Ω, $\omega(x)|_{\partial\Omega} = 0$, $\varepsilon = const > 0$.

Remark 1. The function $\omega(x)$ can be constructed practically for any domain using R-functions theory [14].

Hence,

$$v_1 = \int_\Omega G(x,s)[\lambda v_0^q + v_0^p]ds =$$
$$= \int_\Omega G(x,s)[\lambda\varepsilon^q\omega^q(s) + \varepsilon^p\omega^p(s)]ds =$$
$$= \int_\Omega G(x,s)\varepsilon^q[\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]ds =$$
$$= \varepsilon^q \int_\Omega G(x,s)[\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]ds.$$

From the inequality $v_1 \geq v_0$ it follows that

$$\varepsilon^q \int_\Omega G(x,s)[\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]ds \geq \varepsilon\omega(x)$$

or

$$\int_\Omega G(x,s)[\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]ds \geq$$
$$\geq \frac{\varepsilon\omega(x)}{\varepsilon^q} = \varepsilon^{1-q}\omega(x).$$

Then, squaring the last expression and applying the Cauchy-Schwarz inequality $|(u,v)| \leq \|u\|\|v\|$:

$$\left(\int_\Omega G(x,s)[\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]ds\right)^2 \geq$$
$$\geq \varepsilon^{2(1-q)}\omega^2(x),$$
$$\int_\Omega G^2(x,s)ds \int_\Omega [\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]^2 ds \geq$$
$$\geq \varepsilon^{2(1-q)}\omega^2(x)$$

or

$$\omega^2(x) \leq \varepsilon^{2(q-1)} M \int_\Omega [\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]^2 ds,$$

where: $M = \max_{x\in\Omega} \int_\Omega G^2(x,s)ds.$

Finally, we obtain

$$\max_{x\in\Omega} \omega^2(x) \leq$$
$$\leq \varepsilon^{2(q-1)} M \int_\Omega [\lambda\omega^q(s) + \varepsilon^{p-q}\omega^p(s)]^2 ds \tag{10}$$

and this estimate is satisfied for any domain Ω.

Let us find w_0. First we put $w_0 = \beta = const > 0.$ Then, using the inequality $w_1 \leq w_0$ and scheme (9) we obtain

$$w_1(x) = \int_\Omega G(x,s)[\lambda w_0^q(s) + w_0^p(s)]ds =$$
$$= \int_\Omega G(x,s)[\lambda\beta^q + \beta^p]ds \leq \beta.$$

It now follows that

$$L \leq \frac{\beta}{\lambda\beta^q + \beta^p}, \tag{11}$$

where: $L = \max_{x\in\Omega} \int_\Omega G(x,s)ds.$

Thus, conditions (10) and (11) link parameters *p*, *q* and ε, β. The latter ones define the conical interval

$$< v_0 = \varepsilon\omega(x), w_0 = \beta >.$$

3) In order to show u_0-concavity we will use Definition 7.

Since we have shown how to build $< v_0, w_0 >$, it follows that (7) is satisfied.

Furthermore, from (6) $\lambda > \dfrac{\beta^{p-q}(t-t^p)}{t^q - t}$.

Now we define $g(t) := \dfrac{t-t^p}{t^q-t}$.

Solutions of the equation

$$(1-pt^{p-1})(t^q-t)-(qt^{q-1}-1)(1-t^p)=0 \quad (12)$$

define maximum values of the function $g(t)$ for $t \in (0,1)$.

Let $t_* \in (0,1)$ be a solution of (12). Now note that

$$\lim_{t\to 1} g(t) = \frac{p(p-1)}{q(q-1)}.$$

This implies that the parameters λ, q, p, and the constant β must also satisfy

$$\lambda > \max\left\{\beta^{p-q}\,\frac{p(p-1)}{q(q-1)}, \beta^{p-q}\,\frac{t_*-t_*^p}{t_*^q-t_*}\right\}. \quad (13)$$

This completes the proof of the lemma.

Now, we build an iteration process for equation (8) by the following scheme

$$\begin{aligned} v_{n+1}(x) &= \int_\Omega G(x,s)[\lambda v_n^q(s) + v_n^p(s)]ds, \\ & \qquad n = 0,1,\ldots, \\ w_{n+1}(x) &= \int_\Omega G(x,s)[\lambda w_n^q(s) + w_n^p(s)]ds, \\ & \qquad n = 0,1,\ldots. \end{aligned} \quad (14)$$

The main result of this paper is

Theorem 1. Process (14) converges to $u^*(x)$ from both sides with respect to the norm of space $C(\Omega)$ if λ, q, p, ε, β satisfy (10), (11), and (13), where $u^*(x)$ is an exact positive single solution of equation (5) and

$$v_0 < v_1 < \ldots < u^* < \ldots < w_1 < w_0.$$

Proof. First, we know that the cone $K \in C(\Omega)$ is normal. The operator T is completely continuous $\forall u \in K$, monotone and maps conical interval $< v_0, w_0 >$ into itself by Lemma 1 if λ, q, p, ε, β satisfy (10), (11), and (13). It now follows that the equation has exactly one positive solution [17].

Then since the operator T is also u_0-concave by Lemma 1 and the cone K is normal it follows that process (14) converges to $u^*(x)$ from both sides with respect to the norm of space $C(\Omega)$ [17].

This completes the proof of the theorem.

GREEN'S QUASIFUNCTION

Rvachev V.L. proposed to consider a special function which is close in particular sense to Green's one [14]. It's called Green's quasifunction. Now let's see how it can be established for problem (4).

Let $\psi = 0$ be the normalized boundary equation of the first order on boundary $\partial\Omega$, namely

$$\begin{aligned} \psi(x) = 0, \quad |\nabla\psi| = 1, \quad x \in \partial\Omega, \\ \psi(x) > 0, \quad x \in \Omega. \end{aligned} \quad (15)$$

Now we put

$$\zeta(x,s) = -\frac{1}{2}\ln(r^2 + 4\psi(x)\psi(s)), \quad \Omega \subset R^2,$$

$$\zeta(x,s) = \left[r^2 + 4\psi(x)\psi(s)\right]^{-\frac{1}{2}}, \quad \Omega \subset R^3,$$

where: $r = |x-s|$.

The Green's quasifunction can be established as follows

$$G_2(x,s) = \frac{1}{2\pi}\left[\ln\frac{1}{r} - \zeta(x,s)\right], \quad \Omega \subset R^2,$$

$$G_2(x,s) = \frac{1}{4\pi}\left[\frac{1}{r} - \zeta(x,s)\right], \quad \Omega \subset R^3.$$

Then problem (4) can be reduced to nonlinear integral equation

$$\begin{aligned} u(x) = \int_\Omega G_2(x,s)[\lambda u^q(s) + u^p(s)]ds + \\ + \int_\Omega u(s)K(x,s)ds, \end{aligned} \quad (16)$$

where: $K(x,s) = -\dfrac{1}{2\pi}\left(\dfrac{\partial^2}{\partial s_1^2} + \dfrac{\partial^2}{\partial s_2^2}\right)\zeta(x,s), \Omega \subset R^2,$

$$K(x,s) = -\frac{1}{4\pi}\left(\frac{\partial^2}{\partial s_1^2} + \frac{\partial^2}{\partial s_2^2} + \frac{\partial^2}{\partial s_3^2}\right)\zeta(x,s), \Omega \subset R^3.$$

Now we reduce (16) to a sequence of linear integral equations by applying the method of successive approximations [20]

$$u_{m+1}(x) - \int_\Omega u_{m+1}(s)K(x,s)ds = \\ = \int_\Omega G_2(x,s)[\lambda u_m^q(s) + u_m^p(s)]ds, \quad (17)$$
$$m = 1,2,\ldots,$$

where: $u_1(x) = \delta\psi(x)$, $\delta = const > 0$.

Each of equations (17) can be solved by Bubnov-Galerkin method [21]. In that case we have the following sequence of the solution approximations

$$u_{m,k}(x) = \sum_{i=1}^{k} c_{m,i}\phi_i(x),$$

where: $\phi_i(x)$ is a coordinate sequence, $c_{m,i}$ ($i = \overline{1,k}$, $m = 2,3,\ldots$) is a solution of a system of linear algebraic equations:

$$\sum_{i=1}^{k} c_{2,i}\left[\int_\Omega \phi_i(x)\phi_j(x)dx - \right.$$
$$\left. - \int_{\Omega\Omega}\int K(x,s)\phi_i(s)\phi_j(x)dsdx\right] =$$
$$= \int_{\Omega\Omega}\int G_2(x,s)[\lambda u_1^q(s) + u_1^p(s)]\phi_j(x)dsdx,$$
$$j = \overline{1,k},$$
$$\sum_{i=1}^{k} c_{m,i}\left[\int_\Omega \phi_i(x)\phi_j(x)dx - \right. \quad (18)$$
$$\left. - \int_{\Omega\Omega}\int K(x,s)\phi_i(s)\phi_j(x)dsdx\right] =$$
$$= \int_{\Omega\Omega}\int G_2(x,s)[\lambda u_{m-1,k}^q(s) + \\ + u_{m-1,k}^p(s)]\phi_j(x)dsdx,$$
$$j = \overline{1,k}, \quad m = 3,4,\ldots.$$

COMPUTATIONAL EXPERIMENTS

As an illustration of process (14) we now look at the examples in two domains using both Green's functions and quasifunctions.

Example 1 (Unit Disk for Green's Function). Take

$$\Omega = \left\{x = (x_1, x_2) \mid 1 - x_1^2 - x_2^2 > 0\right\}. \quad (19)$$

The corresponding Green's function in the domain Ω is:

$$G(x,s) = \frac{1}{2\pi}\left(\ln\frac{1}{|x-s|} - \ln\frac{1}{\rho\,|x-s^1|}\right),$$

where: $s \in \Omega$ is a fixed point, s^1 is an 'image point' on the prolonged line segment from the disk center O to s such that $\rho\rho^1 = 1$, ρ is a distance from O to s, ρ^1 is a distance from O to s^1 (see Fig. 1).

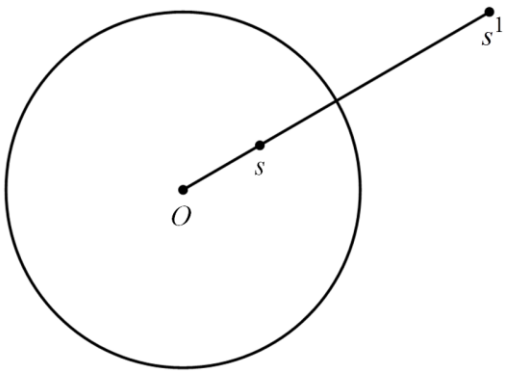


Fig. 1. Points for Green's function expression in Disk domain

The function $\omega(x)$ is:

$$\omega(x) = 1 - x_1^2 - x_2^2.$$

It means that:

$$\max_{x\in\Omega}\omega(x) = 1.$$

Now, by (10), so that:

$$1 < \frac{M\pi}{\varepsilon^{2(1-q)}}\left[\frac{\lambda^2}{2q+1} + \frac{2\lambda\varepsilon^{p-1}}{p+q+1} + \frac{\varepsilon^{2(p-q)}}{2p+1}\right]. \quad (20)$$

We have $M \approx 0.04$, $L \approx 0.25$.

Using (11), (13), and (20) we put $p = 2$, $q = 0.5$, $\lambda = 3$, $\varepsilon = 0.5$, $\beta = 1.5$.

The surface of the upper approximation of the solution w_{18} and its level lines are illustrated in Fig. 2 and Fig. 3 respectively.

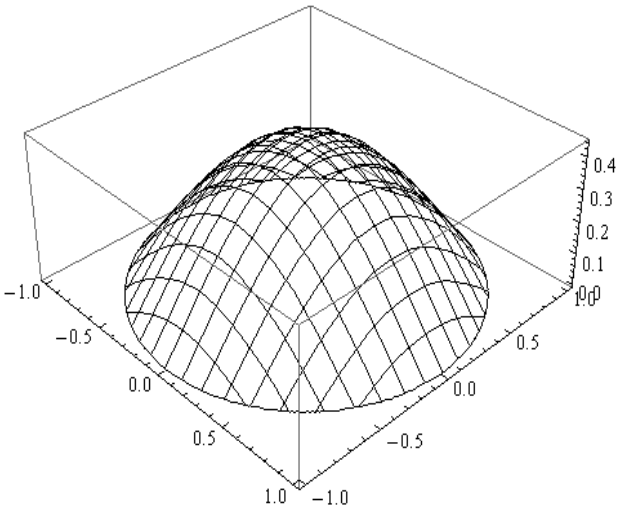


Fig. 2. The surface of w_{18}

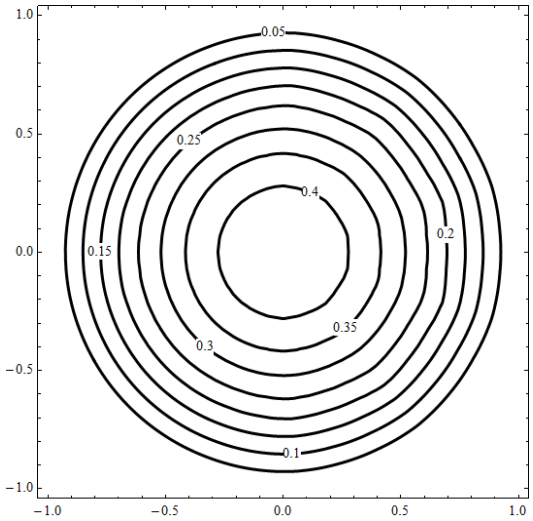


Fig. 3. The level lines of w_{18}

The values of the approximations v_{18} and w_{18} at the points of domain Ω in polar coordinates (ρ_i, ϕ_j), $\rho_i = 0.2i$, $\phi_j = 0.1j\pi$, $i = \overline{1,4}$, $j = \overline{1,5}$ are shown in Table 1.

Table 1. The values of v_{18} and w_{18}

ϕ		ρ			
		0.2	0.4	0.6	0.8
$\frac{\pi}{10}$	w_{18}	0.419061	0.357569	0.258870	0.133876
	v_{18}	0.418969	0.357491	0.258815	0.133848
$\frac{\pi}{5}$	w_{18}	0.419073	0.357322	0.258291	0.133475
	v_{18}	0.418981	0.357244	0.258235	0.133447
$\frac{3\pi}{10}$	w_{18}	0.419096	0.357075	0.257844	0.133076
	v_{18}	0.419004	0.356997	0.257788	0.133048
$\frac{2\pi}{5}$	w_{18}	0.419131	0.357245	0.258149	0.133283
	v_{18}	0.419039	0.357167	0.258094	0.133255
$\frac{\pi}{2}$	w_{18}	0.419188	0.358912	0.261326	0.135752
	v_{18}	0.419095	0.358834	0.261270	0.135723

Example 2 (Unit Disk for Green's Quasifunction). Let

$$\psi(x) = \frac{1}{2}(1 - x_1^2 - x_2^2) \qquad (21)$$

be the normalized boundary equation of the first order on $\partial\Omega$, where Ω is defined by (19). Indeed, conditions (15) are satisfied for (21).

In this case equation (16) will be reduced to (5) which means that Greens' quasifunction is equal to Green's function in Disk region.
Therefore, the results will be the same as in Example 1.

Example 3 (Unit Half Disk for Green's Function). Take

$$\Omega = \left\{x = (x_1, x_2) \mid 1 - x_1^2 - x_2^2 > 0, x_2 > 0\right\} \qquad (22)$$

The corresponding Green's function in the domain Ω is

$$G(x,s)=\frac{1}{2\pi}\left(\ln\frac{1}{|x-s|}-\ln\frac{1}{\rho\,|x-s^1|}-\right.$$
$$\left.-\ln\frac{1}{|x-s'|}+\ln\frac{1}{\rho\,|x-s^{1'}|}\right),$$

where: s, s^1, ρ are described in Example 1, s', $s^{1'}$ are 'image points' corresponding to s and s^1 respectively (see Fig. 4).

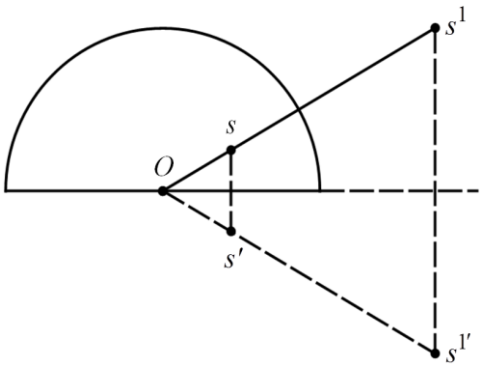


Fig. 4. Points for Green's function expression in Half Disk domain

The function $\omega(x)$ is $\omega(x)=x_2(1-x_1^2-x_2^2)$ meaning that $\max\limits_{x\in\Omega}\omega(x)=1$.

In this case we have $M\approx 0.015,\ L\approx 0.097.$

Using (11), (13), and (20) we put $p=2$, $q=0.5$, $\lambda=8$, $\varepsilon=0.5$, $\beta=1$.

The surface of the upper approximation of the solution w_{15} and its level lines are illustrated in Fig. 5 and Fig. 6 respectively.

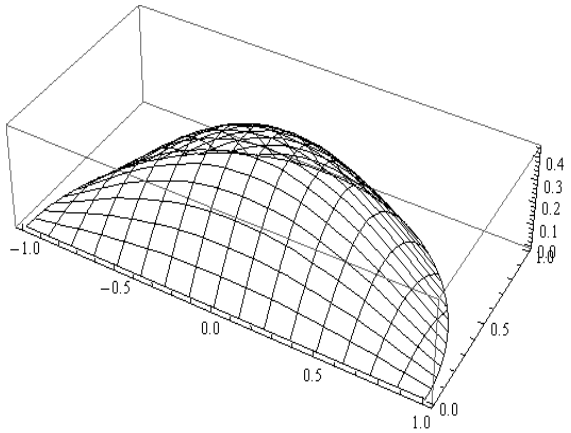


Fig. 5. The surface of w_{15}

The values of the approximations v_{15} and w_{15} at the points of domain Ω in polar coordinates (ρ_i,ϕ_j), $\rho_i=0.2i$, $\phi_j=0.1j\pi$, $i=\overline{1,4}$, $j=\overline{1,5}$ are shown in Table 2.

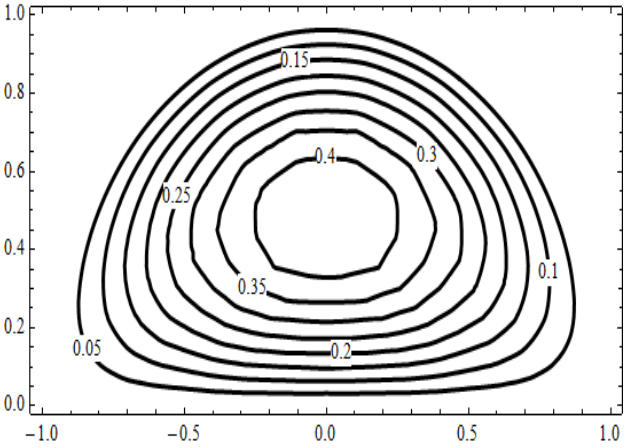


Fig. 6. The level lines of w_{15}

Example 4 (Unit Half Disk for Green's Quasifunction).

Let

$$\psi(x)=\frac{1}{2}(1-x_1^2-x_2^2)x_2 \qquad (23)$$

be the normalized boundary equation of the first order on $\partial\Omega$, where Ω is defined by (22). Indeed, conditions (15) are satisfied for (23).

Now we put $\delta=0.5$, so that

$$u_0(x)=\frac{1}{4}(1-x_1^2-x_2^2)x_2.$$

Then we select the following coordinate sequence

$$\phi_i(x)=\psi(x)P_{i_1}(x_1)P_{i_2}(2x_2-1),$$
$$i_1=\overline{0,2},\quad i_2=\overline{0,2-i_1},$$

where: $i=\overline{1,k}$, $k=6$, $P_m(z)$ are Legendre polynomials

$$P_m(z)=\frac{1}{2^m m!}\frac{d^m}{dz^m}\left[(z^2-1)^m\right]$$

Solving the system of equations (18), we get the approximation of the solution $u_{14,6}$. Its surface and level lines are illustrated in Fig. 7 and Fig. 8 respectively.

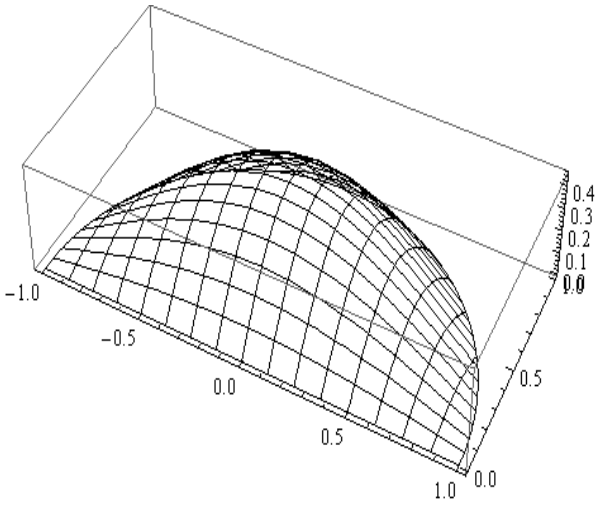


Fig. 7. The surface of $u_{14,6}$

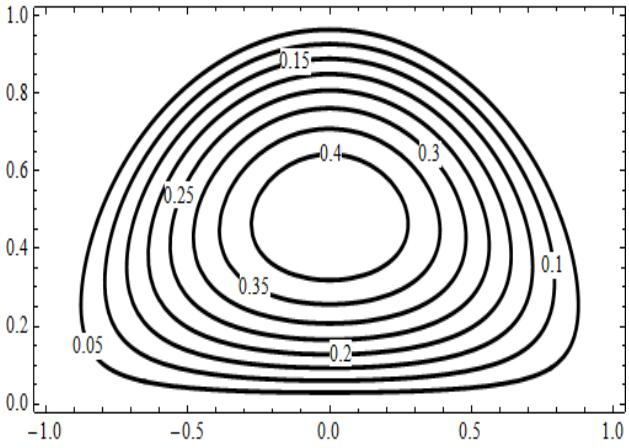


Fig. 8. The level lines of $u_{14,6}$

Table 2. The values of v_{15} and w_{15}

ϕ		ρ			
		0.2	0.4	0.6	0.8
$\frac{\pi}{10}$	w_{15}	0.094379	0.159946	0.167969	0.109479
	v_{15}	0.094362	0.159917	0.167939	0.109459
$\frac{\pi}{5}$	w_{15}	0.175253	0.284471	0.286695	0.179712
	v_{15}	0.175221	0.284419	0.286643	0.179680
$\frac{3\pi}{10}$	w_{15}	0.235852	0.371114	0.363978	0.223259
	v_{15}	0.235808	0.371045	0.363910	0.223218
$\frac{2\pi}{5}$	w_{15}	0.273067	0.421881	0.407777	0.247385
	v_{15}	0.273017	0.421802	0.407701	0.247339
$\frac{\pi}{2}$	w_{15}	0.285590	0.438515	0.421797	0.255014
	v_{15}	0.285537	0.438432	0.421718	0.254967

The values of the approximation $u_{14,6}$ at the points of domain Ω in polar coordinates (ρ_i, ϕ_j), $\rho_i = 0.2i$, $\phi_j = 0.1j\pi$, $i = \overline{1,4}$, $j = \overline{1,5}$ are shown in Table 3.

Table 3. The values of $u_{14,6}$

ϕ	ρ			
	0.2	0.4	0.6	0.8
$\frac{\pi}{10}$	0.097834	0.160302	0.167488	0.111392
$\frac{\pi}{5}$	0.180005	0.285107	0.287184	0.183040
$\frac{3\pi}{10}$	0.241369	0.372598	0.365651	0.226659
$\frac{2\pi}{5}$	0.279068	0.424053	0.410050	0.250792
$\frac{\pi}{2}$	0.291761	0.441007	0.424467	0.258674

CONCLUSIONS

We have built an iteration process that converges to a positive solution of (4) from both sides. Also, we have introduced a new approach for constructing conical intervals, as a left endpoint we $v_0(x) = \varepsilon\omega(x)$ instead of $v_0(x) = 0$, where: $\omega(x) > 0$ in Ω, $\omega(x)|_{\partial\Omega} = 0$, and $\varepsilon = const$.

This approach can be used when the lower approximations don't move from the starting position.

We've obtained a condition that links parameters λ, q, p, ε, β and guarantees existence and uniqueness of a positive solution.

By building the cone segment $\langle v_0, w_0 \rangle$ we provide an a priori estimate of the solution, since $v_0 \le u \le w_0$. The actual two-sided approximations allow us to make a posteriori conclusions.

The algorithm implementation simplicity and relatively small computational resources are the main advantages of the provided method.

Green's quasifunction method has been investigated to compare the results. The functions $\omega(x)$ and $\psi(x)$ can be constructed using R-functions theory [14] in case of domains with complex boundary.

The experimental results in unit disk and unit half disk have shown the efficiency of the provided method. It can be used to solve boundary value problems for stationary heat conduction equations of the form (4) or other problems that are reduced to (4).

REFERENCES

1. Bellman R., Kalaba R., 1968. Quasilinearization and Nonlinear Boundary Value Problems. Moscow:Mir, 186. (In Russian).

2. Frank-Kamenetskii D., 2008. Basics of Macrokinetics. Diffusion and Heat Transfer in Chemical Kinetics. Dolgoprudny:Intellekt Press, 408. (In Russian).

3. Dong Ye, Feng Zhou., 2001. A generalized two dimensional Emden-Fowler equation with exponential nonlinearity. Calculus of Variations and Partial Differential Equations. Volume 13. Issue 2, 141-158.

4. Bozhkov Y., 2005. Noether Symmetries and Critical Exponents. Symmetry, Integrability and Geometry: Methods and Applications. Volume 1. Paper 022, 1-12.

5. Rvachev V., Slesarenko A., Safonov N., 1992. Mathematical modelling of thermal self-ignition under stationary conditions by the R-functions method. Doklady Akademii Nauk Ukrainskoi SSR. Seriya A. n. 12, 24-27. (In Russian).

6. Galakhov E., 2005. Positive Solutions of Quasilinear Elliptic Equations. Mat. Zametki. Volume 78. Issue 2, 202-211. (in Russian).

7. Matinfar M., Nemati K., 2008. A numerical extension on a convex nonlinear elliptic problem. International Mathematical Forum. Volume 3. No. 17, 811-816.

8. Ambrosetti A., Brezis H., Cerami G., 1994. Combined effects of concave and convex nonlinearities in some elliptic problems. Journal of Functional Analysis. Volume 122. Issue 2, 519-543.

9. Baraket S. and Dong Ye, 2001. Singular limit solutions for two-dimensional elliptic problems with exponentially dominated nonlinearity. Chinese Annals of Mathematics. Series B. Volume 22. Issue 03, 287-296.

10. Junping Shi, Miaoxin Yao, 2005. Positive solutions for elliptic equations with singular nonlinearity. Electronic Journal of Differential Equations. Volume 04, 1-11.

11. Kolosova S., Lukhanin V., Sidorov M., 2013. About construction iterative methods of boundary value problems for nonlinear elliptic equations. Visnyk of Zaporizhzhya National University. Physical and mathematical Sciences. № 1, 35-42. (In Russian).

12. Kolosova S., Lukhanin V., Sidorov M., 2015. On the construction of two-sided approximations to the positive solution of the Lane-Emden equation. Visnyk of Zaporizhzhya National University. Physical and mathematical Sciences. № 3, 107-120. (In Russian).

13. Lukhanin V., 2015. On the construction of two-sided approximations to the positive solution of the elliptic boundary value problem with exponential dominant. Radioelektronika i informatika. № 2 (69), 16-18. (In Ukrainian).

14. Rvachev V., 1982. Theory of R-functions and Some Applications. Kiev: Naukova Dumka, 552. (In Russian).

15. Artyukh A., Sidorov M., 2014. Mathematical Modeling and numerical analysis of nonstationary plane-parallel flows of viscous incompressible fluid by R-functions and Galerkin Method. Econtechmod. An International Quarterly Journal. Vol. 3. № 3, 3-11.

16. Lamtyugova S., Sidorov M., 2014. Numerical analysis of the problem of flow past a cylindrical body applying the R-functions method and the Galerkin method. Econtechmod. An International Quarterly Journal. Vol. 3. № 3, 43-50.

17. Krasnosel'skii M., 1962. Positive Solutions of Operator Equations. Moscow:Fizmatgiz, 394. (In Russian).

18. Opojtsev V., 1978. A generalization of the theory of monotone and concave operators. Trans. Mosc. Math. Soc. Volume 36, 237-273. (In Russian).

19. Opojtsev V., Khurodze T., 1984. Nonlinear operators in spaces with a cone. Tbilisi: Tbilis. Gos. Univ., 269. (In Russian).

20. Svirsky I., 1968. Methods of the Bubnov-Galerkin Type and a Sequence of Approximation. Moscow: Nauka, 287. (In Russian).

21. Mikhlin S., 1970. Variational methods in mathematical physics. Moscow: Nauka, 512. (In Russian).

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 21-27

RESEARCH THE POSSIBILITIES OF DIFFERENT FILTERS AND THEIR APPLICATION TO IMAGE RECOGNITION PROBLEMS

V. Brytik, O. Grebinnik, V. Kobziev
Kharkiv National University of Radio Electronics; e-mail: vladimir.brytik@mail.ru

Received September 09.2016: accepted October 12.2016

Abstract. This article is devoted to the analysis from the viewpoint of accuracy allocation of contour points, and maintaining information about the distribution of non-derivative elements. In this article is researched the edge enhancement algorithms such as Prewitt, Sobel, Laplacian, Kirsch and evaluated their work, using some well-known performance measures. Also compared the efficacy of the researched algorithms, which was determined on the selected images with superimposed noise.

Key words: image analysis, contour points, filter, convolution mask

INTRODUCTION

The emergence of a huge number of tools shooting photos and video in the last decade strengthens the role of advanced vision systems and image recognition for processing images of various objects and paintings of the real world. Available collections of photos and video have significantly different technical parameters tools of shooting, storage formats and quality characteristics of images themselves. A variety of applied tasks analysis of static and dynamic images places high demands on the accuracy and speed of processing operations and recognition of visual information. To meet these requirements are need an informed choice of existing image processing methods based on their comprehensive comparison, as well as their modification and development of new methods.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

One of the central problems in pattern recognition is the selection objects on image with a complex background and further interpretation properties and characteristics of these objects [1-5].

Recognition objects on images are one of the most important tasks. Its successful solution depends from many characteristics various objects and methods for their processing [6-10]. Most often we use the characteristic of contour information, which limiting recognizable objects. However, due to the large variability of the contour information, recognition methods don't provide the necessary recognition results. So today on the forefront are coming the structural - linguistic methods, which besides the singular points (contour information) are used as characteristics of the distribution of non-derivative elements (filter response). In the simplest case, as non-derivative element we can use the brightness value and its distribution within the recognizable object can be it's the characteristic.

Based on the foregoing, it becomes obvious that when we solving problem of recognition is necessary to use such filters which after the allocation contour information will not lost information about the distribution of non-derivative elements.

In this paper, we analysis filters in terms how exactly release special points, and how store information about the distribution of non-derivative elements.

A sharp change in brightness occurs for several reasons. First, such abrupt changes often occur at the boundaries of objects - it can be an image of the light object on a dark background or a dark object on a light background. Second, the abrupt changes in brightness are often the result of changes in reflectance on fairly typical structures. For sharp changes images brightness are also include abrupt changes of surface orientation.

Image points in which the brightness changes is particularly strong, often called edges, or edge (contour) points. Contour points in combination with other elements have been used successfully for image analysis and classification in the broad range of applications. Numerous applications and subjective approach to the definition of contour points led to the creation large number operators of their evaluation [11-14], the effectiveness of which depends essentially from the conditions of images formation and objects on the images.

OBJECTIVES

The aim of this work is the comparative study of the quality of set filter (allocation algorithms) contours of images of objects standard geometric forms in the absence and presence of noise with different parameters. Filter evaluation should be performed for each type of object using the known criteria of the quality of work. Furthermore, it is necessary take into consideration the connectivity contour points after performing the filtering of images of objects of each type.

MAIN RESULTS OF THE RESEARCH

To highlight the features of the raster images they are processed by sequential subdivision into groups associated neighboring pixels of two types: 1) a group of pixels arranged successively horizontally (or vertically) - univariate processing methods, 2) a group of pixels arranged successively horizontally in several vertical rows, - two-dimensional processing methods. Typically, the number of pixels in such groups is an odd number (the most commonly used 3 or 5), pixel, which processed at each step, is located at the center of the group.

In a previous work [15], the authors conducted a study of the work of one-dimensional filters for ideal, fuzzy and noise contour models. In this paper, we research a variety of edge enhancement algorithms, and assess their work, using well-known performance measures. As quantitative criteria typically used the following measures:

- percentage of contour points detected on the perfect image contour;
- number of detected contour points that do not coincide with the ideal;
- the ratio of the detected contour points that do not coincide with the ideal circuit to the number of detected points that coincide with the ideal;
- the average width of the detected contour - defined as the ratio of the total number of detected contour points to the number of ideal points of the contour;
- normalized deviation of actual contour points from the ideal points of the contour.

Nowadays we know a large number of papers about the effectiveness of filters for solving allocation contour information. Therefore, we have a question how such information is actually solves the problem of the recognition, in the case when an affine transformation for this object vary insignificantly, and relationship of contour components allow us to identify the object. Such cases are rare in practice. For example, changing the viewing angle is relative to the object which being analyzed, or a change in the lighting conditions which significantly change the description about the contour of objects, so the problem can be considered legitimate about what properties should filter has for the task.

At first, let's research the one-dimensional linear filters. Common form of these filters may be presented as characteristic of brightness $S_1(x,y)$ in pixel with coordinates (x,y) after filtering by next formulas:

$$S_1(x,y) = \sum_{i=-k}^{k} a(x,y) * g(x+i,y), \qquad (1)$$

where: $a(x,y)$ - value of brightness in pixel with coordinates (x,y) before filtering, $g(x+i,y)$ - weight of value in concrete pixel (set of these weights are determine concrete filter), $n = 2*k+1$ – quantity of picture's points, which are considering in concrete filter.

Notation (1) describes the discrete convolution of brightness values $a(x,y)$ with weights $g(x+i,y)$. A set of weights for the adjacent pixel values is called the convolution mask. Pixel is a member of an edge if the intensity of it is greater than that of the members of its surrounding pixels.

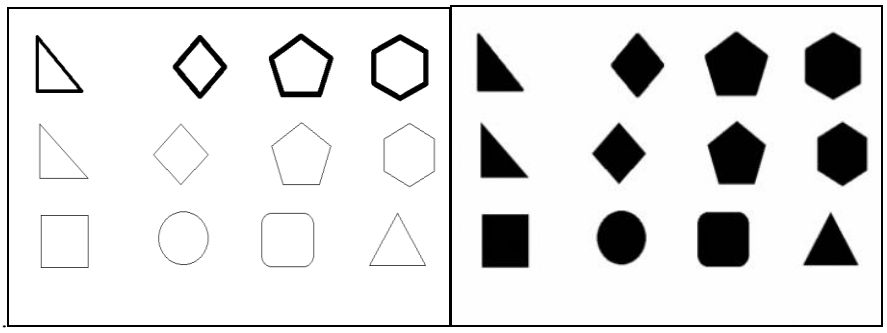
For example consider well-known masks (see Table 1).

Table 1. One-dimensional convolution mask

№	The name of the mask	Principal value of mask n	Values of convolution mask elements
1	Tone mask	3	1 2 1
2	Border mask	3	-1 0 1
3	Spot mask	3	-1 2 -1
4	Wave mask	5	-1 2 0 -2 1
5	Rippling mask	5	1 –4 6 -4 1
6	Vibration mask	7	-1 6 -15 20 -15 6 -1

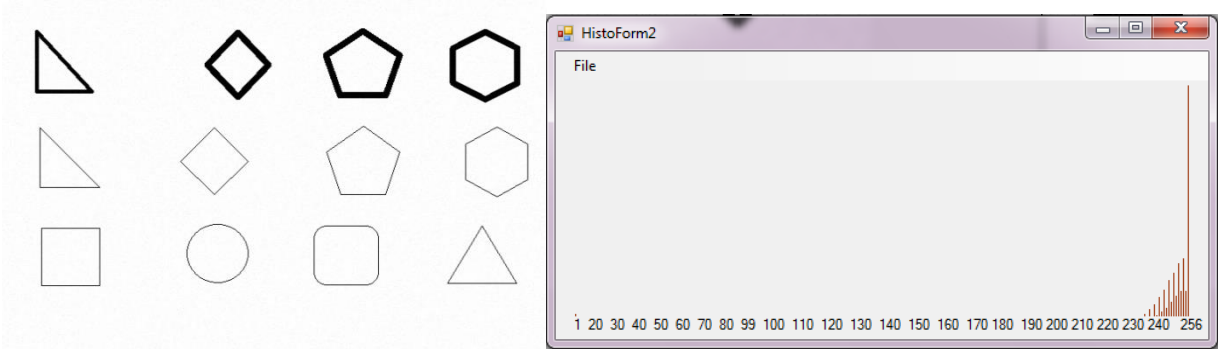
The research of these filters is shown that they can enhance the outline, but at the same time do not lose information about the distribution of brightness values within the object. For example, we can see it on a noisy picture with one from two standard deviation values: $\sigma_1 = 5$, $\sigma_2 = 10$.

After spending experiments and plotting the histograms, we can conclude that after the imposition noise $\sigma = 5$ and $\sigma = 10$ in Fig. 2-5 the difference is not noticeable, but the histogram allows you to track changes on the image.



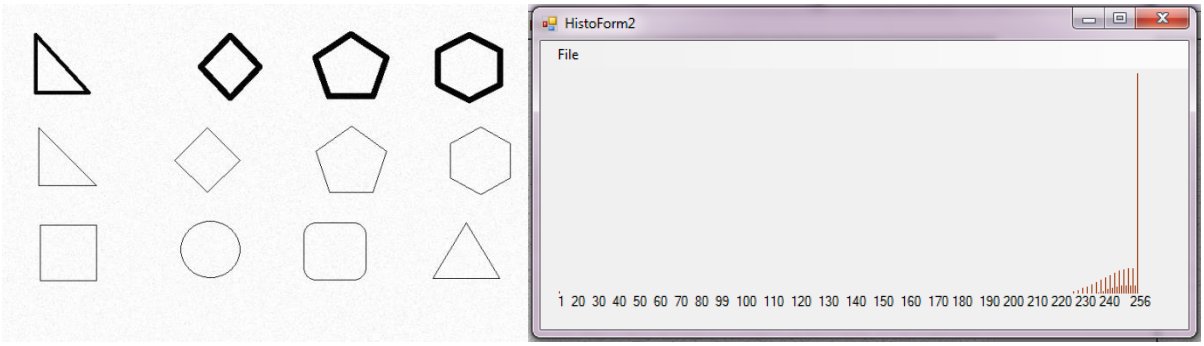
a) b)

Fig. 1. a) The template contour image, b) The template silhouette picture



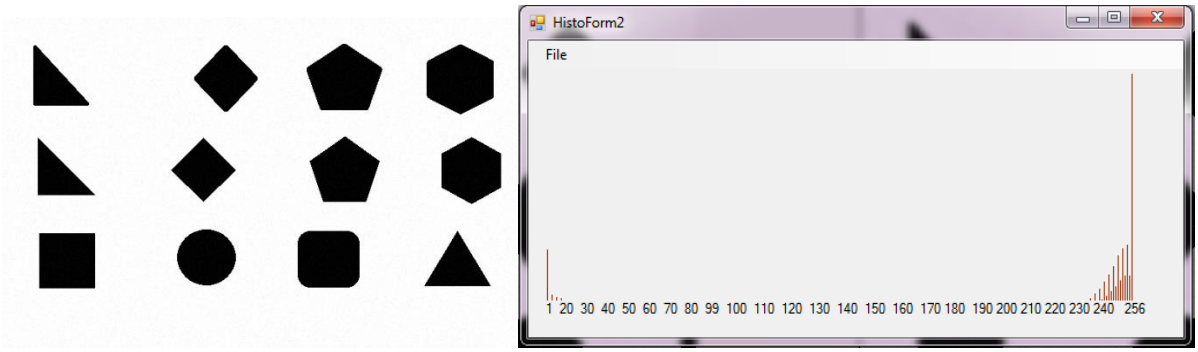
а) б)

Fig.2. Contour image with a gauss noise σ = 5. a) The template; b) A histogram of the template



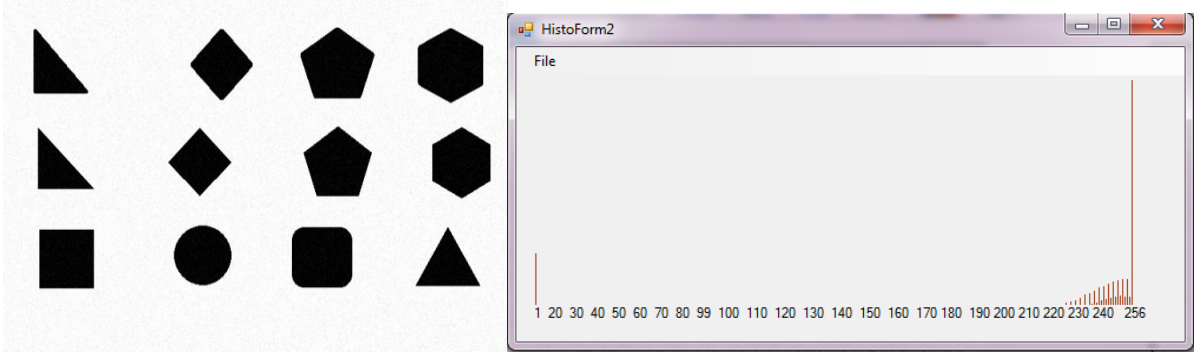
а) б)

Fig.3. Contour image with a gauss noise σ = 10. a) The template; b) A histogram of the template



а) б)

Fig.4. Silhouette image with a gauss noise σ = 5. a) The template; b) A histogram of the template



а) б)

Fig.5. Silhouette image with a gauss noise σ = 10. a) The template; b) A histogram of the template

The experimental results [15] suggest that the one-dimensional filters silhouette images do not allow to allocate contour of silhouette images, but let you receive the distribution of values within an image, and clearly fix Gaussian noise, which is good for the frequency and statistical processing methods.

Two-dimensional filters use different numerical methods to extract contours of objects using the next general formula:

$$S_{loc} = \sum_{i=-k}^{i=k} \sum_{j=-k}^{j=k} a(x, y) \cdot g_k(x+i, y+j), \quad (2)$$

where: x, y – the coordinates of the image plane; k – the dimension of the filter, as a rule $k = 3$ or $k = 5$.

Notation (2) describes the discrete convolution of brightness values $a(x, y)$ in pixel with coordinates (x, y) with weights $g(x+i, y+j)$, $i = \overline{1,k}$, $j = \overline{1,k}$.

There are several known filters, which are combined by the given general formula - filters Sobel, Prewitt, Kirsch, Laplace. For evaluation of the weights in the first three of them are used numerical first-order methods for calculating the gradient of the brightness change within the pixels covering the mask, in the last filter are used second-order method. At each point of the image gradient vector oriented in the direction of the greatest increase in brightness, and its length corresponds to the brightness change.

The components of the gradient vector are derivatives of image brightness on the horizontal and vertical directions:

$$grad(a(x, y)) = (da / dx, da / dy).$$

Sobel filter uses a 3x3 mask to calculate approximate values of derivatives with respect to horizontal and vertical.

Prewitt filter acts like a filter Sobel and considers separately two convolution kernels, but uses other values of the coefficients. The convolution is calculated for each mask separately. Response of the filter in each pixel is equal to the maximum of these two value convolutions.

Kirsch filter is a non-linear edge detector that takes a single kernel mask and rotates it in 45 degree increments through all 8 compass directions. The edge magnitude of the Kirsch operator is calculated as the maximum magnitude across all 8 directions.

Laplace filter uses as a mask Laplacian (the sum of the second order derivatives) in each pixel of the image. There are several mask of this filter.

Results of research with using a two-dimensional filters, such as Sobel, Kirsch, Prewitt, Laplace are presented in Tables 2 and 3. The values of error in these tables specifies the number of the detected contour points which do not coincide with the ideal image.

Table 2. Comparison of edge detection algorithms for contour images with superimposed noise

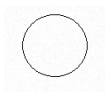
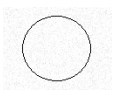
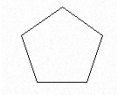
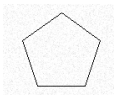
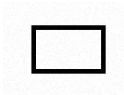
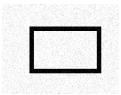
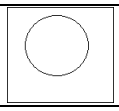
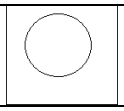
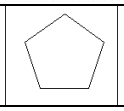
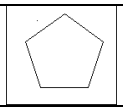
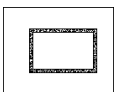
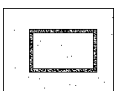
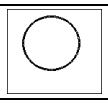
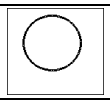
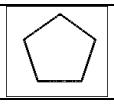
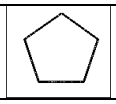
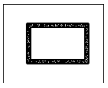
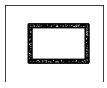
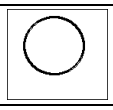
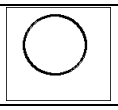
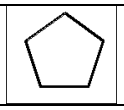
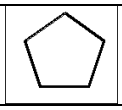
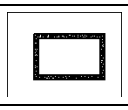
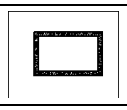
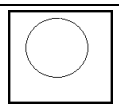
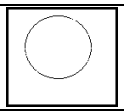
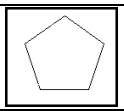
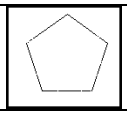
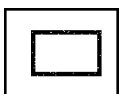
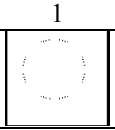
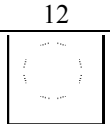
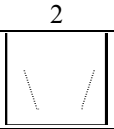
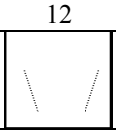
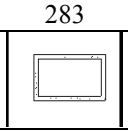
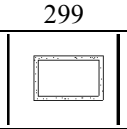
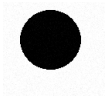
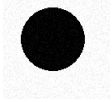
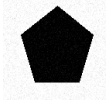
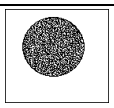
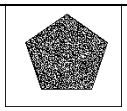
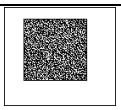
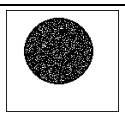
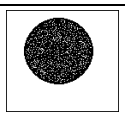
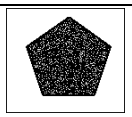
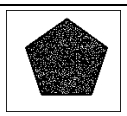
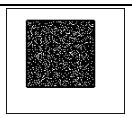
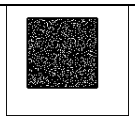
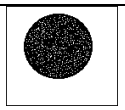
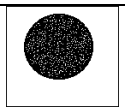
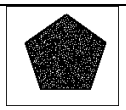
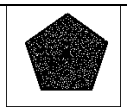
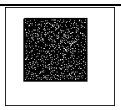
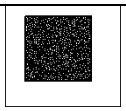
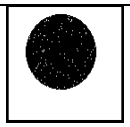
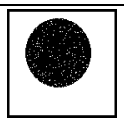
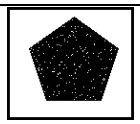
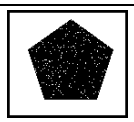
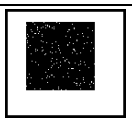
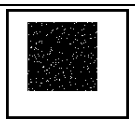
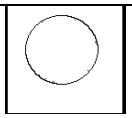
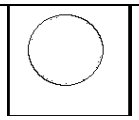
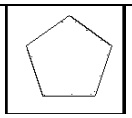
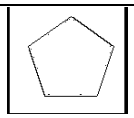
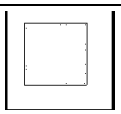
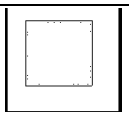
	Noise with a Gaussian distribution with standard deviation σ					
Filter	σ =5	σ =10	σ =5	σ =10	σ =5	σ =10
Original grayscale image						
Prewitt						
error	0	0	0	1	441	435
Sobel						
error	79	88	105	99	614	608
Kirsch						
error	0	0	0	0	647	626
Laplace						
error	1	12	2	12	283	299
Logic						
error	0	0	1001	1002	17	39

Table 3. Comparison of edge detection algorithms for silhouette images with superimposed noise

	Noise with a Gaussian distribution with standard deviation σ					
Filter	σ =5	σ =10	σ =5	σ =10	σ =5	σ =10
Original grayscale image						
Prewitt						
error	2685	2719	3317	3359	3240	3238
Sobel						
error	3609	3621	4471	4520	3382	4403
Kirsch						
error	3810	3817	4708	4734	4589	4629
Laplace						
error	4110	4034	5049	5014	4849	4806
Logic						
error	9	14	49	51	12	18

We can concluded that not all the algorithms are sufficiently effective when dealing with a silhouette image, and their effectiveness depends from the subject. The research has shown that with selected threshold locally adaptive algorithms for contour images such as Prewitt, Sobel, Laplace, Kirsch, Logical work well and allocate contour, but it does not consider the internal distribution. With increasing noise on the contour images the error increases too.

Table 2 shows, that Logical filter worse than all the other filters processes the contour images of circle and pentagon with noise. Image of circles after filtering has only a few unrelated points. Images of pentagon have not lines with small angles of inclination. For rectangles with wide lines, this filter detects two edges between the light and dark areas. Filtering of silhouette images this figures (see Table 3) shows another results.

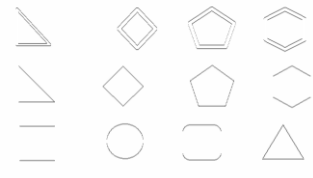
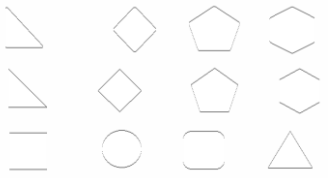
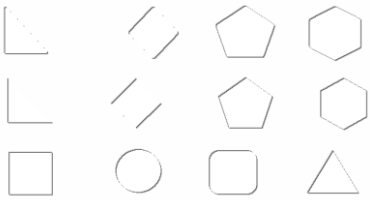
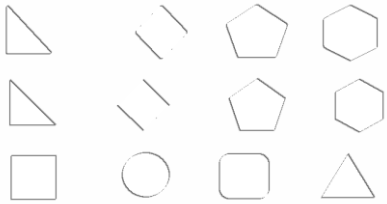
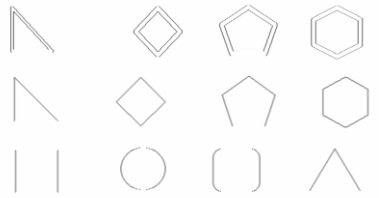
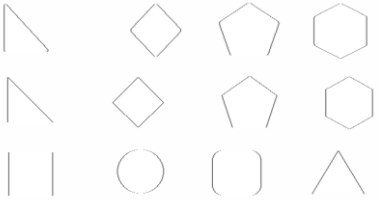
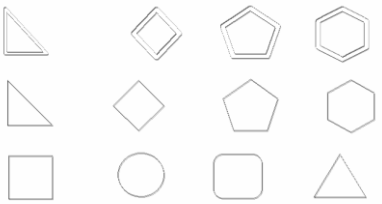
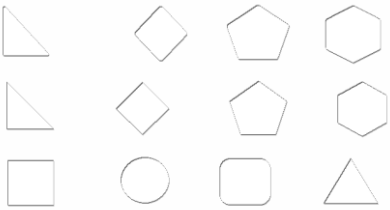
Results of research which using a two-dimensional filters has shown that they are well separated contour information, when properly choose threshold value, but using the threshold leads to losing information on the distribution of pixels brightness that constituting the object. The accuracy of the allocation contour information is largely determined by the accuracy of the selected threshold.

The experience of many studies show that for the edge detection of image elements, which have a complex geometrical shape, it is necessary to take into account the possible noise and use a combination of several filters. It is important to remember that different filters based on an approximation of the first derivative, have different sensitivity to noise. Significantly stronger in the presence of noise react the results of the filters that use the second derivative. In general, the higher the derivative, the more sensitive the operator. It is also useful to consider the results of applying the same filter with masks having larger number of pixels (e.g., 5*5 or 7*7).

This comparing the methods of image recognition and classification leads to the conclusion that for the method of structural recognition is important the accuracy of the allocation boundaries and information within these boundaries. For statistical methods is important to know the internal contours and mutual distribution of brightness values. For feature extraction methods (filters) should allocate contour components, which is typical for some existing methods, and obtain information about the distribution of structural elements outside the boundaries of objects. Losing any of these components, in principle, is not acceptable for decision a wide range of recognition problems.

The accuracy of allocated contour greatly depends by noise components. Even simple correlation methods give a big mistake. Losing the information about distribution inside the object, and its distortion after applying noise should be also used in pattern recognition.

Table 4. Application of two-dimensional filters to contour and silhouette images

Filter mask	The contour inverted image	The silhouette inverted image
$\begin{matrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{matrix}$		
$\begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix}$		
$\begin{matrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{matrix}$		
$\begin{matrix} 0 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{matrix}$		
$\begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{matrix}$		

This table shows that each of the above filters has drawbacks. The first filter is almost does not emit vertical borders; second - lines going from up to down as you move from left to right; the third - the lines going from the bottom-up as you move from left to right; fourth - horizontal border. In each case, these lines are perpendicular to the direction of recording of negative coefficients in the corresponding mask filter.

Last filter represents some combination of the first two. His results detects all directs of boundaries of all geometric shapes that has been processing. Disadvantages of the first two filters are shown much weaker - some of the boundaries are visible with less clarity.

Thus, we come to understand that we need a set of filters that would not lose the information about directions left-right, up-down, etc. We need to find a set of filters to identify each figure. The existence of these figures suggests that they can also be on the complex image, whereas the filter response, we will record the number of this filter. All of this allows us to build a diagram, which will display the number of filters and their sequence for detecting complex form of real image edges.

CONCLUSIONS

This comparing the methods of image recognition and classification leads to the conclusion that for the method of structural recognition is important the accuracy of the allocation boundaries and information within these boundaries. For statistical methods is important to know the internal contours and mutual distribution of brightness values. For feature extraction methods (filters) should allocate contour components, which is typical for some existing methods, and obtain information about the distribution of structural elements outside the boundaries of objects. Losing any of these components, in principle, is not acceptable for decision a wide range of recognition problems.

The accuracy of allocated contour greatly depends by noise components. Even simple correlation methods give a big mistake. Losing the information about distribution inside the object, and its distortion after applying noise should be also used in pattern recognition. Thus, it is necessary to develop rules for creating a set of filters, which can adjust the errors identified above depending on characteristics of the objects at the images.

REFERENCES

1. **Duda R., Hart P., 1973.** Pattern Classification and Scene Analysis. New York: Wiley.
2. **Tou T., Gonzalez R., 1974.** Pattern Recognition Principles. Addison-Wesley, Reading, Mass. 97-104.
3. **Gonzales R., Woods R. 2007**. Digital Image Processing. Prentice Hall, 976.
4. **Shapiro L., Stockman G., 2006.** Computer Vision. Moscow, BYNOM. 752.
5. **Putyatin E., Matat E., 2003.** Information systems technology. Image processing and pattern recognition. Kharkiv National University of Radio Electronics. Kharkiv. 105.
6. **Malik J., Belongie S., Leung T., Shi J., 2001.** Contour and Texture Analysis for Image Segmentation // International Journal of Computer Vision, 43(1), 7–27.
7. **Liu X., Wang D., 2006.** Image and Texture Segmentation Using Local Spectral Histograms // IEEE Transaction on image processing, Vol. 15, No. 10, 3066-3077.
8. **Semenets V., Natalukha Yu., Taranukha O., Tokarev V. 2014.** About One Method of Mathematical Modelling of Human Vision Functions. ECONTECHMOD. An international quarterly journal Vol. 3, №3, 51-59.
9. **Brytik V.I., Zhilina O.Yu., Kobziev V.G. 2014.** Structural Method of Describing The Texture Images. ECONTECHMOD. An international quarterly journal Vol. 3, №3, 89-98.
10. **Anokhin M., Koryttsev I., 2015.** Decision-making rule efficiency estimation with applying similarity metrics. ECONTECHMOD. An international quarterly journal. Vol. 4, №3, 73-78.
11. **Vitcus R., Yaroslavskiy L., 1988.** The adaptive linear filters for image processing. Moscow, Nauka.
12. **Forsyth D., 2004.** Computer vision: a modern approach – Moscow, Publishing House Williams. 928.
13. **Kinoshenko D., Mashtalir V., Shlyakhov V., Yegorova E., 2010.** Nested Partitions Properties for Spatial Content Image Retrieval // International Journal of Digital Library Systems. – Vol. 1, No 3. - 58-89.
14. **Venmathi A., Ganesh E., Kumaratharan N., 2016.** Kirsch Compass Kernel Edge Detection Algorithm for Micro Calcification Clusters in Mammograms. Middle-East Journal of Scientific Research 24 (4). - 1530-1535.
15. **Brytik V., Zhilina E., 2014.** Investigation possibilities of various filters which used in pattern recognition problems // Bionica Intellecta. 2(83), 88-95.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 29-36

COMBINATORIAL APPROACHES TO THE CAPITAL-BUDGETING PROBLEM

O. Pichugina

Kharkiv National University of Radio Electronics; e-mail: pichugina_os@mail.ru

Received September 22.2016: accepted October 20.2016

Abstract. Optimization approaches, combinatorial and continuous, to a capital-budgeting problem (CBP) are presented. This NP-hard problem, traditionally modelled as a linear binary problem, is represented as a biquadratic over an intersection of a sphere and a supersphere. This allows applying nonlinear optimization to it. Also, the method of combinatorial and surface cuttings (MCSC) is adopted to (CBP). For the single constrained version (1CBP), new combinatorial models are introduced based on joint analysis of the constraint, objective function, and feasible region. Equivalence of (1CBP) to the multi-choice knapsack problem (MCKP) is shown. Peculiarities of Branch&Bound techniques to (1CBP) are described.

Key words: capital-budgeting problem, integer programming, knapsack problem, combinatorial optimization, Branch and Bound.

INTRODUCTION

Nowadays, attraction of investment funds are relevant more than ever [1]. However, even more important is their rational management [2]. Capital-budgeting modelling - is a universal tool that allows applying optimization techniques to the current management of (possibly) thousands of capital projects that yields the greatest return on investment and satisfies specified financial, regulatory and project relationship requirements [3, 4], as well as to carry out a rational long-term planning [5]. In general, vision of the potential cash flows is necessary in a direct management, the same as at the stage of developing business plans.

Consider the following capital-budgeting problem (CBP) [5]: select potential investments out of the set $\mathbf{X}=\{X_i\}_{i=\overline{1,n}}$ maximizing total contribution from all investments without exceeding the limited availability of resources $\mathbf{R}=\{R_j\}_{j=\overline{1,m}}$ if partial investments are not permitted and are given: a) limits b_j, $j=\overline{1,m}$ on the resources, b) contribution c_i resulting from the investment X_i, $i=\overline{1,n}$, c) the amount a_{ji} of resource R_j required for the investment X_i, $i=\overline{1,n}$, $j=\overline{1,m}$.

The limited resources might be cash, manpower, time, etc., the investment decisions – a choice among possible plant locations, selecting a configuration of capital equipment, picking a set of research&development projects, and so on.

Another scenario for (CBP) [5] is a long-range planning. In this case:

a) m is the number of periods of planning;

b) $\mathbf{R}$ - are the periods;

c) a_{ji} is the net cash flow from the investment X_i in the period R_j, $i=\overline{1,n}$, $j=\overline{1,m}$;

d) b_j represents the incremental exogenous cash flow in the period R_j, $j=\overline{1,m}$.

All the parameters a_{ji}, c_i, b_j can be arbitrary integers. For instance, in the long-range planning (CBP)-version, $a_{ij}>0$ if the investment X_i requires additional cash in the period R_j, $a_{ij}<0$ if the investment X_i generates cash in the period R_j, while $a_{ij}=0$ if it neither requires nor generates cash. Also, $b_j>0$ if additional funds are made available in the period R_j, $b_j<0$ if funds are withdrawn in this period, otherwise, $b_j=0$. Finally, if $c_i>0$ the investment X_i is beneficial, if $c_i<0$ then it is harmful, otherwise, X_i is neutral.

If a plan of the investments denote:

$$x=(x_i)_{i\in J_n}:\ J_n=\{1,...,n\},$$

$$x_i=\begin{cases}1 \text{ if the investment } \mathrm{R}_i \text{ accepted},\\ 0 \text{ if the investment } \mathrm{R}_i \text{ is rejected},\end{cases} \quad (1)$$

then the problem is formalized as follows: find a boolean vector (1) maximizing $z=c^T x$ subject to constraints that the funds required for investment are enough for the whole planning horizon.

Let x^* is an optimal plan, then the mathematical model of (CBP) is [3, 5]:

$$z^*=max\ c^T x,\ x^*=\arg max\ c^T x, \quad (2)$$

$$x\in B_n=\{0,1\}^n, \quad (3)$$

$$a_j^T x\le b_j,\ j\in J_m, \quad (4)$$

where:

$$a_j, c\in \mathrm{R}^n,\ b_j\in \mathrm{R},\ j\in J_m. \quad (5)$$

If $m>1$ it is a multiply constrained (CBP), (mCBP), if $m=1$ - is single constrained (1CBP) which looks like (2), (3), subject to

$$a^T x \le b, \tag{6}$$

$$a,c \in \mathrm{R}^n,\ b \in \mathrm{R}. \tag{7}$$

Formulas (2)-(5) is a particular case of integer programs, namely, it is a linear constrained binary program. Exactly it is solvable with help of branch&bound (B&B), cutting plane methods, or a combination of both - branch&cut techniqies. Also, it can be solved approximately by heuristics such as tabu search, hill climbing, simulated annealing, evolutionary and genetic algorithms, as well as asymptotically by asymptotic integer algorithms [4-9].

Typically, in (CBP) there are present two types of investments: beneficial, that required resources, and harmful, that generates cash. In this case, (1CBP) is reducible to a knapsack problem, (KP), (0-1KP) [9-12]: n objects with positive values (profits, utilities) c_i and weights a_i ($i \in J_n$) are given and a knapsack of a capacity b is formed from them with maximal total value (profit, utility).

It's mathematical model is (2), (3), (6)

$$a,c \in \mathrm{R}^n_{++},\ b \in \mathrm{R}_{++}.$$

Similarly, (mCBP) becomes the multiple constrained (KP) (mKP) [10,11] if (4) are knapsack constraints [13], i.e., there is holds:

$$a_j, c \in \mathrm{R}^n_{++},\ b_j \in \mathrm{R}_{++},\ j \in J_m. \tag{8}$$

Detecting (KP)-type problems among (CBP) allows applying various solution approaches specific to (KP) exactly and approximately. Among exact approaches are dynamic programming (DP), (B&B), and hybridizations of both; the integer hull search with cutting planes and tightening constraints. Among approximate are heuristics, reduction and asymptotic methods, e.g., greedy and fully polynomial time approximation schemes [9-13].

OBJECTIVES

The purpose of the paper is to present new approaches to (CBP) based on analysis of properties of nonlinear functions, as well as peculiarities of all components of the problem – the feasible discrete set, constraints, and objective function.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

In recent years, heuristic evolutionary and genetic algorithms have been intensively developed in integer programming, in particular, for (KP) [14].

Recent investigations concerned, primarily, methods specific to various KP generalizations such as the multidimensional and multi-objective (KP) [15], generalized assignment and quadratic (KP) [16].

A great success was achieved in approximate (KP)-solving. As reported in [14], instances of dimensions up to 100000 are solvable by DP, greedy and genetic algorithms, of which the first is exact and last shows better results than greedy. Note that execution time of DP is, on average, 10 times more than of the approximate ones. At the same time, B&B handle problems with at most 60000 variables.

From our point of view, a promising way to solve exactly large-size (CBP) is in constructing its new Euclidean combinatorial models [17, 18] on B_n-subsets, investigating properties of the subsets, then applying them in optimization. The optimization approaches can be combinatorial, such as branch&bound and branch&cut techniques [19, 20], as well as continuous based on functional representations of these sets [19, 20] and their inscription into a hypersphere. Among the continuous approaches are cutting plane techniques [21] and equivalent unconstrained reformulations based on extensions of objective functions [19, 20].

THE MAIN RESULTS OF THE RESEARCH

Introduce some terminologies.

A numerical 1-multiset (or a multiset) [17] is a collection of numbers:

$$G = \{g_i\}_{i \in J_n} :\ g_i \in R,\ i \in J_n. \tag{9}$$

Without loss of generality, we can assume that its elements are ordered:

$$g_i \le g_{i+1},\ i \in J_{n-1}. \tag{10}$$

A multiset is defined by a set $S(G)$ of its different elements, a basis, and multiplicities, a G-primary specification $[G]$ [18]:

$$S(G) = \{e_i\}_{i \in J_k} :\ e_i < e_{i+1},\ i \in J_{k-1}; \tag{11}$$

$$[G] = (n_i)_{i \in J_k} : n_i - \text{is a multiplicity of } e_i. \tag{12}$$

Now, G is representable as follows [18]:

$$G = \left\{e_j^{n_j}\right\}_{j \in J_k} : \sum_{j=1}^{k} n_j = n. \tag{13}$$

A 2-multiset is a collection of 2-tuples:

$$G = \{g_i\}_{i \in J_n} :\ g_i = \begin{pmatrix} g_i^1 \\ g_i^2 \end{pmatrix} \in R^2,\ i \in J_n. \tag{14}$$

Here, we assume that the tuples are ordered lexicographically:

$$g_i \le^{lex} g_{i+1},\ i \in J_{n-1}, \tag{15}$$

implying that:

$$\forall i \in J_{n-1}\ g_i^1 \le g_{i+1}^1;\ \text{if}\ g_i^1 = g_{i+1}^1,\ g_i^2 \le g_{i+1}^2. \tag{16}$$

Similarly to a 1-multiset, different tuples of a 2-multiset (14) form its basis $S(G)$ whose elements are strictly lexicographically ordered:

$$S(G) = \{e_j\}_{j\in J_k} :\ e_j = \begin{pmatrix} e_i^1 \\ e_i^2 \end{pmatrix} \in R^2,\ j \in J_k; \tag{17}$$
$$e_j \prec e_{j+1} \Leftrightarrow\ e_j \le^{lex} e_{j+1},\ e_j \ne e_{j+1}.$$

In terms of e_j-coordinates, this means that:

$$\forall i \in J_{k-1}\ e_i^1 \le e_{i+1}^1;\ \text{if}\ e_i^1 = e_{i+1}^1,\ e_i^2 < e_{i+1}^2. \tag{18}$$

Now, similar to a 1-multiset, a G-primary specification is defined by (12) and the 2 multiset is representable in the form (13).

A set $\overline{\mathrm{S}}_k^n(G)$ is a <u>set</u> of n-combinations with repetitions from the multiset (13) with $[G] = (k^n)$ [22]. Its elements are ordered n-samples from G whose coordinates are ordered non-decreasingly:

$$\{\overline{\mathrm{S}}_k^n(G) = x \in R^n :\ x_i \in S(G),\ i \in J_n; \\ \mathrm{x}_i \le x_{i+1},\ i \in J_{n-1}\}. \tag{19}$$

A convex hull of (19) is a <u>polytope</u> $\overline{\mathrm{Q}}_k^n(G)$ of n-combinations with repetitions [22] which is a n-simplex:

$$\overline{\mathrm{Q}}_k^n(G) = conv\overline{\mathrm{S}}_k^n(G) = \\ \{x \in R^n :\ x_1 \ge e_1,\ x_n \le e_k;\ x_i \le x_{i+1}, i\}. \tag{20}$$

After eliminating the constraint on ordering x-coordinates, $\overline{\mathrm{S}}_k^n(G)$, $\overline{\mathrm{Q}}_k^n(G)$ become a <u>set</u> $\overline{E}_k^n(G)$ and a polytope $\overline{\Pi}_k^n(G)$ of permutations with repetitions, respectively [18]:

$$\overline{\mathrm{E}}_k^n(G) = \{x \in R^n :\ \mathrm{x}_i \in S(G),\ i \in J_n\}, \tag{21}$$

$$\overline{\Pi}_k^n(G) = \{x \in R^n :\ \mathbf{e}_1 \le x \le \mathbf{e}_k\}. \tag{22}$$

A particular case of (21), (22) are the Boolean set and unit hypercube [19, 20]:

$$B_n = \{0,1\}^n = \overline{E}_2^n(\{0^n, 1^n\}),$$
$$PB_n = [0,1]^n = \overline{\Pi}_2^n(\{0^n, 1^n\}).$$

If, in a zero-one multiset, multiplicities of $0,1$ can be restricted:

$$G = \{0^{\eta_1}, 1^{\eta_2}\} : 1 \le \eta_1, \eta_2 \le n,\ \eta = \eta_1 + \eta_2 \ge n, \tag{23}$$

the corresponding B_n - subset is a Boolean permutation set $B_n(\eta_2)$ [20] if $\eta = n$ and it is a Boolean partial permutation set $B_n(n-\eta_1, \eta_2)$ [20] if $\eta > n$:

$$\forall \mathrm{a} \in \mathrm{R}\ \mathbf{a} = (a)_{i\in J_n}$$
$$B_n(\eta_2) = \{x \in B_n :\ x^T\mathbf{1} = \eta_2\}, \tag{24}$$
$$B_n(n-\eta_1, \eta_2) = \{x \in B_n :\ n-\eta_1 \le x^T\mathbf{1} \le \eta_2\}.$$

Convex hulls of the sets (24) are $n-1$-hypersimplex and n - hypersimplex [20]:

$$\Delta_{n,\eta_1,\eta_2} = \{x \in B_n :\ n-\eta_1 \le x^T\mathbf{1} \le \eta_2\}. \tag{25}$$

A particular case of (25) is a unit n-simple<u>x</u>:

$$\Delta_{n,\eta_2} = \{x \in B_n :\ x^T\mathbf{1} = \eta_2\},$$

$$\Delta_{n,0,1} = \mathrm{conv}\, B_n(0,1) = \{x \ge \mathbf{0} : x^T\mathbf{1} \le 1\}. \tag{26}$$

One more B_n-subset is a Boolean set of combinations with repetitions $\overline{\mathrm{S}}_2^n(\{0^n, 1^n\})$.

The Cartesian product of combinatorial sets is called a <u>set</u> of the sets' tuples.

Let J_n be partitioned into l subsets:

$$J_n = \bigcup_{j=1}^{l} I_j,\ |I_j| = n_j > 0,\ j \in J_l,\ \sum_{j=1}^{l} n_j = n, \tag{27}$$
$$\overline{n} = (n_j)_{j\in J_l},\ \overline{2} = (2^l), \tag{28}$$

Then, for instance,

$$\bar{\mathbf{S}}_2^{\bar{n}}\left(\left\{0^n,1^n\right\}\right)=\bigotimes_{j=1}^{l}\bar{\mathbf{S}}_2^{n_j}\left(\left\{0^{n_j},1^{n_j}\right\}\right), \quad (29)$$

$$\mathbf{B}_{\bar{n}}(0,1)=\bigotimes_{j=1}^{l}B_{n_j}(0,1)\ - \quad (30)$$

are a set of tuples of the 0-1 combinations with repetitions and a 0-1 set of tuples sum to at most 1, respectively.

Preliminary stage. It is known [10], (1CBP) is reducible to (KP) and after this transformation the problem dimension (possibly) decreases. Recall its stages and then modify them for (mCBP) and apply.

1. Denote

$$I^0=\{i: a_i\ge 0, c_i\le 0\}, I^1=\{i: a_i\le 0, c_i\ge 0\}$$
$$I^+=\{i: a_i, c_i>0\},\ I^-=\{i: a_i, c_i<0\}; \quad (31)$$
$$n^{[.]}=\left|I^{[.]}\right|,\ \sum_{[.]\in\{0,1,+,-\}} n^{[.]}=n. \quad (32)$$

2. Assign $x_i^*=\begin{cases}0,\ i\in I^0,\\ 1,\ i\in I^1,\end{cases}$ and reduce the dimension to $n'=n-n^0-n^1$.

3. Introduce new variables:

$$y_i=\begin{cases}x_i,\ i\in I^+,\\ 1-x_i,\ \forall i\in I^-.\end{cases}.$$

Now, (1CBP) is equivalent to (KP):

$$z^*=max\sum_{i\in I^+\cup I^-}|c_i|y_i+\sum_{i\in I^1\cup I^-}c_i, \quad (33)$$
$$y_i\in\{0,1\},\ i\in I^+\cup I^-. \quad (34)$$
$$\sum_{i\in I^+\cup I^-}|a_i|y_i\le b-\sum_{i\in I^1\cup I^-}a_i, \quad (35)$$

For (mCBP), (31), (32) become:

$$I^0=\{i: c_i\le 0,\ a_{ji}\ge 0, j\in J_m\},$$
$$I^1=\{i: c_i\ge 0,\ a_{ji}\le 0, j\in J_m\}, \quad (36)$$
$$I^-=\{i\notin I^0: c_i<0\},\ \bar{I}=J_n\setminus\{I^0,I^1,I^-\}.$$

Formulas (33)-(35) are transformed into:

$$z^*=max\sum_{i\in\bar{I}\cup I^-}|c_i|y_i+\sum_{i\in I^1\cup I^-}c_i, \quad (37)$$
$$y_i\in\{0,1\},\ i\in\bar{I}\cup I^-, \quad (38)$$
$$\sum_{i\in\bar{I}}a_{ji}y_i-\sum_{i\in I^-}a_{ji}y_i\le b_j-\sum_{i\in I^1\cup I^-}a_{ji},\ j\in J_m. \quad (39)$$

The problem (37)-(39) is (mKP) if, in (36),

$$\bar{I}=I^+=\{i: c_i>0,\ a_{ji}\ge 0, j\in J_m\}, \quad (40)$$

otherwise, it is a general linear binary problem (referred to as (mCBP) again).

Approaches to (mCBP). For this general case, we recommend the following continuous approaches:

1. The method of combinatorial and surface cuttings (MCSC) [21] where a sphere

$$S:\sum_{i=1}^{n}\left(x_i-\frac{1}{2}\right)^2=\frac{n}{4} \quad (41)$$

circumscribed around B_n [19, 20] is used. (41) implies that B_n is polyhedral-spherical [20], therefore we use two continuous relaxations of (mCBP) – spherical and polyhedral [19, 20]:

$$z^S=\max_{x\in S}c^Tx,\ x^S=\arg\max_{x\in S}c^Tx, \quad (42)$$
$$z^P=\max_{x\in P}c^Tx,\ x^P=\arg\max_{x\in P}c^Tx, \quad (43)$$
$$P=convB_n\cap\{x: a_j^Tx\le b_j, j\in J_m\}=$$
$$=\{x\in[0,1]^n: a_j^Tx\le b_j, j\in J_m\}. \quad (44)$$

Assume that $n'=dimP=n$, otherwise, a projection onto n' dimensional space is performed.

Outline (MCSC) in application to (mCBP):

- solve the linear program (43);

if $x^P\in B_n$, then $x^*=x^P$, otherwise, form a right cut for x^P:

- choose n P-edges intersecting at x^P: $\{l_i=[x^p,x^i]\}_{i\in J_n}:\{x^i\}_i\subset vertP$;
- use the relaxation (42) extending the edges toward x^i-x^p, $i\in J_n$, up to an intersection with S and get $Y=\{y^i\}_i\subset S$;
- construct, trough Y, a hyperplane $\Pi=\{x: a_{m+1}^Tx=b_{m+1}\}$ and a cut of x^P $D_{m+1}=\{x: a_{m+1}^Tx\le b_{m+1}\}: a_{m+1}^Tx^P>b_{m+1}$;
- add D_{m+1} to (44), set $m=m+1$, and repeat all these steps iteratively.

2. The Lagrangian and penalty methods based on the following functional representations of B_n [19]:

(R1): $$f_1(x)=\sum_{i=1}^{n}x_i-\sum_{i=1}^{n}x_i^2=0,$$
$$f_2(x)=\sum_{i=1}^{n}\left(x_i-0.5\right)^4-0.625n=0;$$

(R2): $$f_1(x)\le 0,\ f_2(x)\le 0.$$

If $f_0(x)=-c^Tx$, $f_{j+2}(x)=a_j^Tx-b_j$, $j\in J_m$, then an equivalent problem to (mCBP) is:

$$F(x,\lambda)=f_0(x)+\sum_{j=1}^{m+2}\lambda_j f_j(x)\to min, \qquad (45)$$

$$x\in R^n,\ \lambda\in R_+^{m+2}. \qquad (46)$$

Formulas (45), (46) is solvable numerically [23] and yields a local minimum for (mCBP).

Another approach is incorporate all constraints into a penalty function [23], e.g.:

$$\Phi(x,\mu)=f_0(x)+$$
$$\mu\left(f_1^2(x)+f_2^2(x)+\sum_{j=3}^{m+2}min\left(0,-f_j(x)\right)^2\right)\to min$$

solvable numerically for increasing sequence of $\mu\in R$, and get also a local minimum for (mCBP). The Lagrangian $F(x,\lambda)$- and penalty minimization techniques can be combined by the augmented Lagrangian method [23].

Approaches to (1CBP). First, transform the problem into (KP). Assume that we deal with (2)-(4),(8). Preliminary, check the following: a) $\sum_i a_i>b$, otherwise, (4) does not work; b) multiplicities of items allow to put the whole group G_j of items of the same weight e_j in the knapsack: $n_j\le\lfloor b/e_j\rfloor$, $j\in J_l$, where:

$$A=\{a_i\}_i=\left\{e_j^{n_j}\right\}_{j\in J_l}$$
$$a_i\le a_{i+1},\ i\in J_{n-1};\ e_j<e_{j+1},\ j\in J_{l-1}. \qquad (47)$$

Otherwise, $\forall j\in J_l\ n_j^{'}=n_j-\lfloor b/e_j\rfloor$ items of G_j with the smallest values are eliminated from A; c) a capacity of the knapsack does not require each specific G_j: $\sum_{i=1}^{n}a_i-n_je_j>b$, $\forall j\in J_l$, otherwise, $n_j^{''}=\left|\left(b-\sum_{i=1}^{n}a_i\right)/e_j\right|$ items of G_j with the largest values are placed in the knapsack.

To get initial feasible solutions x^{**}, lower z^l and upper z^u bounds on z^*, determine values $r_i=c_i/a_i$, $\forall i$ of the profit per unit weight [10]

$$J_n=\left\{i_j\right\}_j: r_{i_j}\ge r_{i_{j+1}},\ j\in J_{n-1}.$$

Now, the knapsack x^{**} is filled with the items with the largest r_i-values: $x_{i_j}^{**}=1,\ j\le j_0$, $x_{i_j}^{**}=0,\ j\ge j_0$.

A polyhedral relaxation (43) solution [10]:

$$x_{i_j}^P=1,\ j\le j_0;\ x_{i_j}^P=0,\ j>j_0+1;$$
$$x_{i_{j_0}+1}^P=\left(b-\sum_{j=1}^{j_0}a_{i_j}\right)/a_{i_{j_0}+1}.$$

x^{**},x^P yield the bounds $z^l=c^Tx^{**}=\sum_{j=1}^{j_0}c_{i_j}$,
$z^u=\lfloor z^P\rfloor$, $z^P=c^Tx^P=z^l+x_{i_{j_0}+1}^P\cdot c_{i_{j_0}+1}^P$.

Now, a two-sided knapsack constraint [24]:

$$z^l+1\le c^Tx\le z^u \qquad (48)$$

can be added to (KP).

Another two-sided knapsack constraint - on a number k of items in x^* - may be added:

$$k_1\le k=x^T\mathbf{1}\le k_2. \qquad (49)$$

The bounds k_1,k_2 can be found by filling the knapsack with the heaviest and lightest items, respectively:

$$k_2:\sum_{i=1}^{k_2}a_i\le b,\ \sum_{i=1}^{k_2+1}a_i>b; \qquad (50)$$

$$k_1:\sum_{i=1}^{k_1}a_{n-i+1}\le b,\ \sum_{i=1}^{k_1+1}a_{n-i+1}>b. \qquad (51)$$

In terms of the Boolean partial permutation set (see (24)). Now, our (KP) is reformulated as a linear constrained combinatorial problem (referred as (KP.C1)) (2), (6), (48),

$$x\in B_n(k_1,k_2). \qquad (52)$$

Notice, (52) may considerably reduce the search domain in comparison with (3).

A specifics of B&B for (KP.C1). As $B_n(k_1,k_2)$ is decomposed into $B_n(k)$-sets:

$$B_n(k_1,k_2)=\bigcup_{k=k_1}^{k_2} B_n(k), \tag{53}$$

the traditional for binary problems branching scheme based on fixing a coordinate [4-11] (we refer it to as Scheme 1) can be combined with another one (Scheme 2), based on analysis of integrity of $k^P=\mathbf{e}^T y^P$. We recommend the following: if $k^P\in Z$, then (Scheme 1) is applied, otherwise, (Scheme 1) is used. It is based on the fact that the feasible region is divisible into branches:

$$B=\left\{x:\ \mathbf{e}^T\mathrm{x}\le\left\lfloor \mathrm{k}^P\right\rfloor\right\},\ B'=\left\{y:\ \mathbf{e}^T\mathrm{x}\ge\left\lceil \mathrm{k}^P\right\rceil\right\}.$$

Now, (KP.C1) is decomposed into two (KP.C1)-subproblems of the same dimension: (2), (6), (48) on $B_n\left(k_1,\left\lfloor \mathrm{k}^P\right\rfloor\right)$, $B_n\left(\left\lceil \mathrm{k}^P\right\rceil,k_2\right)$, respectively. Since $x^P\notin B,B'$, the polyhedral relaxations on two hypersimplexes (25) need to be solved.

For these subproblems, infeasibility of (KP.C1) and irredundancy of constraints (2), (6) are easily verified. It is due to $B_n(k_1,k_2)$ is a kind of the partial permutation set and a linear problem over (53) is solved explicitly [18].

Remark. Ordering (47) allows (possibly) adding new constraints to (KP.C1). Namely:

$$\forall i\in J_{n-1}\ \text{if}\ j<i: c_j\ge c_i\ \Rightarrow x_i\le x_j, \tag{54}$$

implying a priority of an item that is neither heavier not less valuable than another one.

A model (KP.C2). The observation (54) allows to order variables within each G_j. For that, a 2-multiset $AC=\left\{(a_i,c_i)^T\right\}_{i\in J_n}$ of the items weights and values are ordered: $(a_i,c_i)^T\le^{lex}(a_{i+1},c_{i+1})^T$, $i\in J_{n-1}$.

Now, from (54), there follows: $\forall j\in J_l$:

$n_l>1\ x_i\le x_{i+1},\ i\in J_{n_j^0}\setminus J_{n_{j-1}^0}$, where $n_0^0=0$,

$n_j^0=\sum_{i=1}^{j} n_i$, $j\in J_l$. With (53), this implies (see (19)) that

$$\forall j\in J_l:\ \bar{x}_j=(x_i)_{i\in J_{n_j^0}\setminus J_{n_{j-1}^0}}\in\bar{S}_2^{n_j}\left(\left\{0^{n_j},1^{n_j}\right\}\right), \tag{55}$$

Respectively, according to (27)-(29),

$$x\in\bar{\mathbf{S}}_2^{n}\left(\left\{0^n,1^n\right\}\right). \tag{56}$$

A new (KP)-model (referred as (KP.C2)) is a linear constrained problem (2),(6),(48),(49), (56) on the set of tuples of 0-1-combimations with repetitions.

Notice a peculiarity of B&B for (KP.C2) that (Scheme 1) of fixing a variable within each $G_j:\ n_j>1$, leads to decomposition of the problem into two subproblems of the dimension $n-1,\ n-n_j$. Thus, considering large-size groups first are expected to discard the branches faster.

A model (KP.C3). One more combinatorial model of (KP) will be formed based on the following proposition:

Proposition 1. A linear program (2),(4)

$$x\in\bar{\mathrm{Q}}_k^n(G), \tag{57}$$

is equivalent to a linear problem:

$$z'^*=max\ c'^T y,\ y'^*=\arg max\ c'^T y, \tag{58}$$
$$y\ge\mathbf{0}, \tag{59}$$
$$y^T\mathbf{e}\le e_k-e_1, \tag{60}$$
$$A'y\le b'. \tag{61}$$

Proof. By (20), the polytope $\bar{\mathrm{Q}}_k^n(G)$ is n-simplex given by a system:

$$x_1\ge e_1. \tag{62}$$
$$x_i\le x_{i+1},\ i\in J_{n-1}. \tag{63}$$
$$x_n\le e_k. \tag{64}$$

Introduce a change of variables:

$$y_1=x_1-e_1, \tag{65}$$
$$y_i=x_i-x_{i-1},\ i\in J_n\setminus\{1\}. \tag{66}$$

Formula (65) transforms the (62) into $y_1\ge 0$, (66) with (63) yields $y_i\ge 0,\ i\in J_n\setminus\{1\}$ Hence (59) holds. The inverse change of variables is:

$$x_i=\sum_{j=1}^{i} y_j+e_1,\ i\in J_n. \tag{67}$$

By (67), the constraint (64) becomes:

$$x_n=\sum_{j=1}^{n} y_j+e_1\le e_k\ \text{ or }\ \sum_{j=1}^{n} y_j\le e_k-e_1$$

that is (60). Transform the constraints (4) in the form:

$$\sum_{i=1}^{n} a_{ji} x_i \le b_j,\ j \in J_m. \qquad (68)$$

Applying (67) to (68), we obtain:

$$\sum_{i=1}^{n} a_{ji}\left(\sum_{j'=1}^{i} y_{j'} + e_1\right) = e_1 \sum_{i=1}^{n} a_{ji} + \sum_{i=1}^{n}\sum_{i'=1}^{i} a_{ji} y_{i'} =$$

$$= e_1 \sum_{i=1}^{n} a_{ji} + \sum_{i=1}^{n} y_i \sum_{i'=i}^{n} a_{ji'} \le b_j,\ j \in J_m,$$

wherefrom, (61) is derived with

$$A' = \left(a'_{ji}\right)_{m\times n},\ b' = \left(b'_j\right)_m : b'_j = b_j - e_1 \sum_{i=1}^{n} a_{ji},$$

$$a'_{ji} = \sum_{i'=i}^{n} a_{ji'},\ i \in J_n,\ j \in J_m. \qquad (69)$$

Similarly, (2) becomes (58) with

$$c' = \left(c'_i\right)_n :\ c'_i = \sum_{i'=i}^{n} c_{i'},\ i \in J_n. \qquad (70)$$

Corollary 1. A linear program (2), (4)

$$x \in \overline{\mathrm{Q}}_k^n\left(\left\{0^n, 1^n\right\}\right), \qquad (71)$$

is equivalent to a linear program (58), (61), (69), (70),

$$b' = b,\quad y \in \Delta_{n,0,1}. \qquad (72)$$

If, in the corollary, we move on to a vertex set of $\overline{\mathrm{Q}}_k^n\left(\left\{0^n, 1^n\right\}\right)$, (71), (72) are transformed into:

$$x \in \overline{\mathrm{S}}_k^n\left(\left\{0^n, 1^n\right\}\right),$$

$$y \in B_n(0,1). \qquad (73)$$

Corollary 2. A linear program (58), (61), (73) is equivalent to $n+1$-dimension linear multi-choice knapsack problem (MCKP) [10]:

$$z'^* = max\ \bar{c}^T \bar{y},\ \bar{y}'^* = \arg max\ \bar{c}^T \bar{y}, \qquad (74)$$

$$y \in B_{n+1}(1),$$

$$\bar{y} = (y_i),\ \bar{c} = \left(c'_i\right) \in R^{n+1} : y_{i+1} = 1 - \sum_{i=1}^{n} y_i,$$

subject to (61).

Proposition 2. A linear program (2), (6), (56) is equivalent to linear constrained over (30).

Proof. Combine the linear constraints (6), (48), (49) of (KP.C3) into a system (4) with $m = 5$. Decompose this problem into l subproblems corresponding to each group G_j:

$$z = c^T x = \sum_{j=1}^{l} \bar{c}_j^T \bar{x}_j,\ \bar{c}_j = \left(c_i\right)_{i \in J_{n_j^0} \setminus J_{n_{j-1}^0}},\ j \in J_l,$$

$$a_i^T x = \sum_{j=1}^{l} \bar{a}_{ij}^T \bar{x}_j \le b_i,\ \bar{a}_{ij} = \left(a_{ij}\right)_{i \in J_{n_j^0} \setminus J_{n_{j-1}^0}} \quad \forall i, j.$$

Applying Corollary 1 to vectors (55), they are transformed into

$$\bar{y}_j = B_{n_j}(0,1),\ j \in J_l, \qquad (75)$$

Subject to five common linear constraints, for all these groups, representable in the form (61). By (30), the combinatorial constraints (73) are combined into:

$$y \in \bar{\mathbf{B}}_n(0,1). \qquad (76)$$

The model (58), (61), (74) (referred to as (KP.C3)) is (KP) equivalent reformulation on the 0-1 set of tuples sum to at most 1.

Remark. Further application Corollary 2 to (KP.C3) transforms it into (MCKP) of the dimension $n+l$. Thus we found an equivalent reformulation of (KP) as (MCKP). Now, techniques specific to (MCKP) [9-11] can be applied to the standard (KP), as well as to it's another generalization - (1CBP).

CONCLUSIONS

1. New optimization approaches to the capital-budgeting problem (CBP) are presented. They are based on biquadratic functional representation of B_n. Two of them are continuous and one – combinatorial. These are: an exact cutting plane (MCSC), an approximate based on (CBP)-reformulation as a nonlinear unconstrained problem, and exact – B&B, respectively. The continuous methods are extendable into most (KP)-generalization including nonlinear.

2. A possibility of reducing a feasible region of (1CBP) depending on a presence of repetitions in c-coefficients was studied and three equivalent combinatorial models of (1CBP) were obtained – on $B_n(k_1, k_2)$, $\bar{\mathbf{S}}_2^n\left(\left\{0^n, 1^n\right\}\right)$, and $\bar{\mathbf{B}}_n(0,1)$. A new branching scheme based on B_n-decomposition into $B_n(k)$-sets are recommended to (1CBP).

REFERENCES

1. Paranchuk S., Korbutyak A., 2013. Problems investment support innovative development of the national economy and solutions. Econtechmod. An Internetional Quaterly Journal, 2(4), 53–60.

2. Podolchak N., Melnyk L. and Chepil B., 2014. Improving administrative management costs using optimization modeling. Econtechmod. An Internetional Quaterly Journal, 1(1), 75–80.

3. Markowitz H., Manne A., 1957. On the Solution of Discrete Programming Problems. Econometrica, (1), 84-110.

4. Hoffman K., Padberg M., 2001. Combinatorial and Integer Optimization. Encyclopedia of Operations Research & Management Science, 94–102.

5. Bradley S., Hax A. and Magnanti T., 1977. Applied Mathematical Programming. Reading, Mass: Addison-Wesley, 716.

6. Schrijver A., 1998. Theory of Linear and Integer Programming. Chichester, New-York: Wiley, 484.

7. Wolsey L. and Nemhauser G., 1999. Integer and Combinatorial Optimization (1 edition). New York: Wiley-Interscience, 763.

8. Papadimitriou C., Steiglitz K., 2013. Combinatorial Optimization: Algorithms and Complexity. New York: Dover Publications, 528.

9. Taha H., 2014. Integer Programming: Theory, Applications, and Computations. New York: Academic Press, 392.

10. Martello S. and Toth P., 1990. Knapsack Problems: Algorithms and Computer Implementations. Chichester, New York: Wiley, 308.

11. Kellerer H., Pferschy U., Pisinger D., 2010. Knapsack Problems (Softcover reprint of hardcover 1st ed. 2004 edition). Berlin, New York: Springer, 548.

12. Hiroshi I., 2014. Some Thoughts on the 2-Approximation Algorithm for Knapsack Problems: A Survey. Far East Journal of Applied Mathematics, 87(3), 257–267.

13. Escudero L., Martello S. and Toth P., 1998. On tightening 0-1 programs based on extensions of pure 0-1 knapsack and subset-sum problems. Annals of Operations Research, 81(1–4), 379–404.

14. Shaheen A., Sleit A., 2016. Comparing between different approaches to solve the 0/1 Knapsack problem. International Journal of Computer Science and Network Security, 16(7), 1–10.

15. Lust T., Teghem J., 2012. The multiobjective multidimensional knapsack problem: a survey and a new approach. International Transactions in Operational Research, 19(4), 495–520.

16. Öncan T., 2007. A Survey of the Generalized Assignment Problem and Its Applications. INFOR, 45(3), 123–141.

17. Stoyan Y., Yakovlev S., 1986. Mathematical models and optimization methods in Geometric Design. Kiev: Naukova Dumka, 268. (in Russian).

18. Stoyan Y., Yemets O., 1993. Theory and methods of Euclidean combinatorial optimization. Kiev: ISSE, 188. (in Ukrainian).

19. Pichugina O., Yakovlev S., 2016. Continuous Approaches to the Unconstrained Binary Quadratic Problems. In: Bélair, J. et al. (eds.) Mathematical and Computational Approaches in Advancing Modern Science and Engineering, Springer International Publishing, Switzerland, 689-700.

20. Pichugina O., Yakovlev S., 2016. Convex Extensions and Continuous Functional Representations in Optimization, with Their Applications. J. Coupled Syst. Multiscale Dyn. 4(2), 129–152.

21. Pichugina O., 2016. Surface and Combinatorial Cuttings in Euclidean Combinatorial Optimization Problems. Mathematical and computer modeling. The Series: Physics and Mathematics. 1(13), 144–160. (in Russian).

22. Pichugina O., 1996. The methods and algorithms for a solution of some problems of optimization on arrangements and combinations (Ph.D. dissertation). Institute for Mechanical Engineering Problems, Kharkiv, Ukraine. (in Russian).

23. Bertsekas D., 1999. Nonlinear Programming, 2nd edn. Belmont: Athena Scientific, 708.

24. Trick M., 2003. A Dynamic Programming Approach for Consistency and Propagation for Knapsack Constraints. Annals of Operations Research, 118(1–4), 73–84.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 37-42

INVESTIGATION OF THE DYNAMICS OF SINGULAR PROTECTED SYSTEMS

I. Naumeyko, M. Alja'afreh

Kharkov National University of Radio Electronics; e-mail: pmkaf@kture.kharkov.ua

Received September 28, 2016; accepted November 10, 2016

Abstract. The analysis of asymptotic representations of the systems protected from harmful influences is carried out. Various types of general models of the "man-machine-environment" with protection are considered. Each of them adequately describes some of the practically important qualities of the object, and they all together describe the object in terms of its safe operation. The dynamic properties of complex ergonomic systems, presented in the form of systems of differential equations with a small parameter at the derivative are investigated The methods of reducing the impact on the person of harmful factors are theoretically substantiated. The dynamic protection response speed is considered to be significantly greater than the harmful factor production rate.

Numerical solution of the general problem and the analytical solution for autonomous case is obtained for harmful effects. By using asymptotic the system of equations has been solved in closed form not only for autonomous case, but also for parameters smoothly changing in time. The estimates of the cost of protection was obtained for the various cost-functionals and for different functions in the right-hand side of the equation describing the dynamics of protection. To assess the accuracy of model calculations and for graphic representation of the results mathematical package MAXIMA is used.

Key words: "Man-Machine-Environment" model, non-linear system, singular equations, asymptotic, linearization.

INTRODUCTION

It is known [1], that safety and efficiency are conflicting criteria, because they compete for the same resources. Their union in the single criterion is possible only in the super-system [1, 2]. This approach allowed us to consider the model of the "man-machine-environment-protection" as a well-known general model of competition of two factors – the safety and efficacy [3].

It runs a large number of processes with different time scales. The hierarchy of these times is such that they differ by many orders of magnitude [4]. Usually, various problems of physics and engineering are modeled by means of differential or algebraic equations. And almost always it turns out that they have a high order, and when it comes to systems, they are of large dimensions. To overcome this problem the two diametrically opposite approaches are known.

The essence of the first lies in the fact that if their characteristic elements are similar in the system, they can be considered equal in a first approximation. And we use that symmetry, considering small deviations in the subsequent approximations.

The second is used when the individual elements of the system are very different in their characteristics. In this case, we introduce small parameters representing their attitude and conduct an asymptotic reduction of dimension, i.e., reducing the number of degrees of freedom. The use of asymptotic methods are not always stipulated specifically, and sometimes not even realized in modelling. So, in engineering practice it is extremely widespread to model systems with one degree of freedom. It is clear that the use of such models involves an asymptotic reduction of dimension. If the system in question consists of sets of similar elements, the asymptotic approach does not lead to a reduction of dimension, but rather to increase it. This method is applied to a very important class of models in which discrete systems are replaced by continuous, that is, as in our case, a system of differential equations. As a rule, the solution based on the asymptotic method cannot be expressed in a finite form, but only with the help of some series [4]. It turns out that the perturbation series are not necessarily converge. For example, it often happens that you can use the infinite series that diverges, but have the value in a certain sense. A typical situation is as follows: a function can be expanded in a series of functions and approximation, given by the first few terms of the series. It serves the better, the closer is the independent variable, or a parameter to a certain limit value. In many cases, the values of terms at first decrease rapidly, but then again begin to increase. In mathematical literature such series are called asymptotic series [5].

Here we first consider the overall system "Man-Machine-Environment" under this approach. The input to this system is the information from the superior system (targets, instructions, etc.); the exit of such system is the result of work and a lot of other factors that are harmful to the environment and personnel.

In operation the system changes its internal state. The element "man" has three functional parts: the part, that controls the "machine"; the object of the external environment and the impact of an object from the "machine".

"Machine" element performs basic technological functions: impact on the subject of work and change the parameters of the environment.

The paper discusses different types of common models of "human-machine environment", each of which adequately describes some of the practically important quality of the object, and all together they describe the object in terms of its safe operation [6]. We get further details and results in the well-known and, as well as, some new models of subsystems [7]. This work is devoted to the quantitative analysis of an important model of a system with protection of a human from the harmful effects of the external environment and the impact of sub-system called the "machine".

THE ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

In works [8, 9] a model of dynamic system describing the situation where primary subsystem "produces" a harmful factor, and second sub-system called “protection”, tries to reduce it completely, or at a reasonable price. As the base model – the basis for modification – a system of ordinary differential equations was taken. It describes fundamental laws of competition [10], and also known in ecology as a model of coexistence of species [11 – 15].

We first introduce the basic assumptions, directly following from everyday experience. They are evident so, do not require additional justification but only need to be formalized. A more detailed discussion of these issues are delivered in [8].

We call Bioinfluence U of the harmful factor an increasing function of time t and the intensity of the factor u. In the first approximation, it can be written as an integral

$$U = \int_0^T u\ (t)dt \cdot$$

It also fits the additive property [6].

The following axiom are true:

a) autocumulative;

b) mutual cumulative;

c) intensity of bioinfluence u fits:

– in a regular situation $\frac{\partial}{\partial t}u \le 0$;

– in critical situations (positive feedback) $\frac{\partial}{\partial t}u > 0$.

Protection factor $z(t)$ may be controlled adaptively or programmatically, depending on the value of $u(t)$.

The cost of protecting $C=C(z)$ is natural to consider as a monotonically increasing function of its intensity.

In [6-8], we have conducted a formal description of the model that is under study here.

Suppose there are two types of internal system states: production factors (including the production of harmful substances) U and the impact of protective factors Z. Let f and g be smooth functions, monotonically increasing in both arguments, such that $f(0,Z) = g(U,0) = 0,\ \forall U,Z$. Then it is natural to consider that $U' = f(U,Z)$; $Z' = g(U,Z)$. This is the most common model of dynamics of the system with protection. However, to obtain meaningful results, it must be detailed.

Suffice general case of such a model of the system can be represented as:

$$\begin{cases} u'(t) = \alpha u(t) - \beta z(t)u(t) \\ z'(t) = F(u(t), z(t)) \end{cases}, \qquad (1)$$

with the constrains $u \ge 0,\ z \ge z_c$, where z_c is fixed (stationary) protection value.

The function $F(u, z)$ can take quite arbitrary form [8]. The most common of them are the following three:

1) $F(u(t),\ z(t)) = \gamma u(t)$;

2) $F(u(t), z(t)) = \gamma u(t) - \delta z(t)$;

3) $F(u(t), z(t)) = \gamma_1 u(t) + \gamma_2 u^2(t) - \delta_1 z(t) - \delta_2 z^2(t)$.

Solution of the system of differential equations (1) is not always possible to find analytically. Therefore, to find protection functions and the harmful effects some numerical methods for solving systems of differential equations are used. For the system (1) it is necessary to investigate the stability at different values of the parameters of the protection subsystem

$$F(u(t),\ z(t)).$$

It is also necessary to evaluate the cost of protection for different functions F. In [8] (1) is assumed to be autonomous, and the bifurcation parameters are not dependent on time.

OBJECTIVES

In this paper, in contrast to [8], it is assumed that the parameters of equation (1) depends on the time and takes into account the effect of "boundary layer" [5] near $t = 0$.

Based on the results obtained in the course of the study, an analysis of bifurcations for protection is made, ie, we find a scenario of possible loss of stability [16] and the effectiveness of protection.

THE MAIN RESULTS OF THE RESEARCH

1. Methods for studying the stability of models

The linearization theorem establishes a connection of the phase portrait of the nonlinear system (1) in the neighborhood of a fixed point with the phase portrait of its linearization [16, 17].

In general, if a nonlinear system $\dot{y} = Y(y)$ has a simple fixed point $y = 0$, then in the neighborhood of the origin, the phase portraits of this system and its linearization qualitatively equivalent, unless a fixed point of the linearized system is not the center [17].

Application of the theorem on linearization similarly is considered in the analysis of environmental models just the same way as for competition in economic systems [18-20]. We conclude that the studied in this paper system has a stationary point (0, 0) of the "saddle" type.

2. The problem of fast and slow variables.

Dynamical systems include a large number of processes with different time scales. Given the time hierarchy process reduces the number of differential equations. "Very slow" variables do not change on time scales of these processes and can be regarded as constant parameters. For "fast" variables there can be written algebraic equations for their steady-state values instead of differential equations. The "fast" variables reach their stationary values almost instantly if compared to the "slow" [18]. This difference leads to a singularity by parameter in the second of the equations (1). Note that the asymptotic solution itself obtained in [8], is singularly in *t*. Because of this, the protection features and hazards for the second term of the asymptotic approximation have the form [8]:

$$z(t)=\frac{1}{\beta t}+\varepsilon\frac{1}{2\beta}(\alpha-\frac{2\ln t}{t^2\delta})+\varepsilon^2\frac{1}{12\beta\delta^2t^3}(36-$$
$$-6\alpha\delta t^2+\alpha^2\delta^2t^4-12\ln t+12\ln^2 t);$$
$$u(t)=\frac{\delta}{\beta\gamma t}+\varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta-2\ln t-2)+\varepsilon^2\frac{1}{12\beta\delta\gamma t^3}\times$$
$$\times(24-6\alpha\delta t^2+\alpha^2\delta^2t^4+12\ln t+12\ln^2 t).$$

Therefore, the solution obtained in [8] was adequate only far from the starting point. In this paper we use a generalized asymptotic representation, which takes into account the effect of "boundary layer" in the vicinity of the starting point.

3. Research Algorithm

We use an iterative algorithm with the following three steps.

A) We find, if possible, an analytical solution of the system (1) using the functions included in the standard MAXIMA package, which is distributed on the basis of General Public License. If a solution cannot be found in a general way, then we use numerical methods (in the default package there used sufficiently universal method of Adams [19, 21]) or the asymptotic method proposed below.

B) After a solution of (1) was found, analyze the behavior of the hazard function, at what times, if any, its values exceed its protection features, that is a system of dynamic protection comes to operation.

By finding these intervals we decide:

- to increase the protection against harmful factors that lead to an increased cost of the protection system;
- leave the system without modification;
- if the intensity of harmful factors does not exceed an opportunity of fixed protection, the overall cost of the protection system can be reduced by reducing both fixed and dynamic protection.

C) Selecting the solution and repeat the steps A) - C) until you go beyond the limitations (the time of the system work or its value).

4. Analytical study of the model

Consider a system of differential equations (1) with a small parameter ε:

$$\begin{cases}u'(t)=\varepsilon\alpha u(t)-\beta z(t)u(t)\\ \varepsilon\, z'(t)=\gamma u(t)-\delta z(t)\end{cases}. \qquad (2)$$

The difference of this system from the previously considered is a quasi-stationary harm $u(t)$. Let us solve the system (2) with the asymptotic method by finding a series of terms with $\varepsilon^0,\varepsilon^1,\varepsilon^2$.

To begin, write out the system (2), taking into account the dependence of the functions $u(t,\varepsilon)$ and $z(t,\varepsilon)$ in time and small parameter.

We solve the system (2) for the main asymptotic term - with ε^0 (zero approach).

Write the asymptotic for the functions $u(t,\varepsilon)$ and $z(t,\varepsilon)$ as follows:

$$u(t,\varepsilon)=u_0(t)+O(\varepsilon),\quad z(t,\varepsilon)=z_0(t)+O(\varepsilon).$$

The system (2) for the zero-order approximation takes the form :

$$\begin{cases}u_0'(t)=-\beta u_0(t)z_0(t)\\ 0=\gamma u_0(t)-\delta z_0(t)\end{cases}. \qquad (3)$$

After substitution $u_0(t)=\frac{\delta}{\gamma}z_0(t)$, with stationary protection $z_c=0$ we get:

$$z_0'(t)=-\beta z_0^2(t).\quad z_0(t)=\frac{1}{\beta t},\quad u_0(t)=\frac{\delta}{\beta\gamma t}.$$

The resulting singularity at t=0 indicates the impossibility of such a solution for the Cauchy problem at t_0 =0.

Protection functions and hazard at zero approximation for $z_c>0$ have the form:

$$z(t)=\frac{1}{\beta t+1/z_c},\quad u(t)=\frac{\delta}{\gamma(\beta t+1/z_c)}.$$

Similarly, we solve the system (2) with regard to the term ε^1.

So, we write asymptotic for $u(t,\varepsilon)$ and $z(t,\varepsilon)$. The system (2) for the first approximation takes the form:

$$\begin{cases}u_0'(t)+\varepsilon u_1'(t)=\varepsilon a u_0(t)-\beta u_0(t)z_0(t)-\\ -\varepsilon\beta(u_1(t)z_0(t)+u_0(t)z_1(t))\\ \varepsilon z_0'(t)=\gamma u_0(t)+\varepsilon\gamma u_1(t)-\delta z_0(t)-\varepsilon\delta z_1(t)\end{cases}. \qquad (4)$$

The terms with the factor ε in powers of 2 and higher form the remainder term $O(\varepsilon^2)$.

The obtained system for $z_1(t)$ and $u_1(t)$ has the form:

$$\begin{cases} u_1'(t) = \alpha\dfrac{\delta}{\beta\gamma t} - \beta(u_1(t)\dfrac{1}{\beta t} + \dfrac{\delta}{\beta\gamma t} z_1(t)) \\ -\dfrac{1}{\beta t^2} = \gamma u_1(t) - \delta z_1(t) \end{cases} \quad (5)$$

After replacement $u_1(t) = \dfrac{1}{\gamma}(\delta z_1(t) - \dfrac{1}{\beta t^2})$ we solve the equation:

$$\frac{1}{\gamma}(\delta z_1'(t) + \frac{2}{\beta t^3}) = \alpha\frac{\delta}{\beta\gamma t} -$$
$$-\beta(\frac{1}{\gamma}(\delta z_1(t) - \frac{1}{\beta t^2})\frac{1}{\beta t} + \frac{\delta}{\beta\gamma t} z_1(t)).$$

As a result, the function $z_1(t)$ is found and, with its help, also the function $u_1(t)$, that are the first terms of the asymptotics.

$$z_1(t) = \frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}),$$
$$u_1(t) = \frac{1}{2\beta\gamma t^2}(t^2\alpha\delta - 2\ln t - 2).$$

Then the resulting functions of protection and hazard intensities for the first approximation take the form :

$$z(t) = \frac{1}{\beta t} + \varepsilon\frac{1}{2\beta}(\alpha - \frac{2\ln t}{t^2\delta}),$$
$$u(t) = \frac{\delta}{\beta\gamma t} + \varepsilon\frac{1}{2\beta\gamma t^2}(t^2\alpha\delta - 2\ln t - 2).$$

Similarly to [8] a decision based on the second term of the asymptotics is given above.

5. Selection of the coefficients and the number of expansion terms

We solve the system (2) with the asymptotic method for ε^0, ε^1. It will be shown that, in this study, the first two members of the series will be sufficient to obtain a good approximation.

We present an algorithm for constructing an asymptotic solution of the problem (2) is similar to [18], under the assumption that the function on the right side is sufficiently smooth. We will look for it in the form of the asymptotic expansion

$$z(t,\varepsilon) = \bar{z}(t,\varepsilon) + \Pi z(\tau_0,\varepsilon)$$

where $\bar{z}(t,\varepsilon) = \bar{z}_0(t) + \varepsilon\bar{z}_1(t)\ldots$ is the so-called regular series:

$$\Pi z(\tau,\varepsilon) = \Pi_0 z(\tau) + \varepsilon\Pi_1 z(\tau) + \ldots,$$

that describes boundary layer in the neighborhood of t=0 (τ=t/ε).

We choose the coefficients of system (2) from the physical meaning of the problem :

$$\alpha=0.5,\ \beta=5,\ \gamma=2,\ \delta=1,\ \varepsilon=0.0001.$$

Also, we define the initial conditions at t=0 for numerical and asymptotic solutions as $z_c = z^0 < u^0$, because the system has to emerge from the fixed protection value. Let z^0=2, u^0=3 and T <10 – the time interval for the system. In the first approach the parameters of the system (2) are constant.

6. The asymptotic solution of the problem

We show that for $\beta \in$ (0, 10), the asymptotic solution built for the first two terms of the expansion, is little different from a sufficiently accurate numerical one.

Using the procedure described in [8, 22, 23], we obtain the graph of the solution of problem (2). The results are shown in the figures below. For a better representation of the system behavior near the boundary layer, we draw the schedule not on the whole range of T, but only at the beginning of its section .

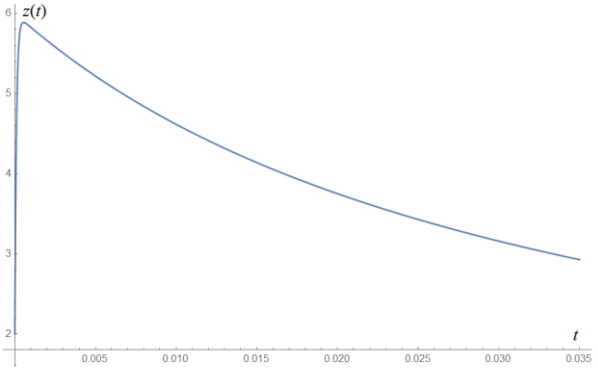
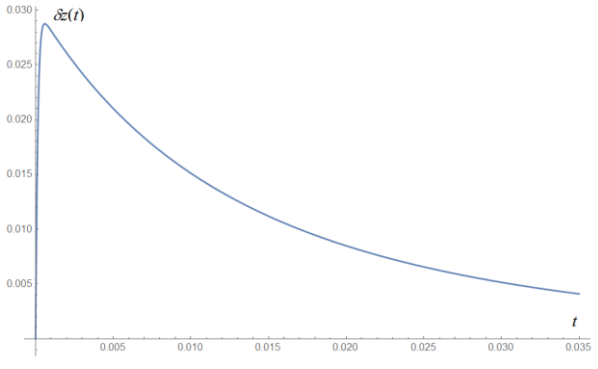


Fig. 1. The schedule of the first approximation (the first two terms of the series) and regular part of the asymptotic behavior at the boundary layer for protection function $z(t, \varepsilon)$



)

Fig. 2. Schedule of regular member of the zero error and the border of the asymptotic solutions for the protection function with respect to the numerical one

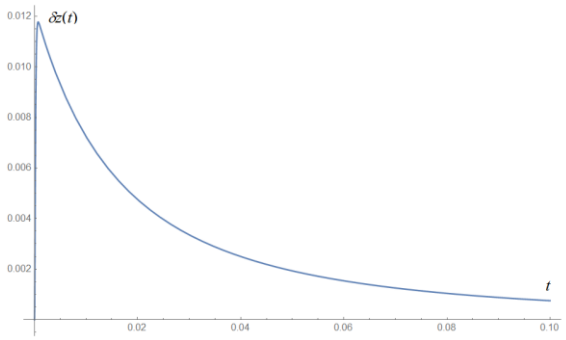


Fig. 3. Graph of error of the first approximation of regular and border of the asymptotic solutions for the protection function with respect to its numerical solution.

As can be seen from the graphs, the error decreases with increasing number of terms in the expansion in powers of ε.

7. Estimates of the cost of protection

We use the function

$$C(T) = \int_0^T c(z - z_0)dt + C_0 \,, \qquad (6)$$

where: C_0 – the cost of fixed protection; z_0 – the value of fixed protection; $c(z)$ – cost function, which can take the form of a), b) and c) below.

We integrate, taking T=6.5 (the time during which the necessary protection of the system will take a value less than z_0) and write down the results:

а) $c(z) = z$, C=1270;

б) $c(z) = z^2$, C=1744;

в) $c(z) = z \ln z$, C=1421.

A disadvantage of the cost function (6) is that the formula did not account for the protection increases with the increasing reaction rate β. Therefore it is suggested the following clarification:

$$c(z) = \int_0^t \left(z(\tau) - z_c + K \max\left[0, z'(\tau)\right] \right) d\tau + C_0, \qquad (7)$$

wherein the coefficient K is selected from considerations of the reaction speed value contribution in the total cost. In the experiments K was chosen, such that the contribution rate of charge and other factors was equivalent (K = 0.01). But as β increases, the integration period is reduced without limit together with the integral value. This does not fit the actual conditions.

Therefore the task of optimizing the cost function that depends not on a one-time operation of the system but also on all the contingencies that can happen for the entire life span of the system as well as the actual cost of purchasing the system.

Then the cost function takes the form:

$$S(\beta, \varepsilon) = n \cdot c\left(z(t)\right) + \varphi(\varepsilon, \beta), \qquad (8)$$

where: $c(z(t))$ is a function (7); $\phi(\varepsilon,\beta)$ is a function of the purchase price of protection system.

A coefficient n is numerically equal to the number of emergency situations in which the protection system goes from a stationary mode and for each adverse factor It is calculated using the formula $n = T \times N$, where T is the average life-term of the protection system, N – the average number of emergencies in a year.

This makes it possible to calculate the minimum cost of the whole system and say what speed parameters β and ε we need to buy it. The problem is reduced to one-dimensional optimization of S (β,ε) for small ε. Here is an example for n = 10. The function in (8) is chosen in the form:

$$\varphi(\varepsilon, \beta) = \frac{\beta}{2\varepsilon}.$$

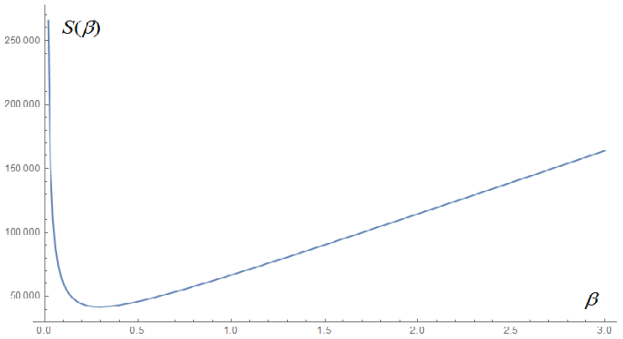


Fig. 4. Schedule of value S on the parameter β at $n = 10$

The minimum is achieved when $\beta \approx 0.294$ and the value of the cost function $S(\beta) \approx 41792$.

CONCLUSIONS

In this paper there are first obtained or improved the following results and methods:

1. For the first time there proposed a dynamic model of the system with protection from harmful factors taking into account the great difference in order of specific operating times and speeds of the subsystems.

2. For the "singular" differential equations of this model there improved and applied the method of asymptotic expansion in small parameter of the solution taking in view of the phenomenon of boundary layer.

3. The method [8] got its further development, which allowed to determine the total cost, depended on the intensity of dynamic protection functions. It uses previously obtained analytical expressions and the variety of cost functions for the specific cases of protection.

The paper also proposes for practical application the approach that saves the total cost of the protection [6]. For this purpose were studied times and the system states when the intensity of harmful factor $u(t)$ does not exceed the threshold of dynamic protection action and hence the value z_0 of static protection may be redundant.

REFERENCES

1. **Alexeev I., Voloshyn O., 2013.** Formation of Compensation Mechanism of Regional Enterprises' Human Resources Regeneration in the Labor Potential Development System. ECONTECHMOD. An International Quarterly Journal. Vol. 2. No. 3, 3-8

2. **Saati T. 1993.** Decisions making. Method of hierarchy analysis. M.: Radio& communication, 282. (in Russian).

3. **Dzundzjuk B., Naumeyko I., Serdyuk N., 2000.** Content model for number of harmful factors of impact on human. Radioelektronika i informatika, №3(12), 127-128. (in Russian).

4. **Haken H., 2004.** Synergetics: introduction and advanced topics. Springer-Verlag, 201.

5. **Vasil'eva A., Butuzov V., 1990** Asymptotic expansions of solutions of singularly perturbed equations. - Moscow: Higher School. 201 (in Russian).

6. **Naumeyko I., Al-Refai V., 2013.** Concerning issue of critical regimes analysis with dynamic defense systems from harmful influence / Information systems and technologies // Materials 2 Intern. Science and Techn. Conf. - Kharkiv. NTTM. 12. (in Russian).

7. **Naumeyko I., 2011.** Critical points of dynamic model for harmful factors distributing. Materials of International scientific conference ISTE 2011. Kharkov-Yalta. October 1-6 2011, 60-61 (in Russian).

8. **Naumeyko I., Alja'afreh M., 2015.** Dynamic balance research of protected systems. ECONTECHMOD. An International Quarterly Journal. vol.4, No 3 – P. 85–90.

9. **Inozemtsev G., 2012.** Scientific and technical preconditions of electric field application at plants protection. ECONTECHMOD. An International Quarterly Journal. Vol. 1. No. 1, 47-50.

10. **Ilyichev V., 2003**. Stabilization and adaptation mechanisms in ecology models: Dis. ... Doctors of technology sciences : 05.13.01, 05.13.18 : Rostov-on-Don, 279. RGB OD, 71:04-5/418. (in Russian).

11. **Nasritdinov G. and Dalimov R., 2010.** Limit cycle, trophic function and the dynamics of intersectoral interaction. Current Research J. of Economic Theory, 2(2), 32–40.

12. **Brauer F., Castillo-Chavez C., 2000.** Mathematical Models in Population Biology and Epidemiology. Springer-Verlag, 412.

13. **Hoppensteadt F., 2006.** Predator-prey model. Scholarpedia, 1(10), 1563.

14. **Jost C., Devulder G., Vucetich J., Peterson R., Arditi R., 2005.** The wolves of Isle Royale display scale-invariant satiation and density dependent predation on moose. J. Anim. Ecol. 74(5), 809–816.

15. **Arditi R., Ginzburg L., 2012.** How Species Interact: Altering the Standard View on Trophic Ecology. Oxford University Press. ISBN 9780199913831.

16. **Sahal D., 1976.** System Complexity: Its Conception and measurement in the Design of Engineering systems. IEEE Trans. Syst. Man. Cybern., SMC 6, 152.

17. **Arrowsmith D., Place C., 1986.** Ordinary differential equations. A qualitative approach with applications. M.: Mir, 243. (in Russian).

18. **Andrianov I., Barantsev R., Manevitch L., 2004.** Asymptotic Mathematics and Synergetics. M.: Publishing house "Editorial URSS", 304. (in Russian).

19. **Sidorov S., 2009.** Mathematical and numerical study of dynamic chaos in dissipative systems of non-linear differential equations: Thesis ... doctor of Mathematics and Physics science : 05.13.18 ; [Place of defense: Moscow state municipal university].- Moscow, 2009, 283 : il. RGB OD, 71 10-1/73. (in Russian).

20. **Amirokov S., 2006.** Numerical methods and simulation experiment in research of dynamics and structure of interacting societies: Thesis ... Candidate of Physical and Mathematical Science : 05.13.18.- Stavropol 2006, 187 : il. RGB OD, 61 06-1/929. (in Russian).

21. **Ilyina V., Silaiev P., 2004** PK MAXIMA computer algebra system for theoretical physicists. M.: Moscow State University. 113. (in Russian).

22. **Koronovskiy A., 2007.** Synchronized behavior, complicated dynamics and transmission processes in self-sustained oscillation systems and standard reference models of non-linear theory of oscillations : thesis ... Doctor of Physics and Mathematics science : 01.04.03; [Place of defense: Saratov State University].- Saratov, 2007, 462 : il. RGB OD, 71 07-1/420. (in Russian).

23. **Latypov V., 2010.** Mathematical models of perturbed motion of a high order of accuracy: thesis ... candidate of Physics and Mathematics science : 05.13.18; [Place of defense: St-Petersburg state university].- St-Petersburg, 2010, 133: il. RGB OD, 61 10-1/736. (in Russian).

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 43-48

RANDOM GENERATION OF COMBINATORIAL SETS WITH SPECIAL PROPERTIES

I. Grebennik, O. Lytvynenko

Kharkiv National University of Radio Electronics; email: igorgrebennik@gmail.com

Received October 14.2016: accepted November 15.2016

Abstract. General approach for solving the problem of random generation of compositional $k-$images of combinatorial sets (*k*-sets) has been proposed. *K*-sets are powerful apparatus that can be applied for solving many scientific and applied problems. Though many literature is dedicated to the problem of generating combinatorial configurations, existing studies deals mostly with simple combinatorial configurations like combinations, permutations etc.

The algorithms of generation both basic combinatorial sets and *k*-sets have been described. Algorithm for random generation of basic sets allows generating various combinatorial sets, and laws of constructing basic combinatorial sets can be pre-set. If identification of the laws fails, the algorithm allows using other algorithms to generate basic sets.

Complexity of described algorithms has been evaluated. The complexity of the algorithm of generation *k*-sets is determined by the complexity of generation of basic sets, as well as the complexity of operations of *n*-substitution and a number of levels of a certain *k*-set.

The described approach to the random generation is very flexible since it allows obtaining various results by varying algorithm parameters. In its turn, it allows adjusting the number of elements for both basic sets and *k*-sets. The developed software allows solving the described problems of random generation of *k* -sets and basic combinatorial sets.

Key words: combinatorial generation, k-set, basic combinatorial set, random generation, complexity.

INTRODUCTION

Generation of various combinatorial objects is frequently required in developing and implementing methods and algorithms for solving many scientific and applied problems [1-6]. Generation is usually understood as a construction of all combinatorial structures of a given type [3]. In these sources, the problem of generation of simple combinatorial objects like permutations, combinations, splits, trees, binary sequences is mainly solved. Solution of generating more complex combinatorial objects is hindered by lack of special structural tools and by significant computational costs caused by redundant results of using well-known methods and algorithms of generation.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

Quite complex combinatorial configurations can be formally described and generated with the help of structural tools of description of compositional $k-$images of combinatorial sets ($k-$sets), proposed in [7]. A combinatorial set is understood as a variety of tuples constructed from a finite set of arbitrary elements (generating elements) in accordance with certain rules [7, 8].

Permutations, combinations, arrangements, binary sequences etc. can serve as examples of classical combinatorial sets. The apparatus of $k-$sets has been widely explored [7–9]: general concepts of their generation are considered in [8], the task of exhaustive generation of $k-$sets was solved in [9], and some special cases of this task were studied in [10].

However, the task of random generation of $k-$sets hasn't been solved yet.

Both random and exhaustive generation of $k-$sets require solving the problem of generation of basic combinatorial sets used to construct $k-$sets. Basic sets can be combinatorial ones with known descriptions and algorithms of generation: either classical combinatorial sets (permutations, combinations etc.) or non-classical, e.g., permutations of tuples, compositions of permutations, permutations with given number of cycles etc. [7-12].

Algorithms of generation of many basic combinatorial sets have been described in many articles [1-3, 5, 13-21]. However, in most cases, each generation algorithm is based on specific properties of combinatorial sets.

OBJECTIVES

Objectives of this paper are:

1. Developing a general approach for solving the problem of random generation of *k*-sets.
2. Developing the algorithms of generation of *k*-sets have been described.
3. Evaluating the complexity of constructed algorithms.

MAIN RESULTS OF THE RESEARCH

Let us briefly remind mathematical description of compositional $k-$images of combinatorial sets ($k-$sets).

COMPOSITION *K*-IMAGES OF COMBINATORIAL SETS (K-SETS)

Let $z^{\beta}=\{z_1^{\beta}, z_2^{\beta},..., z_{n_\beta}^{\beta}\}\subseteq \mathbf{Z}_{\boldsymbol{\beta}_i}$, where $\mathbf{Z}_{\boldsymbol{\beta}_i}$ are sets of arbitrary elements,

$$\beta\in\boldsymbol{\beta}_i,\ i\in J_k^0=\{0,1,...,k\},$$

where:

$$\begin{gathered}\boldsymbol{\beta}_0=\{0\},\ \boldsymbol{\beta}_i=\{\beta_j, j=1,2...,\eta_i\},\\ \beta_j=(\alpha_1,\alpha_2,...,\alpha_i),\ \alpha_1\in J_n,\\ \alpha_2\in J_{n_{\alpha_1}},...,\alpha_i\in J_{n_{\alpha_1...\alpha_{i-1}}},\\ \eta_1=n,\ \eta_2=\sum_{j=1}^{n} n_j,\\ \eta_i=\sum_{\alpha_1=1}^{n}\sum_{\alpha_2=1}^{n_1}...\sum_{\alpha_{i-1}=1}^{n_{1...i-2}} n_{\alpha_1...\alpha_{i-1}},\ i=3,4,...,k.\end{gathered}\tag{1}$$

Let us consider mappings [7, 8]:

$$\Gamma_{\boldsymbol{\beta}_0}:\mathbf{Z}_{\boldsymbol{\beta}_0}\to\mathbf{Y}^0,\ \Gamma_{\boldsymbol{\beta}_i}:\mathbf{Y}^{i-1}\times\mathbf{Z}_{\boldsymbol{\beta}_i}\to\mathbf{Y}^i,$$

where:

$$\begin{gathered}\mathbf{Y}^0=\{Y_{\boldsymbol{\beta}_0}(z^{\beta}),\beta\in\boldsymbol{\beta}_0\},\\ \mathbf{Y}^i=\{Y_{\boldsymbol{\beta}_i}(Y_{\boldsymbol{\beta}_{i-1}},z^{\beta}),\beta\in\boldsymbol{\beta}_i\},\ i\in J_k,\\ J_t=\{1,2,...,t\},\end{gathered}$$

$$Y_{\boldsymbol{\beta}_i}=\Gamma_{\boldsymbol{\beta}_i}(Y_{\boldsymbol{\beta}_{i-1}},z^{\boldsymbol{\beta}_i})=F(Y_{\boldsymbol{\beta}_{i-1}},\tilde{\Gamma}_{\boldsymbol{\beta}_i}(z^{\boldsymbol{\beta}_i})),\ i\in J_k,$$

$F(Y_{\boldsymbol{\beta}_{i-1}},\tilde{\Gamma}_{\boldsymbol{\beta}_i}(z^{\boldsymbol{\beta}_i}))$ – a mapping that realizes n-composition which consists in replacing each generative element of set $Y_{\boldsymbol{\beta}_{i-1}}$ with elements of primary combinatorial sets $Y_\beta=\tilde{\Gamma}_\beta(z^\beta),\ \beta\in\boldsymbol{\beta}_i$, respectively,

$$\tilde{\Gamma}_{\boldsymbol{\beta}_i}(z^{\boldsymbol{\beta}_i})=(\tilde{\Gamma}_\beta(z^\beta),\ \beta\in\boldsymbol{\beta}_i),\ z^{\boldsymbol{\beta}_i}=(z^\beta,\ \beta\in\boldsymbol{\beta}_i),$$

$\tilde{\Gamma}_\beta(z^\beta)$ are the primary mappings [7, 8]. It means that:

$$\begin{gathered}(z_{l_1}^{\beta},z_{l_2}^{\beta},...,z_{l_\beta}^{\beta})\in Y_\beta,\ z_{l_t}^{\beta}\in Y_\delta,\ l_t\in J_{l_\beta},\\ \beta\in\boldsymbol{\beta}_{i-1},\ \delta\in\boldsymbol{\beta}_i,\ i\in J_k.\end{gathered}$$

Let us denote

$$\boldsymbol{\Gamma}_i=\{\Gamma_{\boldsymbol{\beta}_i}\},\ i\in J_k.\tag{2}$$

Definition. Composition *k*-image of combinatorial sets

$$\begin{gathered}Y_0,\ Y_1,Y_2,...,Y_n,\\ Y_{11},Y_{12},...,Y_{1n_1},\ ...,\ \underset{k}{Y_{1...1}},...,Y_{nn_1...n_{k-1}}\end{gathered}$$

($k-$set) generated by sets z^{β_k}, $\beta_k\in\boldsymbol{\beta}_k$ is the combinatorial set [7,8]:

$$W_z=\Gamma_k\circ\Gamma_{k-1}\circ...\circ\Gamma_0(z),\tag{3}$$

where: mappings $\Gamma_i\in\boldsymbol{\Gamma}_i$, $i\in J_k$ are determined by (2).

Cardinality of the set (3) can be obtained [7, 8] as:

$$Card(W_z)=\sum_{\{\gamma_1,\gamma_2,...,\gamma_r\}\subset J_n}\alpha_{\gamma_1,\gamma_2,...,\gamma_r}\cdot\prod_{i=1}^{k}\prod_{\beta_i=(\alpha_1\alpha_2...\alpha_i)} Card\ Y_{\beta_i}\tag{4}$$

As the generation of $k-$sets is based on the generation of basic combinatorial sets, we need an algorithm for generating these sets.

GENERATION OF BASIC COMBINATORIAL SETS

Let us associate the set

$$p(T)=\{A,m,S\}$$

with each basic combinatorial set T, where

$$A=\{a_1,a_2,...,a_n\},\ a_1<a_2<...<a_n$$

is a set of generating elements, m is the length of tuple $t\in T$ (let us consider that all tuples in the set are of the same length), S is a set of parameters describing the set T (e.g., parameters $n_1,n_2,...,n_k$ for permutations with repetitions and other parameters specific for different classes of combinatorial sets). We understand a class of combinatorial sets as its belonging to permutations, combinations, etc.

Let the basic set T and its parameters $p(T)$ be defined. We need to generate all elements $t \in T$, where each element is a tuple of the length m. Let us denote

$$t^i = (t_1, t_2, ..., t_i),\ \forall t_i \in A,\ A \in p(T),\ i \in J_n.$$

It means that $t^0 = (\ )$ is an empty tuple and $t^m = t \in T$.

Firstly, let us recall the main concept of the algorithm of random generation of basic sets [9]. This algorithm is of a recursive nature: at each recursion level $i \in J^0_{m-1}$ it expands current tuple $t^i = (t_1, t_2, ..., t_i)$ by adding the following element t_{i+1} and thus obtaining a tuple $t^{i+1} = (t_1, t_2, ..., t_{i+1})$ at level $i+1$. At level $m \le n$ the algorithm adds tuple $t^m = t$ to the resulting set *T*.

The fact that set *T* belongs to a certain class of combinatorial sets imposes some restrictions on element $t_{i+1} \in A$.

At each level $i \in J^0_{m-1}$ let

$$F^i = \{f_1, f_2, ..., f_k\} \subseteq A$$

denote a set of all generating elements that satisfy these restrictions. In this case, for each $j \in J_k$, the algorithm adds element $t_{i+1} = f_j$ to the "input" tuple

$$t^i = (t_1, t_2, ..., t_i)$$

and calls itself recursively with

$$t^{i+1} = (t_1, t_2, ..., f_j)$$

as an input.

The described algorithm can generate elements of set *T* randomly: for that we just need to do a recursive algorithm call at level i not for all $j \in J_k$, but only for q_i % of them (selected randomly). It should be noted that we can use various mechanisms of random selection (in this work we use the simplest way of selection: we just generate a uniformly distributed random value; though other distributions can also be used).

Let us consider specific features of the construction of set F^i for some classes of combinatorial sets. In the case of arrangements with repetitions, set F^i consists of all generating elements:

$$F^i = A.$$

For arrangements without repetitions and permutations as their special case, set F^i consists of $n-i$ generating elements that have not appeared in t^i:

$$F^i = \{f_1, f_2, ..., f_{n-i}\} \subseteq A: f_l \ne t_j,\ \forall j \in J_i, \forall l \in J_{n-i}.$$

As for the combinations the order of the items is not important, let us generate them in the form of ordered sets where $t_1 < t_2 < ... < t_i$ is for combinations without repetitions and $t_1 \le t_2 \le ... \le t_i$ is for combinations with repetitions.

For combinations without repetitions, set F^i includes all generating elements that have not been included into t^i and are greater than t_i:

$$F^i = \{f_1, f_2, ..., f_k\} \subseteq A: f_l \ne t_j,\ f_l > t_i,\ \forall l \in J_k, \forall j \in J_{i-1}.$$

For combinations with repetitions, set F^i also includes generating element which is equal to t_i:

$$F^i = \{f_1, f_2, ..., f_k\} \subseteq A: f_l \ne t_j,\ f_l \ge t_i,\ \forall l \in J_k, \forall j \in J_{i-1}.$$

Let us describe the algorithm **GenBase** that implements the steps described. **GenBase** input data consist of T - type of a basic set, a set of parameters $p(T)$ and tuple t^i. At each recursion level $i \in J^0_{m-1}$, the algorithm builds the set F^i, then randomly adds selected element F^i to tuple t^i as many as v_i times and recursively calls itself. At each level, the number of recursive calls is determined by

$$v_i = \frac{|F^i| \cdot q_i}{100}.$$

To generate all the elements of set T, **GenBase** should be called with parameters T, $(\ )$, $p(T)$.

Here *random*$(1, |F^i|)$ is a uniformly distributed random number between 1 and $|F^i|$. In order to generate combinatorial sets of other classes with this algorithm (Fig. 1), it is sufficient to identify laws of constructing set F^i.

function GenBase(T , t^i , $p(T)$);

local F^i;

if $i = m$ **then** $T := T \cup t^i$; **exit**;
end if;
case T **of**

$\overline{A}_n^m : F^i = A$;

$A_n^m : F^i = \{f_1, f_2, ..., f_{n-i}\} \subseteq A : f_l \neq t_j, \forall j \in J_i, \forall l \in J_{n-i}$;

$\overline{C}_n^m : F^i = \{f_1, f_2, ..., f_k\} \subseteq A : f_l \neq t_j, f_l \geq t_i, \forall l \in J_k, \forall j \in J_{i-1}$;

$C_n^m : F^i = \{f_1, f_2, ..., f_k\} \subseteq A : f_l \neq t_j, f_l > t_i, \forall l \in J_k, \forall j \in J_{i-1}$;

T_n : **exit;**

end case;

$$v_i = \frac{|F^i| \cdot q_i}{100};$$

for $j = 1, 2, ..., v_i$ **do**

GenBase($t^{i+1} = (t_1, t_2, ..., t_i, f_{random(1,|F^i|)})$);

end for;
end function;

Fig. 1. GenBase algorithm

It should be noted that, to generate combinatorial sets of other classes with this algorithm, it is sufficient to identify laws of constructing set F^i.

Example 1. Let the task is to randomly generate arrangements of 4 to 2. Let $q_0 = 75,\ q_1 = 60$. At zero level, set F^0 includes all generating elements: $F^0 = \{1, 2, 3, 4\}$. Hence, $v_0 = \frac{4 \cdot 75}{100} = 3$. The algorithm randomly selects three elements F^0 (let them be 3, 2 and 4) and makes recursive calls of itself with $t^1 = (3),\ t^1 = (2),\ t^1 = (4)$ respectively.

Let us consider the situation at level 1 for $t^1 = (3)$. In this case, set F^1 includes all elements that have not been not included into t^1, i.e. $F^1 = \{1, 2, 4\}$. Hence, $v_1 = \frac{3 \cdot 60}{100} \approx 2$. The algorithm randomly selects two elements F^1 (let them be 1 and 4); it means that, at level 2, full arrangements (31) and (34) are obtained.

A possible recursion tree of the algorithm operation is provided below (Fig. 2).

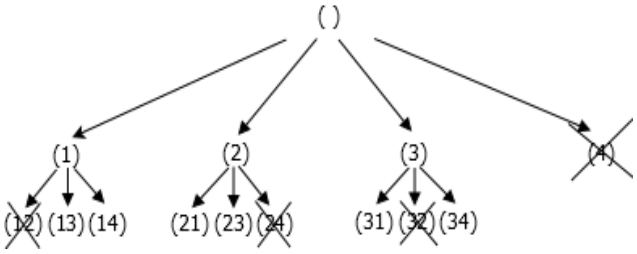


Fig. 2. Example results of **GenBase** algorithm

GENERATION OF *K*-SETS

First, let us briefly describe the algorithm of exhaustive generating *k*-sets [9]. At the beginning, it generates the elements of each basic set using **GenBase** algorithm (values of q_i are individual for each basic set). After that, the algorithm sequentially implements mappings

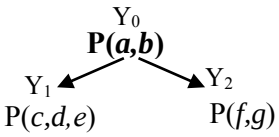
$$\Gamma_{i+1} \circ \Gamma_i \circ .. \circ \Gamma_0(z),\ z = A_0 \in p(Y_0)$$

for each $i \in J_{k-1}^0$. In other words, it implements n-composition, where generating elements of a parent set are replaced by the elements-tuples of its child sets.

Algorithm Get_k-set in its original version in [9], while implementing an operation of *n*-composition, performs a sequential substitution of the element of a parent set by each element of a child set. The proposed algorithm Get_Random_k-set does it not for all, but for some elements of a child set. By analogy with the generation of basic sets, we can put the parameter *Q(Y)* for each basic set *Y* at level $i \in J_k$, which defines the fraction of elements participating in the operation of n-composition. The number of the elements is determined by

$$V(Y) = \frac{|Y| \cdot Q(Y)}{100}.$$

Example 2. Let us describe the generation of the composition of permutations by means of **Get_Random_k-set**. The structure of the corresponding *k*-set can be presented as follows:



The elements "*a*" and "*b*" of set $Y_0 = \{(ab), (ba)\}$ are replaced by the elements of child sets:

$$Y_1 = \{(cde), (ced), (dce),\ (dec), (ecd), (edc)\}$$

and

$$Y_2 = \{(fg), (gf)\},$$

respectively.

Let

$$Q(Y_1) = Q(Y_2) = 50.$$

Hence, $V(Y_1) = 3;\ V(Y_2) = 1$, i.e. element "*a*" is sequentially replaced by three random elements of Y_1, and "*b*" – by one random element of Y_2.

A possible results of the generation is

$$W_z = \{\underbrace{(cdefg),(dcefg),(ecdfg)}_{result\ of\ replacing\ (ab)},$$
$$\underbrace{(gfced),(gfdec),(gfedc)}_{result\ of\ replacing\ (ba)}\}.$$

THE EVALUATION OF THE COMPLEXITY OF ALGORITHM GET_RANDOM_K-SET

The complexity of the algorithm is determined by the complexity of generation of basic sets, as well as the complexity of operations of *n*-substitution and a number of levels of a certain *k*-set. In [9], the evaluation of the complexity of the «full» algorithm **Gen_k-set** is described. This formula can also be used in this case. It is sufficient to replace cardinality *Card(Y)* of each basic set by *V(Y)*. Then, the formula to evaluate **Get_Random_k-set** complexity is:

$$\sum_{i=0}^{k}\sum_{j=1}^{\eta_i} O(Y_{ij}) + \sum_{i=0}^{k-1}(Card(P^i)\prod_{j=1}^{\eta_{i+1}} V(Y_{(i+1)j})),$$

where: $O(Y_{ij})$ is the complexity of generating a basic set Y_{ij} ($\boldsymbol{i}$ is the level of the basic set in *k*-set, $\boldsymbol{j}$ is the sequence number of the set at level $\boldsymbol{i}$); $P^i = \Gamma_i \circ \Gamma_{i-1} \circ ... \circ \Gamma_0(z)$ – "intermediate" *k*-set, which is the parent set at level $\boldsymbol{i}$.

Details about designations and obtaining this formula are given in [9].

CONCLUSIONS

1. We described the general approach for solving the problem of random generation of $k-$sets based on the single approach to random generation of various basic combinatorial sets.

Algorithm for random generation of basic sets allows generating various combinatorial sets, and laws of constructing sets F^i can be pre-set. If identification of the laws fails, the algorithm allows using other algorithms to generate basic sets.

2. The described approach to the random generation is very flexible since it allows obtaining various results by varying parameters q_i and *Q(Y)*. In its turn, this allows adjusting the number of elements for both basic sets and *k*-sets.

3. The developed software allows solving the described problems of generation of *k* -sets and basic sets.

REFERENCES

1. Knuth D., 2005. The Art of Computer Programming, Volume 4, Fascicle 2: Generating All Tuples and Permutations. Addison-Wesley. – 144.

2. Knuth D., 2005. The Art of Computer Programming, Volume 4, Fascicle 3: Generating All Combinations and Partitions. Addison-Wesley.–160.

3. Kreher D., Stinson D., 1999. Combinatorial Algorithms: Generation Enumeration and Search. CRC Press. – 329.

4. Bona M., 2004. Combinatorics of Permutations. Chapman Hall-CRC. – 383.

5. Ruskey F., 2009. Combinatorial generation, Dept. of Computer Science Univ. of Victoria, Canada, 1j-CSC 425/520. – 289.

6. Korsh J., LaFollette P., 2004. Loopless array generation of multiset permutations // The Computer Journal. – Vol. 47, № 5. – 612–621.

7. Stoyan Y., Grebennik I., 2008. Description of Classes of Combinatorial Configurations by Mappings // Proc. of NAS of Ukraine.– 10. - 28-31. (in Russian).

8. Stoyan Y., Grebennik I., 2011. Description and Generation of Combinatorial Sets Having Special Characteristics // International Journal of Biomedical Soft Computing and Human Sciences. Special Volume "Bilevel Programming, Optimization Methods, and Applications to Economics" Vol. 18, №1. – 85-90.

9. Grebennik I., Lytvynenko O., 2012. Generating combinatorial sets with given properties // Cybernetics and Systems Analysis, 48(6). – 890–898.

10. Grebennik I., 2010. Description and generation of permutations containing cycles // Cybernetics and Systems Analysis, 46(6). – 945-952.

11. Grebennik I., Chorna O., 2015. Elements transpositions and their impact on the cyclic structure of permutations. // ECONTECHMOD. An International Quarterly Journal on Economics of Technology and Modelling Processes. – Vol. 4, № 3. – 33–38.

12. Grebennik I., Baranov A., Chorna O., Gorbacheva E., 2016. Optimization of linear function of a cyclic permutation based on the random search. // ECONTECHMOD. An International Quarterly Journal on Economics of Technology and Modelling Processes. – Vol. 5, № 3. – 211–216.

13. Bauslaugh B., Ruskey F., 1990. Generating alternating permutations lexicographically // BIT Numerical Mathematics. 30, 17–26.

14. Poneti M., Vajnovszki V., 2010. Generating restricted classes of involutions, Bell and Stirling permutations // European Journal of Combinatorics. 31, 553–564.

15. Bergeron F., Labelle G., Leroux P., 1998. Combinatorial Species and Tree-Like Structures. University Press, Cambridge.

16. De Bruijn N., 1970. Permutations with given ups and downs // Nieuw Arch. Wisk. 18 (3), 61–65.

17. Carlitz L., 1973. Permutations with prescribed pattern // Math Hachr, vol. 58, No. 1-6, 31–53.

18. Viennot G., 1979. Permutations ayant une forme donne // Discrete Mathematics. 26, 279–284.

19. Etienne G., 1984. Linear extensions of finite posets and a conjecture of G. Kreweras on permutations // Discrete Mathematics. 52, 107–112.

20. Shevelev V., 2012. The number of permutations with prescribed Up-down structure as a function of two variables // Integers. Volume 12.

21. Van Baronaigien R., Ruskey F., Ruskey D., 1992. Generating permutations with given ups and downs // Discrete Applied Mathematics. 36, 57–65.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 49-54

DEVELOPMENT OF DEEP-WATER CONCRETIONS BY A SPIRAL METHOD

M. Sukach

Kyiv National University of Construction and Architecture; e-mail: msukach@ua.fm

Received October 11.2016: accepted November 23.2016

Abstract. These features occurrence and distribution of deep-sea nodules. A development of the bottom of fossil spiral manner. Scheme is shown floating and complex trajectory carriage bottom of the unit. The basic technological calculation unit that moves around the base module and given recommendations on its use.

Key words: floating complex, iron-manganese minerals, spiral method of extraction, trajectory of motion, hose-cable.

INTRODUCTION

The researchers conducted by oceanologists and marine geologists in the past decade have revealed patterns of occurrence of nodules based on landforms, direction of flow, sediment character, hydro chemical conditions and other factors [1, 2]. Usually deposits are elongated shape with compact placement areas where the concentration of nodules in 2...3 times the average [3 – 6]. Nodule deposits are characterized in terms of patchy structure with discrete zones of high placement concentrations [7, 8]. The above makes it necessary to justify the methods of ocean exploration, technology testing, research and industrial mining exploitation [9, 10].

PURPOSE OF WORK

In order to save of resources need to set up a floating complex in such a way that the original data obtained in the pilot mining exploration mining complex would be representative for complex industrial exploitation.

ANALYSIS METHODS OF EXTRACTION

The main element of the exploration mining complex for concretion mining, given the significant depth of development (4...6 km), is a system of concretions raising [11, 12]. Based on the results of tests in natural conditions, conducted by leading foreign companies, to be implemented in the first phase accepted as the most simple pipeline system recovery or airlift pump type [13]. Known technical solution patented by "Lockheed" (USA), which includes craft; descent gear unit pipeline; bottom platform located on its lower end; hose-cable flexible communication and data collection unit [14].

In such a system is extremely difficult to manage the orderly systematic treatment of a loose plot, given the significant difference in the length of the pipeline (6 km) and the width of the working body of the unit fee (a few meters), the masses craft (several hundred thousand tons) and aggregate collection (several tens of tons), the power of influence of natural factors (wind, flow rates of up to 2 m/s) on the pipeline during its movement by floating means (the pressure is several dozens, hundreds of tons) and the aggregate fee (perturbation effects turbid bottom flows on submarine slopes inhibitory effect of flexible communications and ground soil) [15 – 17].

Deprived of such deficiencies on short-term fixation of the lower end of the pipe when lowering the bottom of the bottom platform – bottom base module interacting with flexible communication with the bottom end of the pipeline and quickly movable carriage carrying the tool "carrier" absorption (analog – cleaner containing unit and dust extraction head, interconnected corrugated hose).

SPIRAL WAY NATURE

The proposed technical solution is shown in a design setup is shown in Fig. 1. Bottom base module, which fixes the lower end of the pipeline to the processing area deposits (unit) includes a guided coil connected through hose cable Actuated of carriage, the latter performs a circular motion round the spiral path by bottom base module (Fig. 2).

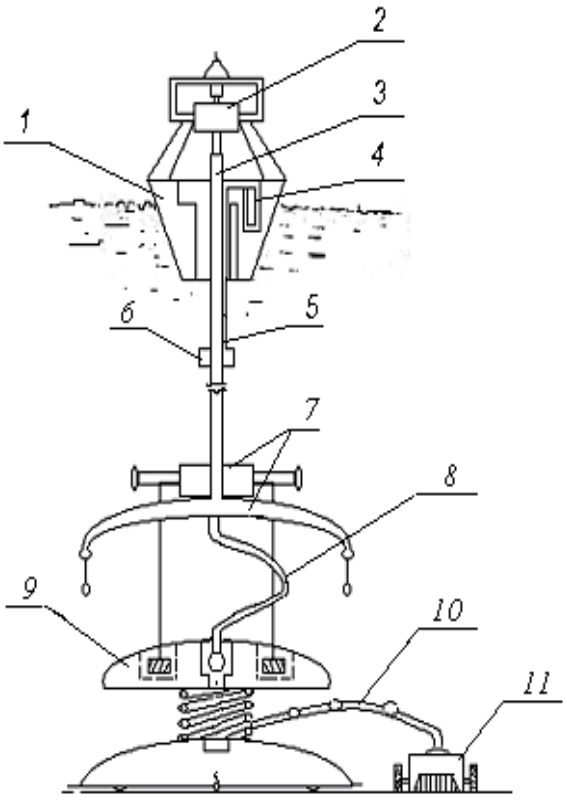


Fig. 1. Scheme complex floating:
1 – craft; *2* – descent gear unit; *3* – pipeline transport artery; *4* – compressor station (airlift system recovery (ESR) or diesel generator – with pumping system recovery (PSR)); *5* – piping for compressed air at ESR or cable at PSR; *6* – mixer (ESR), submersible pumps (PSR); *7* – bottom base module; *8* – flexible pipe; *9* – donna coil; *10* – hose cable; *11* – drive carriage

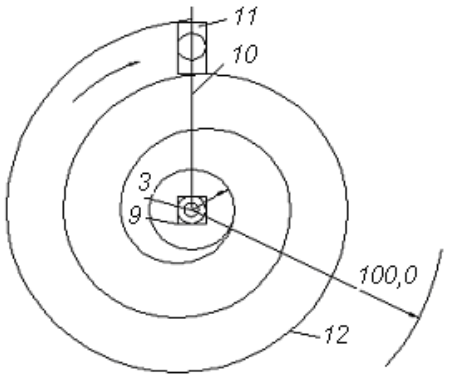


Fig. 2. Trajectory of motion of carriage: denotation *1 – 11* as on Fig. 1; *12* – carriage way

Efficiency of up moving in spiral trajectories with respect to the bottom of the coil is determined as follows (Fig. 3). Carriage with free automatic hose-cable rewinder hose cableya coil describes involute circle, defined by equations in polar coordinates in the parametric form:

$$\rho = R_0\left[1+(\varphi + l_0/R_0)^2\right]^{1/2},$$
$$\alpha = \varphi - \operatorname{arctg}(\varphi + l_0/R_0),$$

where: ρ – polar radius of the point of attachment to the carriage hose-cable; α – polar angle (phase) point of attachment; R_0 – radius of the coil; l_0 – length of the initial segment hose-cable, pulled coil; φ – phase separation of the coil hose cable.

It is assumed that the value l_0 depends on the size of the foundations of the bottom coil, which prevents the carriage freely maneuver close to the coil; apparently, $l_0 \geq R_0$. It is also assumed that the width of the carriage is connected with coil radius ratio $b = 2\pi R_0$ to ensure complete overlap of the treated area between the two coils.

Square shape A_0aA_1 (uncultivated space inside the helix) S_0 consists of the area covered by the polar radius ρ from the point A_0 to point A_1 when deploying hose-cable ($S_0A_0aA_1$), and the area of a triangle ($S_0A_0A_1$):

$$S_0 = S_0A_0aA_1 + S_0A_0A_1,$$

where:

$$S_0A_0aA_1 = \frac{1}{2}\int_{\alpha_0}^{\alpha_1}\rho^2 a\alpha = \frac{1}{2}\int_0^{2\pi}\rho^2\alpha^1 a\varphi =$$

$$= \frac{1}{2}R_0^2\int_0^{2\pi}\left(\tau + \frac{l_0}{R_0}\right)^2 \alpha\tau = \frac{1}{6}\left(\tau + \frac{l_0}{R_0}\right)^3\Bigg|_0^{2\pi}.$$

Denoting:

$$S(\varphi) = \frac{1}{6}R_0^2\ (\varphi + l_0/R_0)^3,$$

we obtain:

$$S_0A_0aA_1 = SR_0^2 = S(\Delta),$$
$$S_0A_0aA_1 = S\left(2\pi - S_{(0)}\right),$$
$$S_0 = S[2\pi - S_0 + S(\Delta)].$$

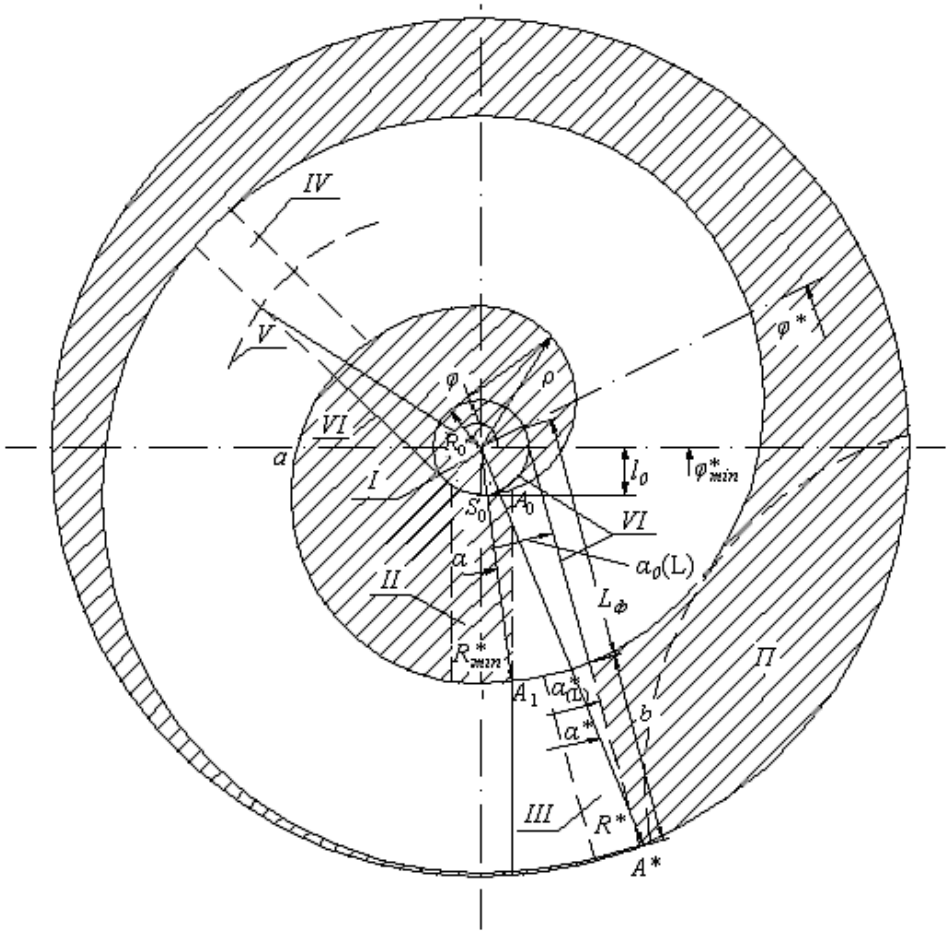


Fig. 3. Diagram spiral trajectory of up bound hose cable coil:
I – coil; *II* ... *IV* – caret position (*II* – original, *III* – end, *IV* – intermediate); *V* – trajectory of the carriage; *VI* – hose-cable; *Q* – overlay zone adjacent blocks

Let processed space is limited circle of radius R^*, associated with the maximum length hose-cable L_Φ, by ratio:

$$R^* = R_0\left[\left(L_\Phi / R_0 + 2\pi\right)^2 + 1\right]^{1/2}.$$

Limit setting R^* will be determined from the expression:

$$\varphi^* = \left(L_\Phi - l_0\right)/R_0 + 2\pi.$$

Note: $\varphi^*_{\min} = 2\pi$, corresponding $R^*_{\min}$. Therefore, the number of complete rotations a carriage:

$$K^* = \left[\varphi^*/2\pi - 1\right].$$

The area that is maintained carriage, in which case:

$$S = \frac{1}{2}\int_{\tau^*-2\pi}^{\tau} \rho^2 \alpha^1 a\tau + S_\Delta - S_0 =$$
$$= S(\tau^*) - S(\tau^* - 2\pi) - S(2\pi) + S_{(0)}.$$

The area of a circle of radius R^*:

$$S^* = \pi R^{*2}.$$

Percentage of area processed,

$$E = \left(S/S^*\right)\cdot 100\,\%.$$

Trajectory, which makes the center of the carriage (see Fig. 2), is described by the equations:

$$\rho = R_0\left[1+\left(\varphi+\left(l_0/R_0\right)+\pi^2\right)\right]^{1/2},$$
$$\alpha = \varphi - \operatorname{arctg}\left(\varphi+\left(l_0/R_0\right)+\pi\right).$$

The path of movement of the carriage:

$$L = \frac{1}{2}\int_{\alpha_0(L)}^{\alpha_L^*}\rho^2 a_X = \int_0^{\varphi^*-2\pi}\rho\ \alpha^1 a\varphi =$$
$$\int_0^{\tau^*-2\pi}\frac{\left(\varphi+\frac{l_0}{R_0}+\pi\right)^2}{\sqrt{1+\left(\varphi+\frac{l_0}{R_0}+\pi^2\right)}}\alpha\varphi.$$

Denoting:

$$z = \varphi+\left(l_0/R_0\right)+\pi,$$

we obtain:

$$L = R_0\int_{\frac{l_0}{R_0}+\pi}^{\tau^*+\frac{l_0}{R_0}-\pi}\frac{Z^2}{\sqrt{1+Z^2}}aZ =$$
$$= \frac{1}{2}R_0\left[Z\sqrt{Z^2+1}-\lg\left(Z+\sqrt{Z^2-1}\right)\right].$$

Then:

$$L = L\left(\varphi^*+\left(l_0/R_0\right)-\pi\right)-L\left(l_0/R_0\right)+\pi.$$

Specific treatment area, id the area that is treated as a unit path traversed by carriage:

$$D = S/L.$$

Using the computer program calculated the values R^*, φ^*, K^*, S, S^*, E, L, D for different lengths of hose-cable (L_Φ = 100...200 м) and radiuses of the coil (R_0 = 0,2...1 м) for a given width of the working body (B = 2,0...10 м) and the length of the initial segment of hose-cable, pulled by coil (l_0 = 1...5 м). According to our calculations, built feasibility dependency mining company (Figs. 4, 5).

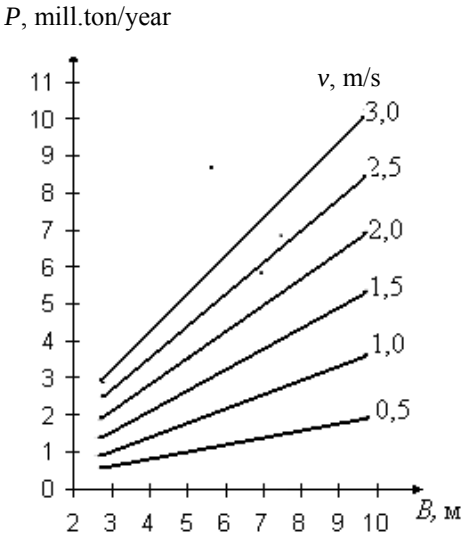


Fig. 4. Graph of the performance of the enterprise P from the width B and speed v of pickup

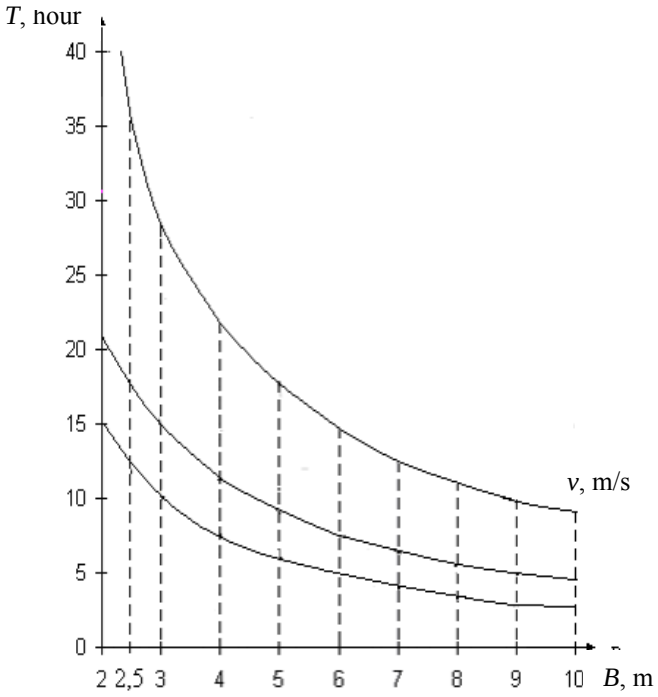


Fig. 5. Dependence term treatment areas T from the width B and speed v of pickup

GUIDELINES FOR POOL DEVELOPMENT

Move the bottom platform – bottom base module in the next section area (block) is at a distance equal to twice the length hose cable. Thus,

the deposits are processed by successive overlap of circular blocks in circuits where excavation is performed spiral (Fig. 6). The variant of the carriage passes through it forward and in reverse on the inside back-spiral.

To improve the efficiency of nodules slot scheme can be used with the trajectory of the carriage for reciprocal trajectories with a fixed rotation angle (210, 240, 270 ° etc.) by operation of reversing switches or program laid down in the onboard memory computer bottom base module. Thus skate chassis up (caret) must be reversed at a small angle (~ 7 °) at the outer side of the coil to create a constant tension in hose-cable that has neutral or slightly positive buoyancy.

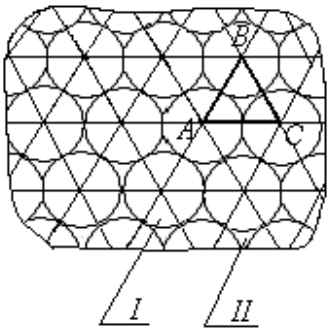


Fig. 6. Scheme working in field plots spiral circular trajectory carriage: *I* – processed, *II* – uncultivated land

Moving the complex area of the new unit should be carried out by the pendulum movement of the lower end of the pipe at elevated above the bottom of the base module at a time when the craft is in the distance between the centers of adjacent range blocks (when included with the drivers, taxiing). More economic office hours – at moving of floating means taking into account influence of superficial flows and wind without including of main propulsion engines.

Lowering the bottom of the module should be performed at the time of stay of the craft center of a new unit (including the effects Angle undercurrents in trubop-rovid). Control mode switching engines, taxiing, carried out by the display location of the bottom base module based on signals received from the bottom sonar beacons.

As can be seen from the graph in Fig. 4 and productivity of industrial enterprises 1...3 million tons of nodules per year can be achieved with acceptable parameters of the speed range up to v = 1...2 m/s and the width of the working body B = 2,5…3 м. And working land area 0,2 км^2 depending on these parameters extended from a few hours to 1,5 days (see Fig. 5). Losses in between a block areas can be substantially reduced by overlapping the blocks in slot pitch contours, for example in the area of "*II*" (see Fig. 3).

CONCLUSIONS

1. A block diagram of practicing deep deposits of ferromanganese nodules sedentary set of fixed at the bottom of the base module and quickly by a movable collector. The latter has a coordinating communication via hose cable with a drive carriage and performs a circular motion around the base module on a spiral trajectory.

2. Productive area of minerals in circular overlapping blocks, where excavation occurs spiral steps. Moving to the next set of traffic control unit and a collector carried by the installed program automatically includes data from sonar beacons.

REFERENCES

1. **Terms** of education and conformity to law of placing of ferrimanganese concretions of the World ocean, **1987**. Edited O.D. Korsakov. Moscow, Nedra. 259.

2. **Ziborov A., 2008**. Prospects and tasks of mastering of marine deposits of mineral raw material. Geology and minerals of the World ocean. Vol.3, 5-18.

3. **Schniukov E., Ziborov A., 2004.** Mineral riches of the Black sea. Kyiv, NANU. 279.

4. **Sukach M., 2012**. Problems of booty of hard minerals from the bottom of the World ocean. Motrol: kom. Mot. Energ. Roln. OL PAN. Vol.14. No 1, 116-122.

5. **Sukach M., 2013.** Prospects of development of marine deposits of mineral raw material. Innovations in science are innovations in education: collection of labours. JuRGPU (NPI). Novocherkassk. 285-291.

6. **Sukach M., 2013**. Ukraine's prospects in development of marine mineral deposits. Econtechmod. An International Quarterly Journal. Vol. 2, No 2. 47-50.

7. **Ferromanganese** concretions of the central part of the Pacific ocean, **1986.** Under editing I.O. Murdmaa, N.C. Skorniakova. Moscow, Nauka, 344.

8. **Fouco I., 1982.** Modern state of technique of booty of deep-water ferromanganese concretions. WCP, № E- 41418. Moscow, 28.12.83, ist. Fusen, Vol. 29. 99-167.

9. **Sukach M., 2012**. Deep-water technique and technology for development of minerals of the

World ocean. Labours VI ISTK "Energia-2012". Simferopol-Alupka. 9.

10. **Ziborov A., 2006.** To obtain or not obtain the black Sea sapropels in Ukraine. Geology and minerals of the World ocean. Vol. 1. 92-99.

11. **Bakurov G., 1988.** Ships for deep-water mountain geological researches. Moscow, Shipbuilding. No 3. 12-16.

12. **Paschkin V., Iakovlev P., Cokolov V., 1999**. Dredging, refuliring- and gidromechanized works: train aid. Odesa, Astroprint. 432.

13. **Nurok G.A., 1985**. Processes and technology of hydromechanization of open mountain works. Moscow, Nedra. 471.

14. **Patent** of the USA № 4232903. **1980**.

15. **Sukach M., 2004.** Workflows of deep machines. Kyiv, Naukova dumka. 364.

16. **Sukach M., Lysak S., Sosnowski S., 2010**. Kinematic analysis of the working process of trencher. TEKA: kom. Mot. Energ. Roln. OL PAN. Vol.10. 425-431.

17. **Sukach M., Lytvynenko I., Bondar D., 2009**. System of the automated management and measuring of parameters of technological processes. Motrol: kom. Mot. Energ. Roln. OL PAN. Vol.11B, 186-189.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 55-60

INFORMATION TECHNOLOGY FOR TREATMENT OF RESULTS EXPERT ESTIMATION WITH FUZZY CHARACTER INPUT DATA

Yu. Ulianovskaya

University of Customs and Finance, Dnipro, Ukraine; e-mail: uyv@rambler.ru

Received September 23.2016: accepted November 24.2016

Annotation. The questions of processing expert information provided by the group of experts are considered. In the treatment group of expert opinion there is a problem obtaining a generalized result. It is assumed that the information is a set of qualitative and quantitative features (alternatives), described by linguistic concepts. Statistical methods of expert's data processing are quite complicated when the expert's answers have form of ranking or separation, and quite simple, if the answers are the results of independent pairwise comparisons. In this article are proposed to carry out processing of expert information by methods of the fuzzy sets theory. Using this theory are developed a method for determining the qualifications of the expert based on his length of service and number of expertise conducted by him, the results of which are accurate. The developed method is based on the construction of a system on fuzzy logic with fuzzifikator and defuzzifikator. For processing expert's estimates are suggested each alternative presented in the linguistic variable form and evaluate it by assigning a group of experts of membership functions each term by the direct method. Obtaining a generalized assessment based on all expert's estimates going on with regard to their competence. In this paper a method of ranking fuzzy alternatives are propose. Based on the developed methods of data processing was designed an automated system that allows to determine the experts qualification, generalized result of expert group evaluation and of ranking alternatives. The developed technology is applicable to any subject area, where it is necessary to analyze alternatives for many of the criteria based on processing of expert estimations.

Key words: expert information, linguistic variable, fuzzy logic, ordering alternatives, qualification of experts, information technology.

INTRODUCTION

Decigion making by man in most areas of activity in which he has some knowledge and experience, requires a deep understanding of various properties of the considered system. Necessary information may be absent or its getting too expensive, then decigion making takes place with the use of intelligent decision support systems [1, 2].

In these systems, to analyze and propose different methods of data mining, knowledge discovery in databases and knowledge, and other techniques developed within artificial intelligence [3, 4]. Systems built by combining the databases and fuzzy logic can significantly extend the functionality and range of tasks of data mining. The theory of fuzzy databases is not yet complete from a mathematical point of view, and there are still many issues that require resolution [5]. The formation of databases intelligent systems takes place on the basis of expert information. Much attention is paid to the problem of the choice of methods for presenting expertise and processing this information [6-7]. At the same time there is a problem processing a group of expert opinions and getting the generalized results of the expert assessment. Improper use of the results of expert assessment can lead to erroneous conclusions.

Expert information often is a combination of qualitative and quantitative traits can be expressed in terms of a natural language, can possess incompleteness and unclearness. Classical statistical methods does not always give an adequate result, when applied to this type of data. In particular, problems arise when trying to averaging the opinions of experts. Statistical methods of processing depend on the mathematical nature of the experts' answers. The choice of method is determined by the nature of the information in question.

Today there are many methods for processing quantitative expert data, as opposed to qualitative. Therefore, an important task is to combine the theory of classical processing methods quantitative expert information with processing techniques qualitative data.

Analyzing the above, we can conclude that in spite of the extensive development of methods of expert assessments, a number of issues related to the fuzzy expert estimations remain open. Not sufficiently developed methods for comparative assessment of expert evaluations, processing and obtaining the views of the group, as well as methods for ranking fuzzy alternatives when they are formulated as fuzzy linguistic concepts. The results obtained in the form of a generalized opinion must take into account the competence of the experts. With a large number of experts and a large variety of alternatives there is a necessity in the use of computer technology for the storage, retrieval and processing of expert information.

THE ANALYSIS OF RESENT RESEARCHES AND PUBLICATIONS

Processes application of computer and telecommunications equipment for the storage, retrieval, transmission and processing data forms information technology [8]. Let's consider the existing methods of experts' data processing. Methods of expert evaluations are scientific methods of analyzing complex problems. Experts conduct an intuitive and logical analysis of the problem with quantification of judgments and with the

formal processing of the results Their general opinion obtained as a result of the processing of individual assessments is adopted as a solution to the problem. To receive qualitative evaluations are used pairwise comparisons, multiple comparisons, ranking methods, etc. For obtain quantitative estimates are used direct numerical evaluation of the alternatives, method Churchman-Akoff and others [9-11]. These methods allow using the experience and knowledge of a person compensate the incompleteness of the information when obtaining it by other means is problematic, demands enough long period, and is quite expensive.

For information, expressed in terms of a natural language and having fuzziness, are developed the methods based on Bayesian probability, confidence coefficients, fuzzy logic and others [12, 13]. It is experimentally shown that fuzzy control gives better results compared with those obtained by conventional control algorithms. For example, fuzzy logic controllers used in various control systems, are an important application of fuzzy set theory. These controllers use the experts' integrated knowledge and describe them by linguistic variables and fuzzy sets [14].

Confidence to results of expert evaluation largely depends on the competence of the experts. This problem can be solved by using an interpolation process. The main idea of this calculation is the suggestion that the competence of the experts must be assessed on the degree of coordination of their evaluations with the group evaluation of objects [15]. The problem of definition of experts' competence can be solved on the basis of the axioms of unbiasedness. But this approach is very cumbersome to calculate and can be implemented only by using an automated system. It is not an absolute full as well as all other methods for assessing subjective characteristics [16].

OBJECTIVES

The aim of this work is to develop information technology for generalization of the results of the expert evaluation, which includes the processing method of fuzzy expert information with expert's qualifications for ranking alternatives according to several criteria and automated information system implementing the developed methods.

THE MAIN RESULTS OF THE RESEARCH

Suppose there are k experts $X = \{x_l\} = \{x_1, x_2, \ldots, x_k\}$. The experts were asked to evaluate a set of alternatives $A = \{a_i\} = \{a_1, a_2, \ldots, a_n\}$, determining their by values term–set $T = \{t_j\} = \{t_1, t_2, \ldots t_m\}$.

The expert in this case indicates the number of μ in the interval [0, 1], which characterizes the degree of compliance by alternative a_j to the selected term t_j. In this case, we get a discrete membership function, built by the direct method for the expert group. Expert opinions in this case may coincide or be not concordant. Consider the method received data processing.

In work [17] it is proposed to form the opinions of all experts, obtained by this method about each an alternative a_i, in form next matrix:

$$\mathrm{M}(a_i) = \begin{array}{c|ccc} & t_1 & \ldots & t_m \\ \hline x_1 & \mu_1^1(a_i) & \ldots & \mu_m^1(a_i) \\ x_2 & \mu_1^2(a_i) & \ldots & \mu_m^2(a_i) \\ \ldots & \ldots & \ldots & \ldots \\ x_k & \mu_1^k(a_i) & \ldots & \mu_m^k(a_i) \end{array} \quad (1)$$

where: $x_1, \ldots, x_k$ – experts, t_j - the term from term-set of linguistic values, describing the alternative, $\mu_j^l(a_i)$ - an membership function alternative a_i to the term t_j in accordance with the opinion of the expert x_l.

With this approach it is necessary to take into account the competence of the experts, however, in direct methods of building membership functions for the expert group there is one delicate question: who assigns weights to the experts estimates [13].

To solve this problem in work [20] a method was proposed for determining the competence of the expert (W) on the basis of its work experience (S) and frequency correct evaluation results (P) by means of constructing a system of fuzzy logic inference type Mamdani with fuzzifikator and defuzzifikator [18, 19].

A linguistic variable is defined by the five attributes:

$$\langle x, T, U, G, M \rangle,$$

where: x - a variable name; T - term-set, each element which (term) is represented as a fuzzy set on a universal set U; G - syntax rules, often in the form of a grammar, generating the name terms; M - semantic rules that define the membership functions of fuzzy terms supplies generated by syntactic rules of G [13, 18].

Let's consider variable S. We define it five attributes:

$$\langle s, T_s, U_s, G_s, M_s \rangle,$$

where: s - "work experience" (hereinafter referred - experience), $T_s = \{t_s^1, t_s^2, t_s^3\}$={"small", "medium", "high"}, U_s = {0, 1, ..., 50}, G_s - syntax rules, generating new terms with quantifiers "no", "very" and "more or less", M_s - trapezoidal membership function, an analytical expression for which has the following form:

$$\mu(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & d \le x \end{cases}, \quad (2)$$

where: the parameters $[a, b, c, d]$ has values t_s^1 = [-18, -2, 5, 20], t_s^2 = [5, 20, 30, 45], t_s^3 = [30, 45, 52, 68].

Consider variable *P*. We define it five attributes:

$$\langle p, T_p, U_p, G_p, M_p \rangle,$$

where: *p* - "frequency of positive results" (for simplicity - frequency), $T_p = \{t_p^1, t_p^2, t_p^3\}$ = {"rare," "medium frequency", "often"}, U_p = [0 ... 1], *G* - syntax rules, generating new terms with quantifiers "no", "very" and "more or less", M_s - trapezoidal membership function, an analytical expression for which is given by equation (1). The parameters [*a, b, c, d*] is t_p^1 = [-0.4, -0.1, 0.1, 0.4], t_p^2 = [0.1, 0.4, 0.6, 0.9], t_p^3 = [0.6, 0.9, 1.1, 1.6].

The output variable *W* define the five attributes:

$$\langle w, T_w, U_w, G, M_w \rangle,$$

where: *w* - level expert competence, U_w = [0,…,1], $T_w = \{t_w^1, t_w^2, t_w^3, t_w^4, t_w^5\}$ = {"low", "below average", "average", "above average", "high"}, G_w - syntax rules, generating new terms with quantifiers "no", "very" and "more or less", M_w - triangular membership function, an analytical expression for which has the following form:

$$\mu(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & c \le x \end{cases}, \qquad (3)$$

where: the parameters *a, c* - the ends of the carrier corresponding fuzzy set linguistic variable value, *b* - value, in which the membership function takes a value of 1. For the elements of the set T, these parameters have the following values:

t_w^1 : *a*=-0,25, *c*=0,25, *b*=0,
t_w^2 : *a*=0, *c*=0,5, *b*=0,25,
t_w^3 : *a*=0,25, *c*=0,75, *b*=0,5,
t_w^4 : *a*=0,5, *c*=1, *b*=0,75,
t_w^5 : *a*=0,75, *c*=1,25, *b*=1.

One of the main methods of knowledge representation in fuzzy logic systems are production rules that allow to get closer to the style of human thinking.

Usually, these production rules are given as operator logical expression IF – THEN:

$$\begin{aligned} &\text{IF } x_1 \in F_1{}^p \text{ and...} \\ &\text{and } x_n \in F_n{}^p \text{ THEN } y \in G^p, \end{aligned} \qquad (4)$$

where: the rule's condition (logical expression) is a statement about the content of the knowledge base, and the consequence (operator) suggests what we should do when this production rule is activated [14]; $F_1{}^p$ and G^p – fuzzy sets; $\bar{x} = (x_1, \ldots, x_n)^T \in X$ and $y \in Y$ – variable inlet and outlet, respectively; $p = \overline{1, m}$.

We form the fuzzy rule system in accordance with the expression (4):

$$\begin{aligned} &\Pi 1: \text{ IF } S = t_s^1 \text{ AND } P = t_p^1 \text{ THEN } W = t_w^1; \\ &\Pi 2: \text{ IF } S = t_s^1 \text{ AND } P = t_p^2 \text{ THEN } W = t_w^1; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\Pi 10: \text{ IF } S = t_s^3 \text{ AND } P = t_p^3 \text{ THEN } W = t_w^5. \end{aligned} \qquad (5)$$

The value of the output variable is found by using the method of defuzzification according to the center of gravity [13, 18, 21]. In the application of the proposed method with next input data: the experience of work is 34 year and the probability of a correct answer is 0.3, according to the method of defuzzification by center of gravity the expert's qualification are estimated by coefficient equal 0,488.

The weight of expert x_l is denoted by w_l, wherein $0 \le w_l \le 1$.

Generalized evaluation of the membership function of each term t_j for alternative a_i may be to compute with the help of expression:

$$\mu_{ij} = \frac{\sum_{l=1}^{k} w_l \mu_j^l(a_i)}{k}, i = 1, \ldots, n, j = 1, \ldots, m, \qquad (6)$$

where: k – number of experts , $\mu_j^l(a_i)$ is identified in matrix (1).

As each alternative has its own matrix (1) after its treatment with the expression (6) we obtain a representation of an alternative in the form:

$$a_j = \{< \mu_{j1} / t_1 >, < \mu_{j2} / t_2 >, \ldots, < \mu_{jk_j} / t_{k_j} >\}. \qquad (7)$$

Processing results for each of the alternatives defined in the form (7) can be grouped into the next matrix:

$$A_{proc} = \begin{array}{c|cccc} & t_1 & t_2 & \ldots & t_k \\ \hline a_1 & \mu_{11} & \mu_{12} & \ldots & \mu_{1k} \\ a_2 & \mu_{21} & \mu_{22} & \ldots & \mu_{2k} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_n & \mu_{n1} & \mu_{n2} & \ldots & \mu_{nk} \end{array} \qquad (8)$$

where: μ_i- affiliation function of alternative a_i to term t_j.

For each linguistic value t_j let's construct non-decreasing sequence alternatives $\{a_i\}$, such that:

$$\forall a_i \in \{a_i\}_{i=1}^n : a_i \ge a_{i+1} \leftrightarrow \mu_{ij} \ge \mu_{i+1\,j}. \qquad (9)$$

In constructed in this way sequence in the first place will be alternative a_i that has the maximum value membership function μ_{ij} to term t_j. Let's form matrix $KOL = \{kol_j(a_i)\}$, j=1,2,…,m, i=1,2,…,n with help of variable $kol_j(a_i)$ equal to the number of occurrences of alternative a_i in the j-th place in this sequence. Let's denote $kol^{\max}$ the maximum value of the variable $kol_i(a_i)$ in each column:

$$kol_j^{\max} = \max_i kol_j(a_i), \quad j = 1,\ldots,m. \qquad (10)$$

We arrange alternatives as follows. Through r_i denote rating alternatives a_i. Establish a one-to-one correspondence between the sequence $\{r_i\}$ and $\{a_i\}$ as follows:

$$r_i = a_i \leftrightarrow kol_j^{\max} = kol_j(a_i). \qquad (11)$$

Thus, it is necessary to repeat this comparison is not all alternatives will be ordered.

Let, for clarity, there are six alternatives $A = \{a_i\} = \{a_1, a_2, \ldots, a_6\}$ and six term-sets of linguistic values $T = \{t_1, t_2, \ldots, t_6\}$. Let's write this alternatives and values of membership functions in the next matrix:

$A =$

	t_1	t_2	t_3	t_4	t_5	t_6
a_1	0,5	0,6	0,7	0,8	0	0,1
a_2	0,4	0,2	0	0,3	0	0,4
a_3	0,1	0,5	0	0,2	0,9	0,6
a_4	0	0,1	0,9	0,3	0,2	0,5
a_5	0,7	0,2	0,2	0,3	0,4	0,7
a_6	0,1	0,9	0,4	0,3	0,8	0,6

Carry out a ranking for each of values of the term-set by ordering all alternatives in descending respective values membership function:

t_1: $< a_5 >, < a_1 >, < a_2 >, < a_3, a_6 >, < a_4 >$
t_2: $< a_6 >, < a_1 >, < a_3 >, < a_2, a_5 >, < a_4 >$
t_3: $< a_4 >, < a_1 >, < a_6 >, < a_5 >, < a_2, a_3 >$
t_4: $< a_1 >, < a_2, a_4, a_5, a_6 >, < a_3 >$
t_5: $< a_3 >, < a_6 >, < a_5 >, < a_4 >, < a_2, a_1 >$
t_6: $< a_5 >, < a_6, a_3 >, < a_4 >, < a_2 >, < a_1 >$

The numbers appearance of alternatives in each column are given in next matrix:

$KOL =$

	kol_1	kol_2	kol_3	kol_4	kol_5
a_1	1	3	0	0	2
a_2	0	1	1	2	2
a_3	1	1	2	1	1
a_4	1	1	1	1	2
a_5	2	1	1	2	0
a_6	1	3	1	1	0

After analyzing the resulting table, we order the alternatives in order to increase the rating. The strongest is the alternative a_5, because it takes the maximum value of the membership function for two terms ($kol_1 = 2$), so it has the highest rating, and is ranked first in ordering. Alternatives a_1 and a_6 have the same value of the variable $kol_2 = 3$, which indicates that these alternatives for three terms have value of membership function following after the maximum value. And for the one term they have maximal meaning. However alternative a_6 has one in a neighboring column to the right, therefore we assume it stronger than alternative a_1, and a_6 in the ranking stands before a_1.

After similar analysis of all columns of the matrix all alternatives will be arranged in order descending of values of the evaluated characteristics as follows:

$$a_5,\ a_6,\ a_1,\ a_3,\ a_2,\ a_4.$$

An integral component of information technology is the use of computer technology for data processing and usage. Consider a software implementation of the proposed approaches. The proposed information system, in accordance with the considered the methods of processing expert information, performs the following functions: input of information about experts, formation of alternatives and terms, input of expert estimations, calculation of the experts qualification, ranking alternatives, viewing and saving the results work of the system, saving (in the case of need) input data to a file for later reference.

Let's illustrate the work of system. On the Fig. 1 is shown program screen shot for the process determining the qualifications of experts. Values of the variable *S* (experience) and *P* (positive frequency of results) are introduced by the system user, value of qualification (*W*) is calculated by the system.

At the same time we have the opportunity to see the right side of the screen the rules of the system (5) that have been used in concrete case (the degree of the truth of the rule is greater than zero).

Vector degrees of competence for the three experts in this example was calculated program way accordance to methods described above and has next values:

$$W = \{w_1; w_2; w_3\} = \{0{,}307;\ 0.095;\ 0.92\}.$$

On Fig. 2 illustrated process the formation the matrix (1) experts estimates for three alternatives, which serve famous higher educational institutions of city Dnepr: the Dnepr National University named after Honchar (DNU), the National Mining University (NGU) and the Dnepr Medical Academy (DMA).

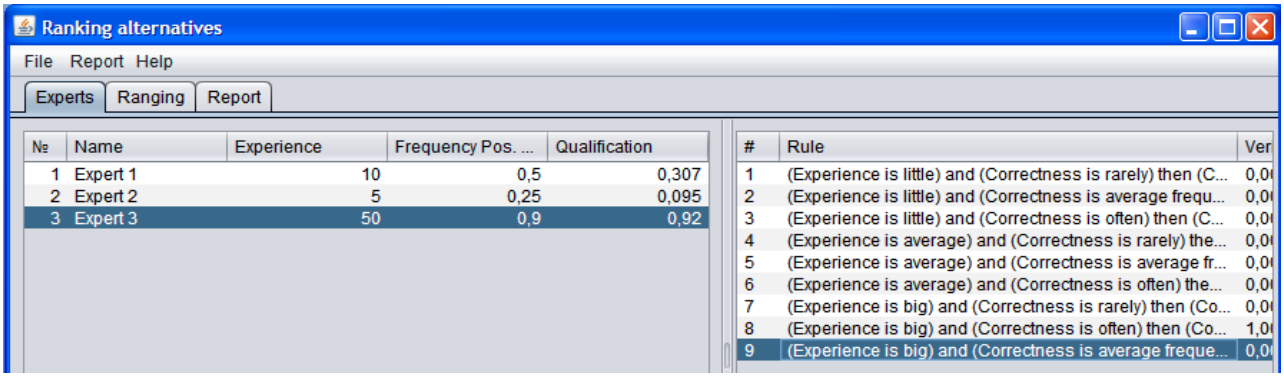


Fig. 1. Program screen form for expert's qualification definition

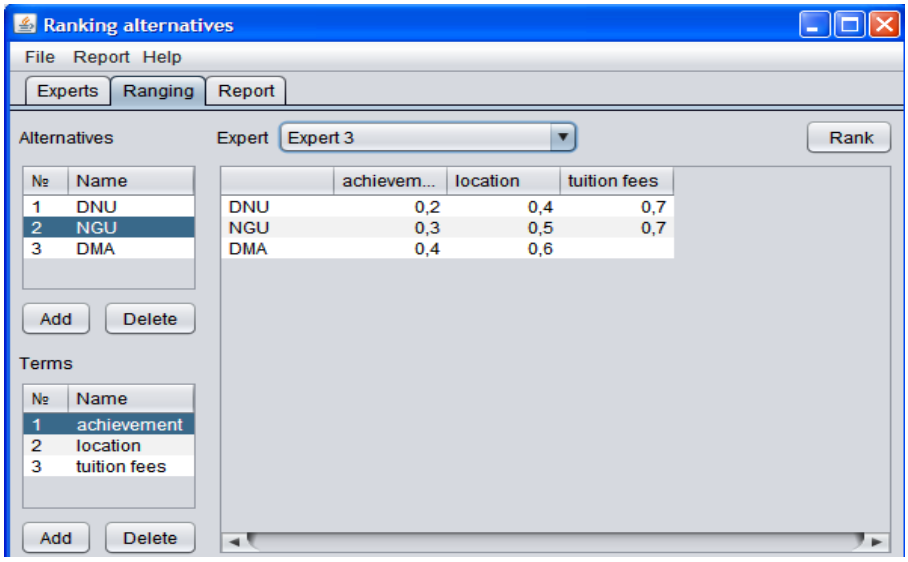


Fig. 2. Program screen form for formation of expert estimates

According to the introduced in this article notations we have a set X that consists of three experts. Set A includes three alternatives:

$$a_1 = \text{DNU},\ a_2 = \text{NGU},\ a_3 = \text{DMA},$$

which to be ranked according to expert estimates of the terms from the set

$$T = \{t_1; t_2; t_3\},$$

where: t_1 = «Permormance rating», t_2 =«Location», t_3 =«Training Price».

On the basis of imposed expert assessments obtain a matrix estimates for each alternative. Then weight's vector of expert's competence

$$W=\{w_1; w_2; w_3\}=\{0{,}307;\ 0,\ 95;\ 0{,}92\}.$$

Let's apply formula (6) to the obtained data of expert evaluations, we obtain an overall assessment, which allows to record every alternative in accordance with expression (7) as a fuzzy variable:

$$a_1=\{<0{,}09/t_1>, <0{,}19/t_2>, <0{,}3/t_3>\},$$
$$a_2=\{<0{,}13/t_1>, <0{,}24/t_2>, <0{,}24/t_3>\},$$
$$a_3=\{<0{,}17/t_1>, <0{,}24/t_2>, <0{,}14/t_3>\}.$$

After applying to alternatives the method of ranking which is proposed in this article and specified by the expressions (9) - (11) we obtain ordering the alternatives that given in Table. 1.

Table 1. Ranging alternatives

Terms	Alternatives		
Permormance rating	<DSMA>	<NSU>	<DNU>
location	<NSU, DSMA>	<DNU>	
Price of training	<DNU>	<NSU>	<DSMA>

This paper presents a software product AltRanging, which is written in object-oriented style of the Java programming language. All objects are described by classes. Each class is stored in separate file with the name of the class. Classes are grouped into packets. This software product has two packages: com.acsu.altranging - contains the basic classes, program logic and GUI, com.acsu.fuzzy - classes and fuzzy logic libraries, which are used to determine the qualifications of the experts. In addition to standard Java packages also applies package org.json , which provides a library JSON. JSON - text format, completely independent of the implementation

language, but it uses the agreements, programs on C-like languages, such as C, C++, C#, Java, JavaScript, Perl, Python and many others. These properties make JSON ideal language data exchange [22].

CONCLUSION

1. The information technology is to combine the author previously developed methods of processing expert information provided by the group of experts and the methods for determining the qualifications of experts in a single systematic approach to the treatment of expert information for the purpose of ranking of alternatives and automate this process with the help of an automated information processing system. The disadvantage of this approach is the need for statistical data on the activities of the people taking part in the expertises.
2. The scientific value of the proposed approaches consist in improving the previously known methods for determining the competence of experts and ranking of alternatives by using elements of fuzzy sets theory.
3. The practical significance of the results consist in the possibility of processing a large enough array of expert data and obtaining generalized results of the expertise. This eliminates the possibility of subjective influence some experts to the opinion of others.
4. Information technology, developed by the author, is applicable for any number of experts and any subject area, a description of which is formulated in linguistic terms.

REFERENCES

1. Angeli C., 2010. Diagnostic Expert Systems: From Expert's Knowledge to Real-Time Systems, TMRF e-Book Advanced Knowledge Based Systems: Model, Applications & Research. Vol. 1, 50–73.

2. Lytvyn V., 2013. Design of intelligent decision support systems using ontological approach. ECONTECHMOD. An international quarterly journal Vol. 2, №1. 31-37.

3. Korneev V., Gareyev A., Vasjutin S., Reich V., 2000. *Databases. Intelligent processing of information*. M.: Knowledge, 352. (in Russian).

4. Kaklauskasa A., Zavadskasbc E., Gargasaited L., 2004. Expert and knowledge systems and databases of the best practice. Vol. 10, 88-95.

5. Filatov V., 2014. Fuzzy models presentation and realization by means of relational systems. ECONTECHMOD. An international quarterly journal Vol. 3, №3. 99-102.

6. Ivanek J., 1991. Representation of expert knowledge as a fuzzy axiomatic theory. International Journal of General Systems. Vol. 20, 55–58.

7. Gong J., Liu L., 2010. Representing and measuring experts knowledge based on knowledge network. Studies in Science of Science. 28 (10), 1521–1530.

8. Daintith J., 2009. IT. A Dictionary of Physics, Oxford University Press, retrieved 1 August 2012.

9. Orlov A., 2004. Applied Statistics. M.: Exam, 656. (in Russian).

10. Ryabushkin T., Baklanov G., Volkov A., 1997. Statistical methods of analysis of expert estimations. Proc. Sciences. Art. Scientific notes on statistics. Vol. 29. M.: Science, 385. (in Russian).

11. Litvak B., 1982. Expert information: Methods for the preparation and analysis. M.: Radio and Communications, 184. (in Russian).

12. Hughes C., 1989. The representation of uncertainty in medical expert systems. Medical Informatics. Vol. 14, 269-279.

13. Melihov A., Bernstein L. and Korovin S., 1990. Situational advising system with fuzzy logic. M.: Science. 272. (in Russian).

14. Kovalyshyn O., Gabriel Yu., 2014. Development of a management systems model of automatic control by using fuzzy logic. ECONTECHMOD. An international quarterly journal. Vol. 3, №. 4. 87-90.

15. Pavlov A., Sokolov B., 2005. Methods of processing expert information: study guide. St. Petersburg: State University of Aerospace Instrumentation. 42. (in Russian).

16. Snityuk V., Rifat Mohammed Ali, 2000. Model methods for determining the competence of experts on the basis of the axioms of unbiasedness. Bulletin of Cherkasy Engineering and Technological Institute. №4, 121-126. (in Russian).

17. Ulianovskaya Yu., 2013. Ranking method of fuzzy alternatives in processes of transfer and information processing in computer systems. Herald of Khmelnytskyi national university. №1(197), 130-134. (in Russian).

18. Shtovba S., 2007. Design of fuzzy systems by means of MATLAB. M.: Hotline – Telecom, 288. (in Russian).

19. Zadeh L., 1976. The concept of linguistic variable and its application to take close resheniy. -M.: Mir, 165. (in Russian).

20. Ulianovskaya Yu., **2014**. Method for determination of qualifications of the experts participating in the expert survey. Herald of Kherson national technical university. № 1(48), 96-101. (in Russian).

21. Rybytska O., Vovk M., 2014. An Application of the Fuzzy Set Theory and Fuzzy Logic to the Problem of Predicting the Value of Goods Rests. ECONTECHMOD. An International Quarterly Journal. Vol. 3, №2, 65-69.

22. Introducing JSON. Available online at: <http://json.org/>

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 61-67

INVESTIGATION OF AGENT-BASED SIMULATION OF MALICIOUS SOFTWARE

D. Chumachenko, S. Yakovlev

National Aerospace University "Kharkiv Aviation Institute"; dichumachenko@gmail.com

Received November 08.2016: accepted December 09.2016

Abstract. Epidemics of malicious software are actual problem and network worms are one of the most important issues. Identifying trends in network worm distribution, finding the factors that influence the spread of the Internet worm will help to identify the effective preventive and precautionary measures to prevent epidemics of malicious software. To solve the problem of the development of advanced security mechanisms against network worms, different approaches to modeling the spreading of worms have been studied. Deterministic models of propagation of computer viruses in a heterogeneous network, taking into account its topological and architectural features have been analyzed and improved. Agent-based model of network worm propagation have been developed. Simulated model is based on epidemic approach to modeling. SAIDR structure of agent-based model has been used for simulation of malicious software of "network worm" type. A comparative study of developed mathematical models has been conducted. Comparative graphs of the dependence of the infected nodes number on the time of the computer system functioning in the propagation of the epidemic have been built. Research carried out by the example of the Code Red worm propagation.

Key words: agent-based simulation, imitation model, malicious software, network worm, Code Red.

INTRODUCTION

The widespread use and easy access to the Internet makes it a prime target for malicious activity. In particular, the Internet has become a powerful mechanism for spreading of malicious software. Network worms, self-contained programs that propagate via computer networks using automatic search, attacks and infection of remote computers, has being developed almost 30 years since the first Morris worm. Under modern conditions, the computer infrastructure is more vulnerable than ever before, because the speed of technological development is much higher than the development of protective measures speed [1]. Incidents of Code Red and Nimda worms in 2001 showed how vulnerable computer networks and how fast can worm spread. Moreover, Weaver introduced certain principles of worm development, using which they could spread even more rapidly [2]. In order to protect from possible attacks of worms in the future, you must understand the different properties: patterns of spread of worms throughout their life cycle, development of patches, awareness and other human countermeasures, network topology, etc.

Development of an accurate model of Internet worm will give an idea about its behavior. This will reveal weaknesses in the dynamics of a network worm, as well as forecast of its distribution to assess the damage from the activities of the worm. In epidemiological studies, there are a number of deterministic and stochastic models for spreading of viral diseases [3], as well, some models exist to simulate the spread of Internet worms.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

Kephart, White and Chess from IBM have conducted a series of network worm simulation experiments from 1991 to 1993, based on epidemiological models of virus infections [4]. All traditional epidemiological models have homogeneous nature, i.e., any infected host can equiprobable infect any susceptible host [3]. Taking into account the local interactions of viruses at that time [5], epidemiological models have been applied to some non-homogeneous networks: random graphs, two-dimensional lattice and a hierarchical graph of type "tree". Despite the fact that at that time the hypothesis of local interaction was correct because of the sharing of information storages, in the modern world it is not suitable for simulating the behavior of network worms, since the vast majority of worms are distributed via the Internet and can hit the target directly. In addition for modeling, model Susceptible-Infected-Susceptible (SIS), which suggests that healed computer can be infected again immediately, has been studied.

Wang and his colleagues presented the results of a simulation of a simple network worm in clustering and hierarchical tree-like networks [6]. They have shown that selective immunization in some network topologies can significantly slow down the spread of the virus. On the other hand, their conclusion was based on a hierarchical tree topology, which is not suitable for the Internet network.

The epidemic of the Code Red worm in July 2001 has stimulated active research activities of simulation and analysis of the worm on the Internet. Staniford and colleagues used classical epidemic simple equations for simulating spreading of the Code Red worm immediately after the incident, July 19th [7]. Their model fairly accurately matched the actual data with the limited observations. Moore has provided the observed data, and the analysis of the behavior of the worm Code Red [8]. Weaver suggested some of the approaches in the design of worms, which can be used to spread malicious code faster than Code Red and Nimda worms [2].

Previous studies on modeling of spreading of network worms neglected the dynamic effects of human countermeasures to the epidemic. Wang and his colleagues looked at protection against the epidemic by means of immunization. However, they only take into account the static immunization, which means that part of the hosts is vaccinated prior to the spreading of network worm. But in real conditions, human countermeasures are dynamic actions which play a major role in reducing the rate of network worm propagation and prevent outbreaks. Many new viruses and worms come out almost every day. Most of them, however, go out without infecting a large number of computers due to the timely implementation of human countermeasures.

The simulation of epidemic process assumes that the level of viral infection is constant. The epidemic modeling process assumed that the level of viral infection is constant. The above-described models of Internet viruses and network worms estimate the time required to search for target of infection of host, regardless of whether it is already infected or not, as a constant. Weaver considered the level of infection, as a random variable, given the unsuccessful attempts to IP-scan of the network worm. Weaver considered the level of infection, as a random variable, given the unsuccessful attempts to scan the IP-worm. However, the average value of rate of infection propagation is still assumed as constant, which is true for simulating epidemics of disease, but it may not be true for Internet viruses and worms.

OBJECTIVES

Modeling of propagation of network worm will identify trends in its distribution, find the factors that influence the spread of the Internet worm. Also model helps to identify the effective preventive and precautionary measures to prevent epidemics of malicious software. To solve the problem of the development of advanced security mechanisms against network worms, different approaches to modeling the spreading of worms were studied. The aim of given research is to develop the agent-based model of malicious software of network worm type by example of Code Red worm.

THE MAIN RESULTS OF THE RESEARCH

SEIQR model. According to this model structure, objects are divided into five groups: Susceptible (S), Infected (I), Exposed (E), Removed (R), Quarantine (Q) (Fig. 1). Here, transitions of states can happen in the following ways:

– from Exposed to Removed,
– from Exposed to Infected,
– from Infected to Quarantine and subsequently to Removed,
– from Infected to Removed.

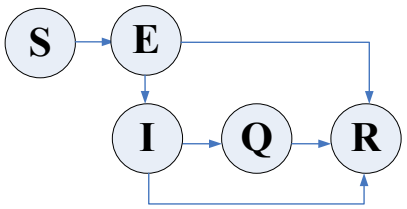


Fig. 1. State changes in SEIQR model

Dynamics of system is described by following differential equations:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\dfrac{\beta I(t)}{N} S(t), \\ \dfrac{dE(t)}{dt} = \dfrac{\beta I(t)}{N} S(t) - (\alpha + k)E(t), \\ \dfrac{dI(t)}{dt} = \alpha E(t) - (\gamma + \delta)I(t), \\ \dfrac{dQ(t)}{dt} = \delta I(t) - \gamma Q(t), \\ \dfrac{dR(t)}{dt} = kE(t) + \gamma(Q(t) + I(t)). \end{cases} \quad (1)$$

where: β is the infection rate, δ is quarantine rate, k, γ are treatment rates, α is transition rate from a latent state E to the infected I (i.e., the average time spent in a latent state).

Results of the Code Red Worm propagation are shown in Fig. 2. Results show that the implementation of additional types of object management and the possibility of implementation of network nodes in quarantine improves the accuracy of the final result on the condition that the anti-virus software is updated.

Progressive SIDR model. Progressive SIDR model or PSIDR model takes into account two factors (Fig. 3):

1. The classical SI model works at early stage of epidemic distribution. Its duration is π.

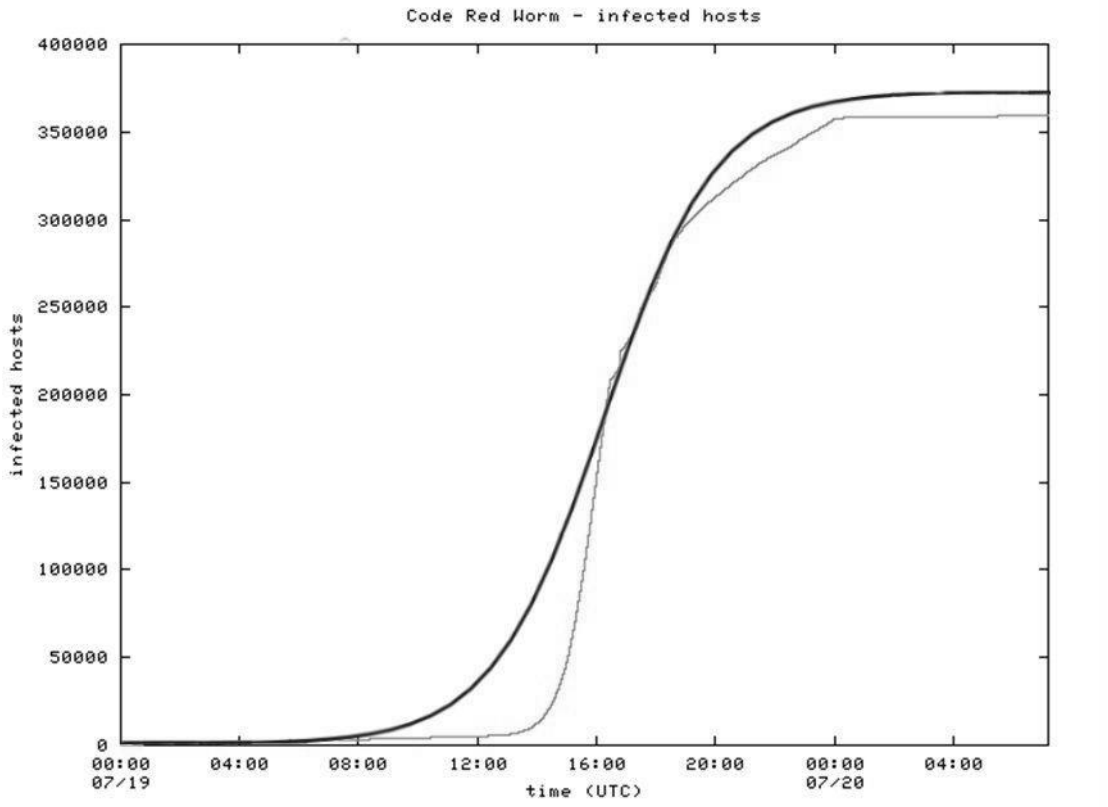


Fig. 2. SEIQR model of Code Red propagation (black) and actual Code Red propagation (grey)

2. Further development of the epidemics is described by introducing an additional state *D* (Detected), in addition to the states *S*, *I* and *R*. The node is said to be in the state D, when the presence of the worm was already discovered, but active counteraction has not yet begun.

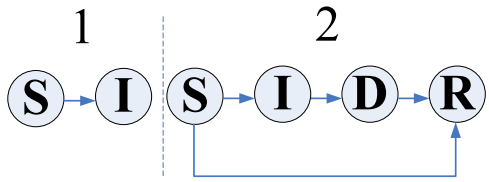


Fig. 3. State changes in PSIDR model

Infection and recovery of network nodes are performed as before, with the average speed β and γ, a transition from state *I* to an "intermediate" state *D* with the speed of μ nodes per time unit.

At stage 1, when $t < \pi$ the following condition applies:

$$S(t) + I(t) = N, \tag{2}$$

and the system is described by following equations:

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI, \\ \dfrac{dI}{dt} = \beta SI. \end{cases} \tag{3}$$

At stage 2, when $t \geq \pi$ the following condition applies:

$$S(t) + I(t) + D(t) + R(t) = N, \tag{4}$$

and the system is described by following equations:

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI - \mu S, \\ \dfrac{dI}{dt} = \beta SI - \mu I, \\ \dfrac{dD}{dt} = \mu I - \delta D, \\ \dfrac{dR}{dt} = \delta D + \mu S. \end{cases} \tag{5}$$

Results of the Code Red Worm propagation are shown in Fig. 4.

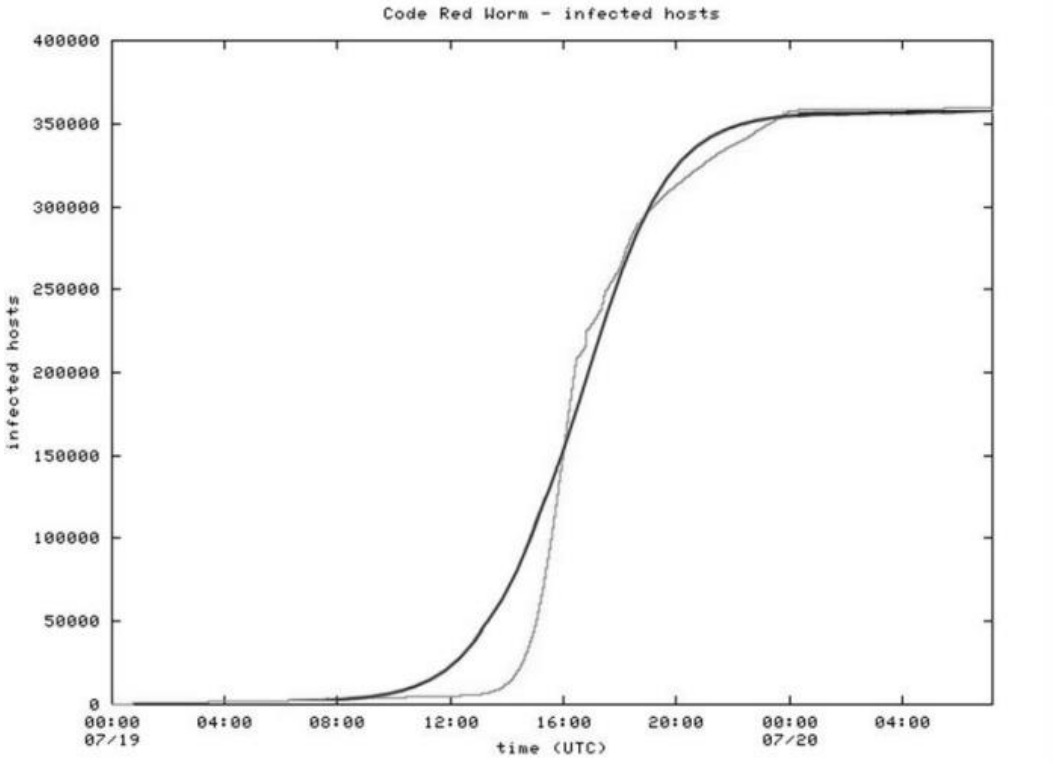


Fig. 4. PSIDR model of Code Red propagation (black) and actual Code Red propagation (grey)

Agent-based simulation. The first research on the epidemic process spreading in medicine, which allowed to make a step forward compared to the classical models, was presented in [9] (Baroyan-Rvachev Model). Actually, this model is the same as the classic models as it is based on the system of differential equations. However, it uses the concept of the model time and takes into account the length of a particular state. However, the abovementioned disadvantages of the model were not fully resolved.

To eliminate these drawbacks, the agent-based simulation was developed. It allows taking into account a large number of factors influencing the process of malware distribution [10-14].

Adequacy of the simulation model is highly dependent on the number of agents in the system. The use of large populations and detalization of properties of agents leads to the need of using the most modern information tools and technologies, particularly the algorithms that are optimal in the number of operations performed by the machine. In agent-approach the simulation process is based on the construction and processing of the event queue, which can be divided into two types:

1. Changing the agent state from the point of view of the external environment (the physical position of agent);
2. Changing the internal state of the agent. Events of this type arise from the interaction of the agent with other agents, as well as with the environment.

The objective is to find and use such a set of properties and methods of agents, which would allow the use of agent-based approach in the greatest way.

In this paper, we propose a formal description of the constructed agent-based model of the Code Red worm propagation. Agent can be viewed as a set of attributes:

$$a = \langle s, s_t, c, l \rangle, \; a \in A, s \in S, c \in C, \qquad (6)$$

where: s_t is time that agent is in state s;

A – amount of agents;

S – set of possible states;

l – life duration;

C – set of cells of working area.

The set of states of the agent is determined beforehand and is permanent. In our model we define set of states as:

$$S = \{Susceptible, Antidotal, Infected, Detected, Recovered\}.$$

This set of states is based on the idea of dividing the entire population into subsets based on their states according to epidemic status. The proposed set describes the model as an analog of expanded SAIDR model.

Fig. 5 shows the transitions between states:

- Susceptible – agent is not infected. This state is only applicable to agents that can be infected with a particular worm.
- Antidotal – agent is not infected. Agents in that state have the installed antivirus software, so it cannot be infected.
- Infected – agent is infected and can propagate infection to the other hosts.
- Detected – agent's infection is detected by antivirus software and isolated from the network.
- Recovered – agent is cured and is not vulnerable to this particular worm anymore.

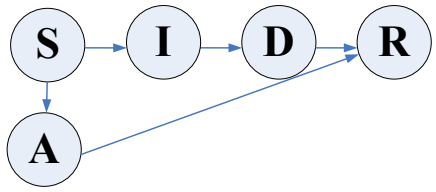


Fig. 5. State changes in expanded SAIDR model for agent-based system

Network is described as a set of cells. Decomposition of workspace leads to a set of cells as the contingent of abstract objects, that include a set of agents and one infected agent.

Thus, a cell can be defined as:

$$c = \left\langle z, \widetilde{A}, i \right\rangle, z \in Z, \widetilde{A} \subset A, i \in I, \qquad (7)$$

where: I is a set of all infected agents; Z is a set of all zones.

In our model, infection is possible only directly from one agent to another. If we treat propagation as a direct communication from one host to another, computation will be inefficient. Processing of such communication is very hard. To simplify the computations, we suggest treating the infection of agent as a fact of agents belonging to the same cell. It will allow taking into account the interaction between agents and significantly reducing the performance loss.

The propagation is implemented in the following way. At first, the possibility of communication between two hosts is checked. The communication and subsequent infection is considered to be possible if a cell of the currently processed agent includes other agents. Each pair of agents of the cell is processed and if at least one of them is infected, it is considered that the communication happens with a certain probability.

Method of agents operating in the crucial network consists of three sub-functions: perception, decision and transformation.

Perception sub-function:

$$Per : CE \rightarrow F_{in} \qquad (8)$$

provides a selection of information from the environment and the assignment of values of input attributes.

Decision sub-function

$$Dec : F_{in} \rightarrow F_{out} \qquad (9)$$

determines the values of the output variables from the values of the input.

Transformation sub-function

$$Tran : F_{out} \rightarrow CE' \qquad (10)$$

changes the state of the environment by performing the translation operation of sets of elements from one to another in accordance with the rules, as well as removes the elements of the sets.

The general structure of the constructed agent-based system can be expressed by tuple:

$$MAS=\{Ind, Prp, Atr, Inp, Out, Str\}, \qquad (11)$$

where: *Ind* is the name of the system, *Prp* is the aims of the systems, *Atr* is general system descriptions, *Inp* is the entrance of system, *Out* is the output of the system, *Str* is a structure of the system *Str={CE, R}*, and *CE* are component elements of the system, *R* are communications of components.

The prototype of developed agent-based model of the Code Red worm spreading was realized using NetLogo agent-based software (Fig. 6).

The advantage of using a given model is its simple interface. As you can see from screenshot (Fig, 6), user of developed model can change input parameters (number of hubs in the given network, infection rate, test frequency etc.) and observe plots of worm dynamics and system behavior in real time. However, the implementation of the proposed model with a large number of agents in NetLogo software requires very high processing power. This does not allow simulating the real processes occurring in a network with a large number of hosts. To solve this problem, the model was developed in Python programming language. The simulation results show the behavior that is similar to real statistical data (Fig. 7).

This research allows us to suggest a hypothesis that time series corresponding to the number of infected hosts must not exceed a certain threshold value for the non-arising of the epidemic. Thus, the dynamical system describing the agent-based model should be the reflection of a finite set I in itself, *MAS:* $I \rightarrow I$, that is a sign of chaotic dynamics for nonlinear mappings. Thus, it seems to be relevant to create conditions in relations (8)-(10), by which it is possible to judge about the stability of the dynamics of agent-based system.

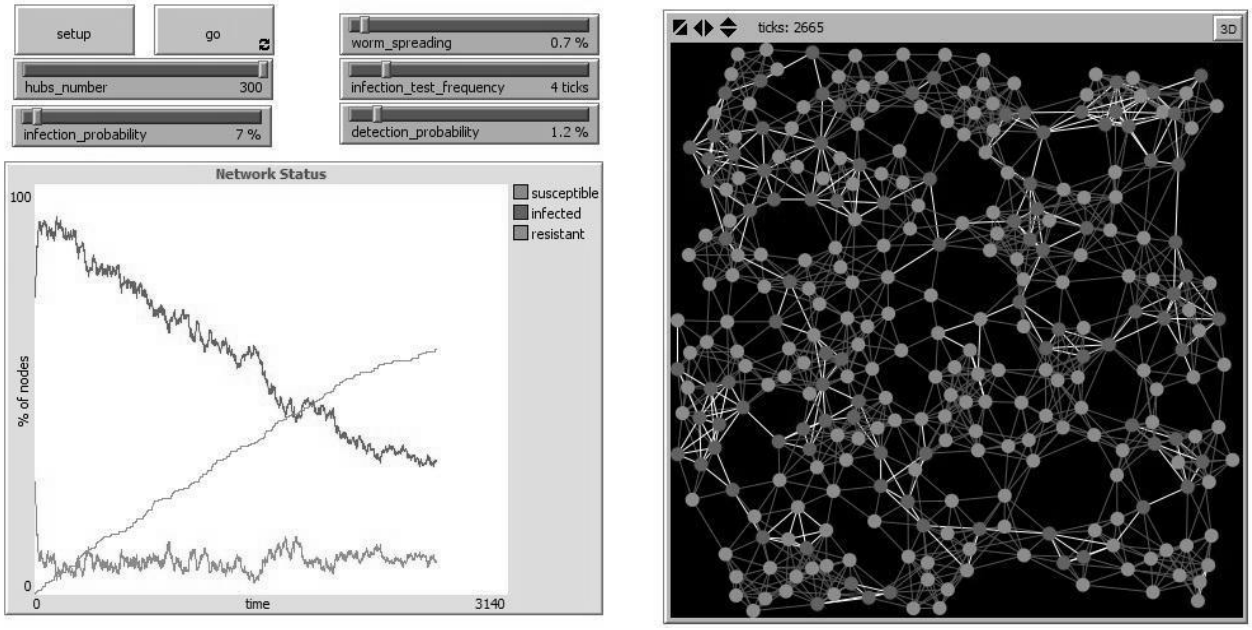


Fig. 6. Model in NetLogo

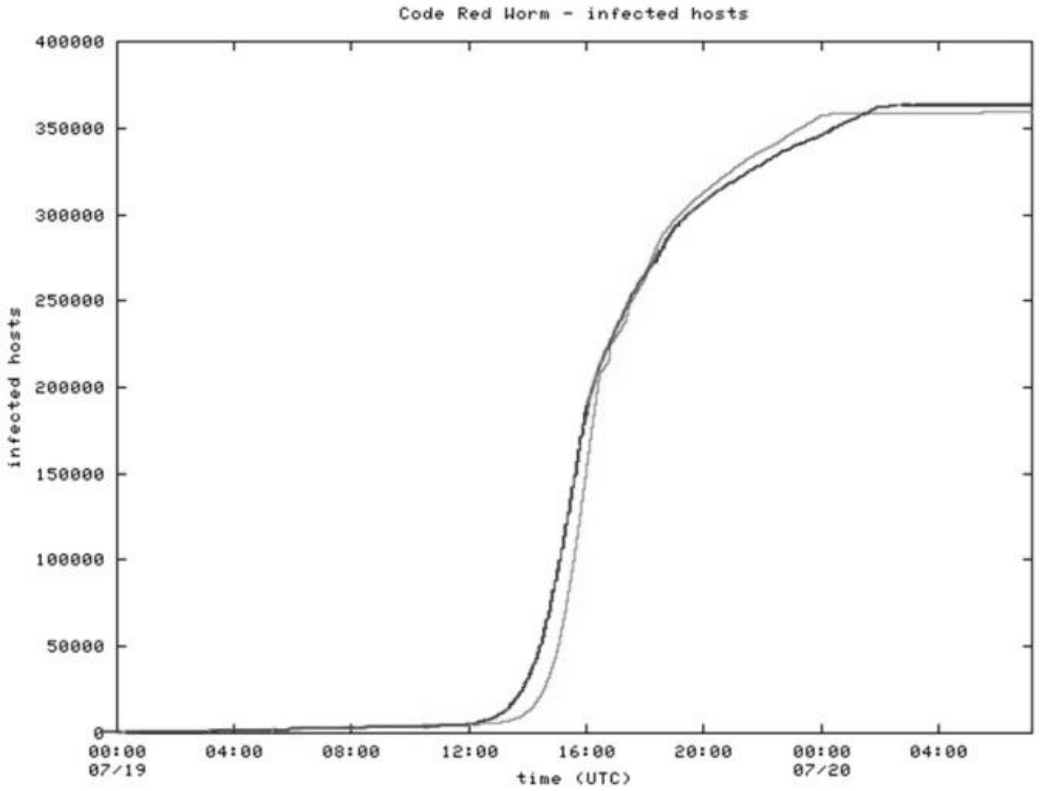


Fig. 7. Agent-based model of Code Red propagation (black) and actual Code Red propagation (grey)

CONCLUSIONS

1. Agent-based model of the malicious software have been developed on the basis of existing models and modeling approaches by the example of the Code Red Worm. Agent-based simulation showed the most close to the real data results.

2. Developed agent-based model is universal for analyzing the dynamics of the behavior of any kind of malicious software of type network worm. In given study the dynamics is shown by the example of the Code Red worm. The adequacy of the model has been checked with real statistics of the Code Red worm incidence.

3. Analysis and comparative research of developed mathematical models led to the conclusion about improving of their accuracy in comparison with the known mathematical models. That led to the conclusion about the appropriateness of using developed model in computer networks. At the same time agent-based simulation results allow practically repeat a real worm behavior.

REFERENCES

1. **Nachenberg C., 2000.** The Evolving Virus Threat. 23rd NISSC Proceedings.

2. **Weaver N., 2001.** Warhol Worms: The Potential for Very Fast Internet Plagues.

3. **Chernyshev Y., Chumachenko T., Chumachenko D., Tovstik A., 2012.** System of Simulation of Epidemic Diseases Spreading. Proceedings of East West Fuzzy Colloquium 2012 (19th Zittau Fuzzy Colloquium, September 5 – 7, 2012), 154 – 161.

4. **Kephart J., Chess D., White S., 1993.** Computers and Epidemiology. IEEE Spectrum 30 (5). 20 – 26.

5. **Kephart J., White S., 1993.** Measuring and Modeling Computer Virus Prevalence. Proceedings of the IEEE Symposium on Security and Privacy. 2 – 15.

6. **Wang C., Knight J., ElderM., 2000.** On Viral Propagation and the Effect of Immunization. Proceedings of 16th ACM Annual Computer Applications Conference. 246 – 256.

7. **Staniford S., Paxson V., Weaver N., 2002.** How to Own the Internet in Your Spare Time. 11-th Usenix Security Symposium. 149 – 167.

8. **Moore D., Shannon C., 2003.** The Spread of the Code-Red Worm. Center for Applied Internet Data Analysis.

9. **Baroyan O., Rvachev L., 1978.** Forecasting of influenza epidemics in USSR. Medicine #2.131-137. (in Russian).

10. **Wooldridge M., 2009.** An Introduction To Multiagent Systems. 468.

11. **Chumachenko T., Chumachenko D., Sokolov O., 2013.** Multiagent simulation of the hepatitis B epidemic process. Online journal of public health informatics, 5 (1).

12. **Boyko N.**, **Kutyuk O., 2016.** Basic concepts of evolution in agents calculating and agents system. ECONTECHMOD. An International Quarterly Journal, Vol. 05, No. 2. 69-76.

13. **Chernyshev Y., Chumachenko D., Tovstik A., 2013.** Development of intelligent agents for simulation of hepatitis B epidemic process. Proceedings of East West Fuzzy Colloquium 2013 (20th Zittau Fuzzy Colloquium, September 25 – 27, 2013). 161 – 168.

14. **Bobalo Y., Politanskyi R., Klymash M., 2015.** Traffic simulation in a telecommunication system based on queuing systems with different input flows. ECONTECHMOD. An International Quarterly Journal, Vol. 04, No. 1. 11-16.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 69-72

CONCEPT OF USING HEURISTIC METHODS IN THE OPTIMIZATION OF ELECTRIC POWER SYSTEMS

M. Gryniewicz-Jaworska

Lublin University of Technology, Faculty of Electrical Engineering and Computer Science, Institute of Computer Science; email: michalina.gryniewicz.jaworska@vp.pl

Received November 21.2016: accepted December 20.2016

Abstract. The dynamic development of information technologies has significantly improved the process of planning and controlling the operation of electrical power systems, nevertheless, we are still looking for methods and solutions which will allow to optimize EPS. The article describes heuristic methods which can be used in electrical power engineering and which will be used to solve OPF (Optimal Power Flow) problems.

Key words: electrical power system, optimization, heuristics.

INTRODUCTION

The attempts to optimize power systems both at the stage of planning the development and controlling their operation focused mainly on looking for a proper distribution of the generated power into the generating units minimizing the total cost of its generation [14]. This task was associated with the minimization of the objective function, which was the sum of the generation costs, and was referred to as the Economic Load Dispatch (ELD). In order to solve the ELD problem, it was necessary to know the characteristics of the particular source costs, which usually take a non-linear form, whereas the equality constraint resulted from the power balance in the system. In order to compute the non-linear objective function minimum with the equality constraint, the Lagrange function with the λ multiplier is used, where the values of the source capacity ensuring the objective function minimum are computed by equaling partial derivatives of the Lagrange function to zero [11, 15]. The computational problem arises when inequality constraints are taken into account, as well as the technical minima, maximum values of sources capacity, and also after consideration of the balance equality constraints caused by transmission losses of the whole network where the power sources and loads are connected:

$$P_{Gi\,min} \le P_{Gi} \le P_{Gi\,max} \tag{1}$$

Taking into consideration the above constraints, the problem in question can be solved in a similar way, but in order to fully optimize the electric power system operation, it is necessary to take into account the full conditions of an industrial grid operation and the constraints associated with it [7].

Introducing the designation of three vectors – the state vector – including the modules of source voltages U and their arguments δ

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\delta} \end{bmatrix} \tag{2}$$

which meets the grid equation taking into account the enforcement vector w (power output collected in nodes)

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{P}_{\mathrm{L}} \\ \boldsymbol{Q}_{\mathrm{L}} \end{bmatrix} \tag{3}$$

and steering vector s (power generated in nodes)

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{P}_{\mathrm{G}} \\ \boldsymbol{Q}_{\mathrm{G}} \end{bmatrix} \tag{4}$$

the optimizing problem can be presented in a general form

$$F_c(x, w, s) \longrightarrow min \tag{5}$$

with equality constraints

$$g(x, w, s) = 0 \tag{6}$$

and inequality constraints

$$h(x, w, s) \ge 0 \tag{7}$$

In order to calculate the minimum EPS balancing costs, the objective function of an OPF problem should be drawn as follows:

$$F_c(\boldsymbol{s}) = \sum_{j=1}^{N_s} c \cdot P_{Gj} \tag{8}$$

where: P_{Gj} is active power generated by the source connected to the node j.

The objective function presented above includes the summation of the steering vector elements corresponding to the relevant grid nodes, and their number is defined as N_z. The elements of the steering vector are capacities of all sources operating in it. The detailed nature of equality and inequality constraints results from the formulas of a typical load-flow problem [7, 8] i.e.:

– inequality constraint resulting from the system technical minimum,

– inequality constraint resulting from the permissible limb capacities,

– inequality constraint resulting from the permissible foreign exchange balance,

– inequality constraint resulting from the permissible values of nodal voltages in the grid,

– inequality constraint resulting from the balance of the active and passive power generated and taken,

– constraint resulting from the consideration of the *N-1* criterion [7, 9].

HEURISTIC AND EVOLUTIONARY METHODS IN ELECTRIC POWER ENGINEERING

Due to computational difficulties in solving the OPF and SCOPF problems with traditional methods, alternative methods of optimization: heuristic methods and genetic algorithms are used more and more often. In contrast to the traditional methods, heuristic methods do not require the knowledge of the derivative of the objective function, they are not affected by the lack of continuity of a function or "getting stuck" of the computational process in a local minimum [7]. At the same time, evolutionary algorithms are more and more often used to solve difficult problems in various fields of technical sciences and turn out to be most useful in case of a great number of solutions in the Pareto sense [10, 13].

The task of the evolutionary algorithm is to analyze alternative solutions in order to choose the best or potentially best ones. Searching is done using the mechanisms of evolution and natural selection, which is associated with remembering for some time the selected parts of the history of this process [16, 17]. The principle of the evolutionary algorithm consists in the processing of a population of individuals, each of which is a proposal of a solution to a specific problem**.** All individuals are assigned a value, referred to as the fitness of an individual, moreover, they are equipped with a genotype, on the grounds of which a phenotype is created. Thus, the principle of the algorithm consists in making multiple loops where reproduction (genetic operations) is followed by assessment and succession [4]. Both the heuristic methods and evolutionary algorithms are universal methods which can be used for computations with any objective function, for example, analyzing a power flow problem which is connected with a time-consuming iterative process determining elements of a state vector. In spite of the fact that an objective function has the form of summation and as such it is easy to optimize, one of the computation groups – limb constraints, permissible current capacity of a line and transformers power rating – can be checked only on the grounds of a state vector which is hard to determine. In some sense, during computation these constraints are not visible and when they are included in the objective function it is difficult to determine which shape is assumed by the new objective function which is created in this way and which is subject to minimization [8]. A few heuristic methods which can be used to solve optimization problems in electrical power engineering are listed and described below.

THE VEGA ALGORITHM (SCHAFFER'S VECTOR EVALUATED GENETIC ALGORITHM)

The author of this method is Schaffer and it is used to solve multi-criteria problems. In case of this algorithm, the population of permissible individuals is divided into numerous subsets, the number of which is determined by the number of the accepted criteria. In each of these subsets, the best individuals are selected, but from the point of view of only one criterion in each subset of another one. (Fig. 1). Thus, it can be said that a multi-criteria problem has been decomposed into a number of autonomous single-criterion problems, which, however, are not explicitly related to one another. The selected best individuals are moved to a temporary population P', where they are mutated and modified through the use of crossover and mutation operations.

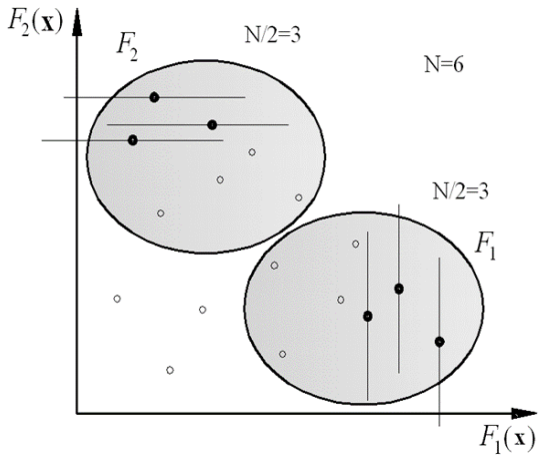


Fig. 1. Selection in the VEGA method for double-criteria maximization [12]

The steps of VEGA algorithm:

1. The PG population is divided into *k*-subsets where k stands for the number of criteria.
2. Selection of the best individuals in each *k* subsets, taking into consideration only one of *N/k* criteria, (where *N* – is the number of all selected individuals).
3. The resulting individuals are moved to a temporary population *P'*.
4. The crossover and mutation operation takes place in the population *P'*. A new population – *PG* is obtained, where *G=G+1*.
5. The procedures described in points 1 – 4 are repeated until the algorithm is terminated (e.g. $G = 400$).

THE HLGA ALGORITHM (HAJELA AND LIN'S WEIGHTING-BASED GENETIC ALGORITHM)

The algorithm was developed by Hajela and Lin and is based on the method of weighted criteria. The weights are coded together with individuals, thanks to which each individual evolves with differently defined criteria, which at the same time makes it possible to ensure many directions of the optimization task [5].

The steps of the HLGA algorithm:

1. Selection of an individual out of the present population PG.
2. Determination of weights of criteria validity and fitness.
3. Selection of the best-fitted individuals and adding them to the temporary population P'.
4. Generating new individuals - both as a result of crossover operations and of mutation.
5. Determination of the new population PG, where $G=G+1$.
6. The procedures described in points 1–5 are repeated until the algorithm is terminated (e.g. $G < 400$).

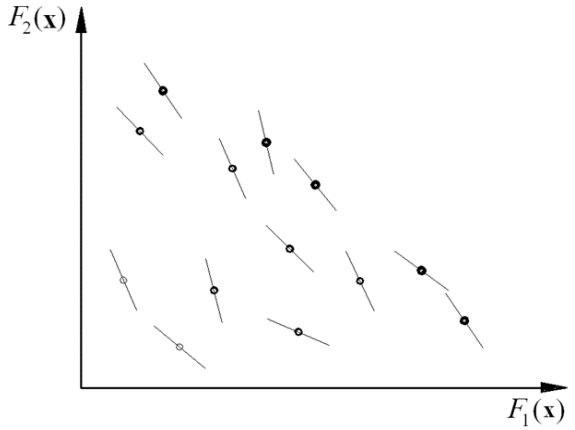


Fig. 2. HLGA method– assigning weights of criteria validity to the individuals [12]

FFGA ALGORITHM (FONSECA AND FLEMING'S MULTIOBJECTIVE GENETIC ALGORITHM)

The algorithm was developed by Fonseca and Fleming, its concept is very similar to the NSGA algorithm, because it also assigns ranks to non-dominated individuals from the set/subset of non-dominated solutions. The major difference, however, is that in the subsequent steps of the selection, the value of fitness depends not only on the number of the step, the value of the latest assigned rank, but also on the number of solutions which dominate them by the solutions forming an earlier front of the non-dominated solutions. There are numerous articles which present several ways of assigning ranks to particular individuals.

The paper [2, 4] uses a formula where the rank of the individual in question corresponds to the number of individuals dominating this individual, plus one. Such a procedure results in a diversity of ranks of the individuals belonging to the subsequent fronts. The individuals dominated by two individuals of the front 1 are assigned rank 3. When we use such a procedure where only individuals belonging to the previous front are taken into consideration, the diversity of ranks of the subsequent individuals will be smaller than in case of the formula described above.

In this situation the individuals of the subsequent front must have worse ranks than the individuals of the previous front, so they are assigned values of at least 4, but the individuals dominated by two individuals of front 2 are assigned a rank of 5. Transferring this procedure to the individuals forming the subsequent front, the first of them is assigned rank 6 and the other one – rank 7.

Steps of the FFGA algorithm:

1. We determine the ranks of all individuals in the population of permissible solutions P_G, remembering to temporary eliminate from the set of permissible solutions the individuals that have already been assigned ranks.
2. We sort the population according to the ranks assigned (ranging from the best to the worst one) and we assign values of initial fitness (according to the formula: *1/rank* value, e.g. ¼, $^1/_7$) to particular individuals.
3. We select the best-fitted individuals and add them to the temporary population P'.
4. We generate new individuals by crossover and mutation operations.
5. We determine a new population P_{G+1}.
6. The procedures described in points 1–5 are repeated until the algorithm is terminated (e.g.. $G < 400$).

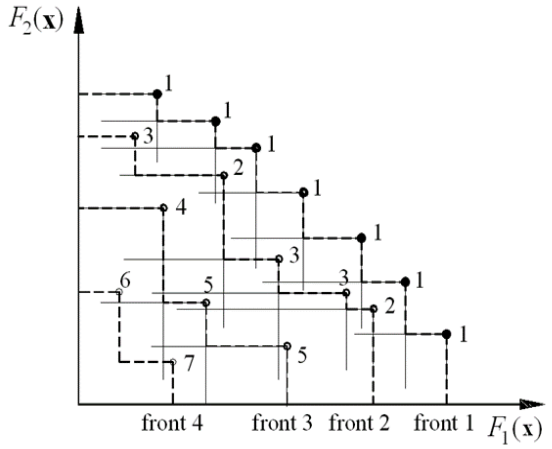


Fig. 3. Assigning ranks in a double-criteria maximizing problem – the FFGA algorithm [12]

THE NPGA ALGORITHM (NICHED PARETO GENETIC ALGORITHM)

The authors of the method are Horn and Nafpliotis who developed an algorithm involving a selection which combines an analysis of individuals domination with a simultaneous tournament selection [1, 3]. This method involves a creation of a comparative set consisting of approximately 10% of the present population. We always take two individuals from the present set of population to create a tournament and each of these individuals is compared to particular individuals from the temporary set.

In such a case when the first individual is dominated by the individuals from the comparative set, and the other one is not, the latter is selected for the reproduction and goes to the temporary population. When both individuals dominate the elements of the comparative subset, the result of the tournament is decided through the method of fitness sharing and both individuals are shifted to the temporary population P'. The same happens when both individuals are dominated by the elements of the comparative subset [6, 12].

According to the authors, such procedures ensure the creation of stable subpopulations along the front of the Pareto-optimal solutions.

The steps of the NPGA algorithm:

1. A set of comparative individuals is picked up randomly from the PG set and the number of them is indicated by the "dominance pressure" value.

2. The individual dominating the random set is shifted to the temporary population P', the individual dominated by the elements of the temporary set is rejected.

3. In the temporary population P', the crossover and mutation operations take place, which leads to a new population – $PG+1$.

4. The procedures described in points 1–5 are repeated until the termination of the algorithm commences (e.g. $G = 150$).

CONCLUSIONS

The dynamic development of the electric power engineering and the associated increase in the number of connected power loads and sources enforces the use of new, non-traditional methods allowing for a real control of the power grid operation. Thus, the minimization of the balancing costs of electric power systems becomes an essential problem. It is an issue, which combines the OPF problem with reliability of EPS functioning. In practice, controlling the operation of small networks does not involve any serious problems, nevertheless, in case of large systems it is necessary to take advantage of algorithms with solid mathematical foundations. However, in order to solve optimization problems, energy experts more and more often use heuristic methods, which allow to work out satisfactory results.

REFERENCES

1. **Horn J., Nafpliotis N., 1993.** Multiobjective optimization using the niched pare-to genetic algorithm. IlliGALTechnical Report 93005, IllinoisGenetic Algorithms Laboratory. University of Illinois, Urbana, Illinois.
2. **Kacejko P., 2010.** Electrical engineering and information technology in modern energy technologies. PAN. 45-58.
3. **Kacejko P., Machowski J., Pijarski P., 2011.** Standing phase angle reduction at switching operation in transmission network. Rynek Energii nr 5(96)-2011r., 24-35.
4. **Kacejko P., Pijarski P, Sobierajski M., Robak S., 2011.** Assessment of power system ability to connect new energy sources: Part III – nonlinear optimization of wind generation. Energy market. 44-51.
5. **Kacejko P., Pijarski P., 2008.** Dynamic Fitting of Generation Level to Thermal Capacity of Overhead Lines. Przegląd Elektrotechniczny, R.84 NR 5/2008, s. 80-83.
6. **Kacejko P., Pijarski P., 2009.** Connecting of wind farms – limitations instead of oversized investment. Rynek Energii nr 1 (80), 10-15.
7. **Kacejko P., Pijarski P., 2014.** Heuristic optimization methods applied to minimize the electric power system balancing costs. Electrical Review. 165-168.
8. **Karpukhin A., Gritsiv D., Tkachenko A. 2014.** Mathematical simulation of infobommunication networks applying chaos theory. ECONTECHMOD. An International Quarterly Journal. Vol. 3, No.3, 33-42.
9. **Konak A., Coit D., Smith A., 2006.** Multi-objective optimization using ge-netic algorithms: A tutorial. Realiability Engineering and System Safety. vol. 91. Elservier. 992-1007.
10. **Kremens Z., Sobierajski M., 1996.** Analysis of power systems. WNT. Warsaw. 55-58.
11. **Montusiewicz J., Gryniewicz-Jaworska M., 2014.** Multi-criteria approach in evolutionary optimization. The problems of modern engineering. Information technology. Publisher Technical University of Lublin. 76-80.
12. **Oliskevych M., 2015.** Optimization of information flows of logistic supply chain. ECONTECHMOD. An International Quarterly Journal. Vol. 4. No. 4, 71-76.
13. **Osyczka A., 2002.** Evolutionary Algorithms for Single and Multicriteria Design Optimization. Physica-Verlag, Haidelberg, New York.25-35.
14. **Pasierbek A. Polomski M, Sokol R., 2013.** Comparison of selected algorithms optimize power flow in the power system. Elektryka. 68-72.
15. **Pijarski P. Gryniewicz-Jaworska M., 2015.** Optimization in the electric power engineering - problems and challenges. TEKA. Commission of Motorization and Energetics in Agriculture Polish Academy of Sciences. 19-22.
16. **Plonka S., Lorek R., 2013.** Multi-criteria optimization of manufacturing processes of machine parts. WNT, Warsaw. 56-68.
17. **Quintana V., Torres G., 1999.** Introduction to interior-point methods. IEEE PICA. Santa Clara, CA.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 73-78

DISTRIBUTION SYSTEM FOR GENERATING SLOWLY VARYING MAGNETIC FIELDS

G. Zaguła, C. Puchalski, M. Czernicka, M. Bajcar, B. Saletnik

Department of Bioenergy Technology, Rzeszów University; e-mail: g_zagula@ur.edu.pl

Received November 21.2016: accepted December 21.2016

Abstract. The presented system describes a concept for generating slowly varying signals to stimulate biological objects with magnetic field. The system enables independent control of up to 10 induction coil and it generates changeable signal inducing magnetic fields in the range from 0.1 to 150μT and the frequencies from 0.1 to 100 Hz. The proposed system was applied to investigate the influence of magnetic fields with the aforementioned parameters on physical and chemical properties in selected types of fruits.

Key words: magnetic fields, magnetic stimulation, coils.

INTRODUCTION

The developed system of magnetic stimulation, based on inducing slowly varying magnetic fields and applying a microcomputer control module, enables generation of fields with induction ranging from 0.1 to 150μT at varied frequencies from 0.1 to 100Hz. Magnetic fields induced in this way were used to magnetically stimulate three groups of fruit, i.e. apples, strawberries and tomatoes.

Today, there are no doubts that magnetic fields affect living organisms. The effects, briefly speaking, depend only on magnetic field strength and on the characteristics of the living organism [1].

There is a close link between magnetic field changes and the pathways of essential life processes on Earth, manifested in the rhythms of physiological, biochemical, genetic and biophysical processes. Hence, magnetic fields may favourably impact and enhance vegetation of crops [5].

This is because plant organisms may be treated as specific antennas receiving electromagnetic waves from the external environment. As a result of this phenomenon enhanced by resonance effect, frequently very low energy leads to significantly stronger effects [15].

Numerous studies suggest that cell membranes play a part in the perception of the geomagnetic field. A magnetic field, in the range from 0.2Hz to 100Hz, also produces changes in membrane permeability. According to Davson–Danielli model from 1935, cell membrane may be treated as an electrical condenser, filled with an imperfect dielectric, built of two lipid layers, which are enriched, in accordance with Singer and Nicolson model from 1972, with carrier protein molecules floating in it [1].

The magnetic component of a slowly changing field causes ordering action in magnetic dipoles, interacts with moving charges as well as neural networks, and induces generation of rotating currents providing the resonance effect which involves opening of ion channels; it also affects uncompensated electron spins and liquid crystals contained in organisms [16].

Futures were many investigations about magnetic field systems and their influence on living organisms. Gut in 2007 used coil with autotransformer for potatos stimulation [4]. Kornarzynski et al. in 2004, 2005 and 2008 used electromagnet equipment for seeds stimulation [5-8]. Ciesla et al. in 2015 presents their superconducting electromagnet to provide very high magnetic field induction [2]. Marks and Szecowka in 2011 built solenoid system with glycol cooling for potatoes seedlings stimulation [9].

Many researchers using also magnetic systems for water condition improving. Systems like this were presents by: Podlesny and

Gendarz [12, 13], Matwiejczyk et al. [10], Pietruszewski et al. [11], Podsiadlo and Lesniak [14,], also Coey and Cass [3].

The developed magnetic field generator system fills a gap in the market by providing mobile laboratory equipment designed for magnetic stimulation carrying out about stability of magnetic field using the resistant temperature coefficient.

MFG SYSTEM, PARAMETERS AND PURPOSE

The developed MFG (Magnetic Field Generator) system is designed to generate sinusoidal slowly varying magnetic fields, and consists of: CompactRIO-9074 controller; NI 9265 current source modules; magnetic field generating solenoid coils with double-pull four-layer copper wire winding with double insulation, with the cross-section of 0.4 mm; NI PS-15 AC adapter with output power of 120W; a computer with web browser or installed LabVIEW environment; connection cables.

The system enables the generation of magnetic fields with the amplitudes of field-B induction from 0.1 to 150μT, with frequency f from 0.1 to 100 Hz. It also controls up to 10 coils, whose parameters can be programmed independently.

Description of MFG system operation

The operator (client) uses a PC with a web browser (installation of free software Run Time Engine from National Instruments is required) to open the Front Panel of MFG.vi control program.

The operator can change the settings of frequency *f* and magnetic induction *B*, and set the time during which magnetic field will be generated; they can read or save the settings, and check the operating accuracy of the current modules and the power supply of the coil.

The heart of the system is CompactRIO-9074 controller which combines an embedded real-time processor and programmable FPGA matrix. The processor is clocked at the frequency of 400Hz, and has 128MB of primary storage and 256MB of non-volatile storage.

The real-time system is supported by FPGA Spartan-3 provided with 46,080 logic cells, which yields two million logic gates. The controller has 8 slots for interchangeable input/output modules, which are operated by FPGA; that results in high speed, synchronization and the ability to reprogram the individual inputs/outputs.

The system uses five NI 9265 output modules (4-channel current source of 0-24mA and 100k Samples/s).

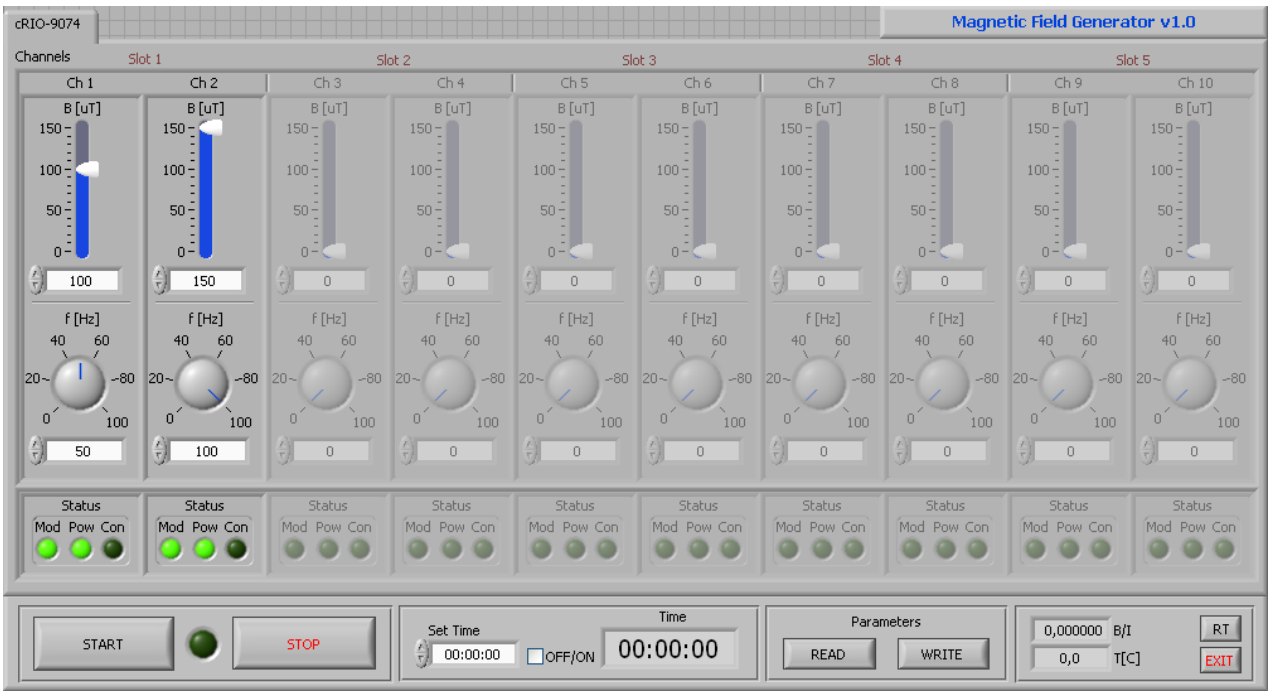


Fig. 1. Control panel for system parameters

Brief description of LabVIEW graphic environment

Programs written in LabVIEW, known as virtual instruments ("vi"), consist of two interconnected layers. The first one, called Front Panel, is an equivalent of the front wall of the device, which features input/output controls: switches, knobs, displays, etc. The second layer called Block Diagram is the actual code of the program, and it is an equivalent of the internal structure of the device, its electric circuit. It shows the interrelations occurring between the Front Panel elements. These relations can be either simple mathematical-logical operations or highly complex functions. Controls and functions are represented by icons linked together by wires, which are used for data exchange between various components. The flow of data along the wires determines the sequence in which the specific parts of the program code are carried out.

LabVIEW provides a convenient and rapid method for programming control and measuring devices. cRIO-9074 controlling software was developed in LabVIEW 2010 environment from National Instruments.

The code of program providing control in the real-time system was created with the use of additional LabVIEW Real-Time Module, and the FPGA matrix was programmed with the use of LabVIEW FPGA Module.

Software description

This software, installed on cRIO-9074, can be divided into two basic parts. The first one is responsible for Host control, real-time system and its main task is to communicate with the client using a built-in web server, to download the settings and to publish the updated parameters on the Front Panel, as well as to pass them on to the code executed by the hardware in the FPGA matrix. The code of this part was implemented in the state machine architecture with two loops responsible for handling the keys and refreshing the readings of the current module status as well as the temperature inside the controller (Fig. 2).

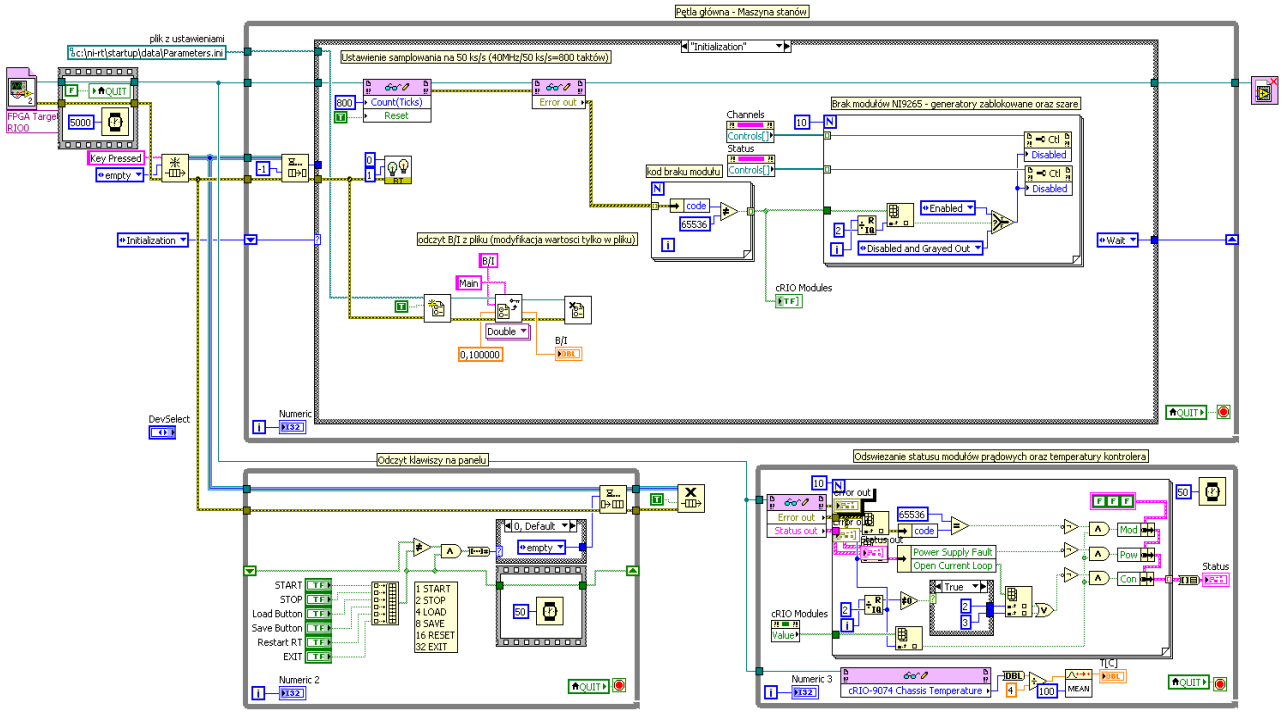


Fig. 2. Code in the state machine architecture responsible for Host control

The second part is the code used for programming the FPGA matrix which produces hardware-managed code generating 10 pairs of independent sinusoidal waveforms with amplitudes and frequencies set by the Host software (Fig. 3).

Samples of the sinusoidal waveform curve can be found in the lookup table. One period contains 1,024 samples. Since the NI 9265 module does not have the feature of generated current direction change, the charge of the current sample (positive or negative value) determines which of the double-pull coil windings is powered. The operating frequency of FPGA system is 40MHz and the main loop executed on it waits 800 clock cycles and is performed at the frequency of 50kHz.

This is the frequency with which current source modules "refresh" their outputs, regardless of the programmed frequency f. This results in smooth sinusoids, simultaneously for all the waveforms, even at extreme settings of frequency f.

Application of the system and preliminary findings

The proposed system of magnetic stimulation was used during the vegetation of crops whereby their fruits were magnetically stimulated a few times, with the use of six 5-minute long doses and with intervals of five days between the consecutive treatment cycles.

The fruits were placed centrally within the coils to ensure uniform distribution of the magnetic field affecting the object (Fig. 4). The findings were analyzed with the use of Vizimag 3.19.

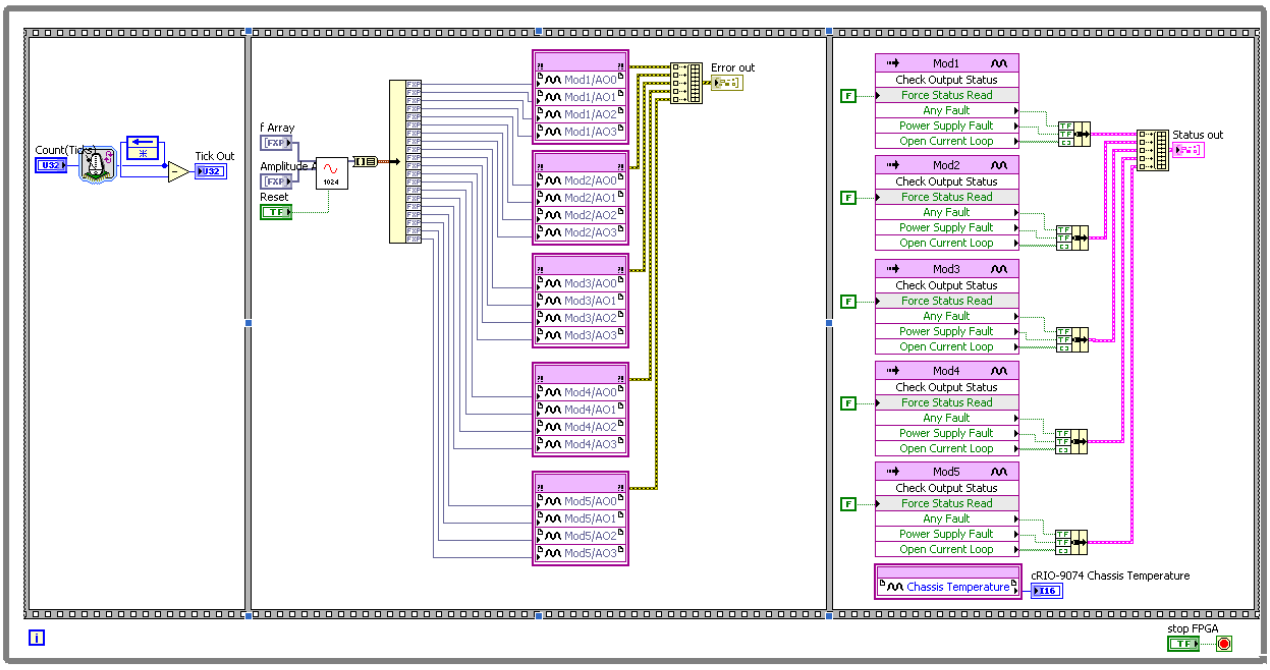


Fig 3. Generating code for the FPGA matrix

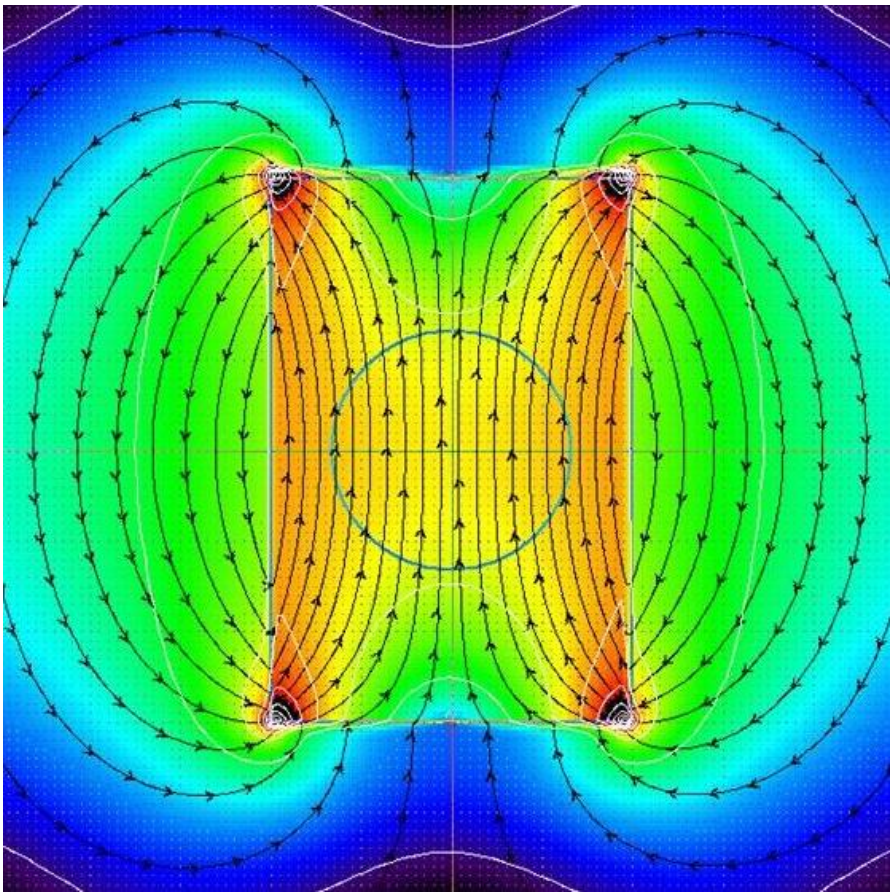


Fig.4. Model distribution of magnetic field in the applied solenoid

CONCLUSIONS

The stimulation treatment led to improved taste of the fruits resulting from increased contents of simple sugars, i.e. glucose and fructose, which were analyzed using a near-infrared Fourier spectrophotometer FT-NIR MPA from Bruker Optics. The proposed magnetic field generation concept has been patented as an invention: A device generating variable magnetic signals to stimulate biological material, under the number P.399624.

REFERENCES

1. **Byczkowska-Lipinska L., Wojnarowska A., 2001.** Problems of electromagnetic compatibility: Electromagnetic effects in biological objects. PTZE publishing, 114-121.

2. **Ciesla A., Kraszewski W., Skowron M., Syrek P., 2015.** Effect of magnetic field on the germination of seeds. Electrical Review. 91(1), 125-128.

3. **Coey J.M.D., Cass S., 2000.** Magnetic Water Treatment. Journal of Magnetism and Magnetic Materials. 209(1), 71-74.

4. **Gut M., 2007.** Impact of alternating electric field on the growth and yield of potato tubers. Agricultural Engineering. 8(96), 73-79.

5. **Kornarzynski K., Gladyszewska B., Pietruszewski S., Segit Z., Lacek R., 2004.** Assessment of the effect of alternating magnetic field on germination of hard wheat caryopses. Acta Agrophysica. 4(1), 59-68.

6. **Kornarzynski K., Pietruszewski S., 2005.** The influence of large doses of alternating magnetic fields on germination of seeds of durum wheat. Agricultural Engineering. 4(2), 11-20.

7. **Kornarzynski K., Pietruszewski S., 2008.** Influence of alternating magnetic field on the germination of low germination. Acta Agrophysica. 11(2), 429-435.

8. **Kornarzynski K., Pietruszewski S., Segit Z., Szwed-Urbas K., Lacek R., 2004.** Preliminary study on the impact of direct and

alternating magnetic field on the rate of increase in wheat germ. Acta Agrophysica. 3(3), 521-528.

9. **Marks N., Szecowka P., 2011.** The stimulation effect of seed alternating magnetic field on the yield of potatoes. Acta Agrophysica. 17(1), 135-150.

10. **Matwijczuk A., Kornarzynski K., Pietruszewski S., Krupa M., 2012.** The influence of the magnetic field and magnetically treated water on selected morphological and chemical composition of sunflower seedlings (Helianthus annuus L.). Acta Agrophysica. 19(3), 621-630.

11. **Pietruszewski S., Kornarzynski K., Lopucki M., 2007.** Magnetic water, some of its properties physical and application. Telecommunication Review. 80(9), 675-682.

12. **Podlesny J., Gendarz M., 2008.** The impact of magnetically treated water on the growth, development and yielding of two pea genotypes. Acta Agrophysica. 12(3), 101-110.

13. **Podlesny J., Gendarz M., 2008.** The impact of magnetically treated water on the growth and yield of two pea genotypes, Agricultural Engineering. 12 (3), 767-776.

14. **Podsiadlo C., Lesniak E. 2009.** The impact of magnetically treated water on germination and initial growth of selected plant species. Infrastructure and Environment Rural. 3, 213-221.

15. **Terlecki J., 2001.** Biophysics. PZWL publishing. 668-695.

16. **Wadas R., 1978.** Biomagnetism. PWN publishing. 66-69.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 79-84

RESEARCH INTO ELECTRONIC CONTROL SYSTEMS EDC

M. Dziubinski, A. Drozd
Lublin University of Technology, e-mail: m.dziubinski@pollub.pl

Received November 21.2016: accepted December 22.2016

Abstract. The article present proposed diagnostic procedures of the EDC 1.2.9 system made it possible to carry out tests and compare the results with the results provided by the manufacturer. As part of the research prior to diagnostic tests, verification tests of Siemens and Bosch pumps were carried out. Experimental study, test procedures making it possible to check particular elements of the EDC system were developed. The developed research program included the fuel system and tests were conducted for: the air intake system, engine sensors, vehicle sensors, the exhaust fumes emission system, transmitters and system elements on the controller connection. The second system, which was subject to experimental testing, was the injection system Common Rail (CR), on which tests of three sets of injectors were conducted. The following values were recorded: engine rotational speed, driver injection dose, pre-injection time, fuel dose in the main injection, fuel dose correction, the adjustment of idle running for the first, second, third and fourth cylinder. The conducted diagnostic tests make possible to predict the state of particular EDC systems reliability.

Key words: electronic diesel control, common rail, injection dose, diesel fuel pump.

INTRODUCTION

The electronic control system EDC (Electronic Diesel Control) meets high demands which modern injection systems face. Unlike in vehicles with diesel engines equipped with conventional row or distributor injection pumps, in the EDC system the driver has no direct influence on the dose of the injected fuel, e.g. by pressing the accelerator pedal cooperating with the pump through the pull system.

The appropriate injection dose is calculated by the system on the basis of the obtained information, e.g. about the state of engine work, the decision of the driver, the emission of exhaust fumes etc. The concept of safety realized by the system consists in identifying common errors and introducing appropriate corrections, e.g. reducing torque or emergency operation in idle running rotational speed. The electronic system of engine control also enables exchanging data with other electronic systems of the vehicle (with the ASR system and the system of gearbox electronic control) for the purpose of increasing driving comfort and safety [7, 8, 9, 13, 16].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

Modern engines have to meet increasingly strict standards concerning the reduction in the emission of harmful fuel components and noise. This means higher requirements concerning the electronic injection system EDC and its regulation:

– high injection pressure,
– shaping the course of injection,
– variable injection start,
– variable injection advance,
– adjusting injection dose, charge pressure and injection start to each state of engine work,
– starting dose adjusted to temperature,
– regulation of idling rotational speed independent of load,
– driving speed control,
– controlled exhaust fumes recirculation,
– preserving small tolerances and high precision of engine parts [3-5].

Signals from sensors are fed to the EDC controller by the following circuits:

– analog input signals, information from analog sensors concerning the amount of sucked-in air, pressure, engine temperature, sucked-in air temperature, battery voltage; they are converted into digital values in the microprocessor of the controller;
– digital input signals, impulses of Hall generator rotational speed, which can be processed directly in the microprocessor;
– pulse input signals from induction sensors with information about rotational speed, positions or references, which are prepared in a part of the driver electric circuit for the purpose of removing interference, and converting them into square-wave signals.

Final elements of the EDC are protected against fault to frame, higher battery voltage as well as damage due to current overload. The occurrence of this type of damage and interruptions in circuits is recognized by final elements and transmitted to the engine controller. Selected input signals of the controller are also transmitted to other systems.

The diagnostic system included in the controller is one of the basic electronic blocks of engine control. In addition to self-diagnostics, the driver supervises input and output signals as well as the communication between controllers [1, 2, 15, 17, 20, 25, 32].

Damages which cannot be detected by the board diagnostic system can be located by means of control measurements. These additional functions of the system diagnostics are built into the engine EDC controller or into a diagnostic tester and may be carried out by means of a diagnostic tester.

Measurements of basic electric values such as current, voltage and resistance can be performed with the use an all-purpose gauge or a diagnostic device. The use of an oscilloscope can make it possible to perform the verification of signals for regulator units. This is particularly important for regulator units which have not been programmed in the diagnostic block. Diagnosing increases the control of sensors and steering regulator units.

VERIFICATION OF COMMON RAIL SYSTEM ELEMENTS

As part of the research prior to diagnostic tests, verification tests of Siemens and Bosch pumps were carried out. The most common damages to the pumps are caused by the pollution of fuel. After dismantling, one can observe the damaged parts which were affected by corrosion, such as pressure sections, the shaft, the cam. Such damages occur when fuel polluted by a small amount of acids is used.

Damages, which occur most frequently, include:

1 - Damages to section valves,

2 - Damages to sections that are subject to blocking or uneven wear (Fig. 1),

3 - Damage to the pump housing (Fig. 2),

4 - Damages to the shaft as a result of friction, visible grooves (Fig. 3),

5 - Damage to the section cover, microcracks occur,

6 - Damage to the roller (Fig. 4).

Fig.1. Damage to the pressure section – uneven wear

Fig.2. Damage to the pump housing – indentation visible

Fig.3. Damage to the shaft – grooves visible

Fig.4. Surface damage to the roller – deformation visible

EXPERIMENTAL STUDY EDC SYSTEM

As part of the experimental study, test procedures making it possible to check particular elements of the EDC system were developed. The subject of the study was the EDC 1.2.9 system (wiring diagram – Fig. 6) mounted on the test bench (Fig. 5).

The developed research program included the fuel system and its results are presented in Table 1. Tests were conducted for: the air intake system, engine sensors, vehicle sensors, the exhaust fumes emission system, transmitters and system elements on the controller connection.

The second system which was subject to experimental testing was the injection system Common Rail (CR) shown in Fig. 7, on which tests of three sets of injectors were conducted. The following values were recorded with the use of the KTS 500 device: engine rotational speed, driver injection dose, pre-injection time, fuel dose in the main injection, fuel dose correction, the adjustment of idle running for the first, second, third and fourth cylinder. By varying rotation speed from 800 rpm. to 1600 rpm, particular values shown in Table 2 were recorded, and on the basis of them, the characteristics of fuel dose in the main injection (Fig. 8) and in the drive injection (Fig.9) were determined. The examined injectors have an efficient electromagnetic coil, which can be recognized by characteristic chirring. Only in one of them injection was observed, and in the other three the fuel dose was directed to a spill.

Fig.5. Test bench for testing the EDC system (1-battery, 2-controller engine, 3-fuel tank, 4-pump drive motor, 5- speed control system, 6- elements of the electronic control, 7-injection pump and fuel injectors)

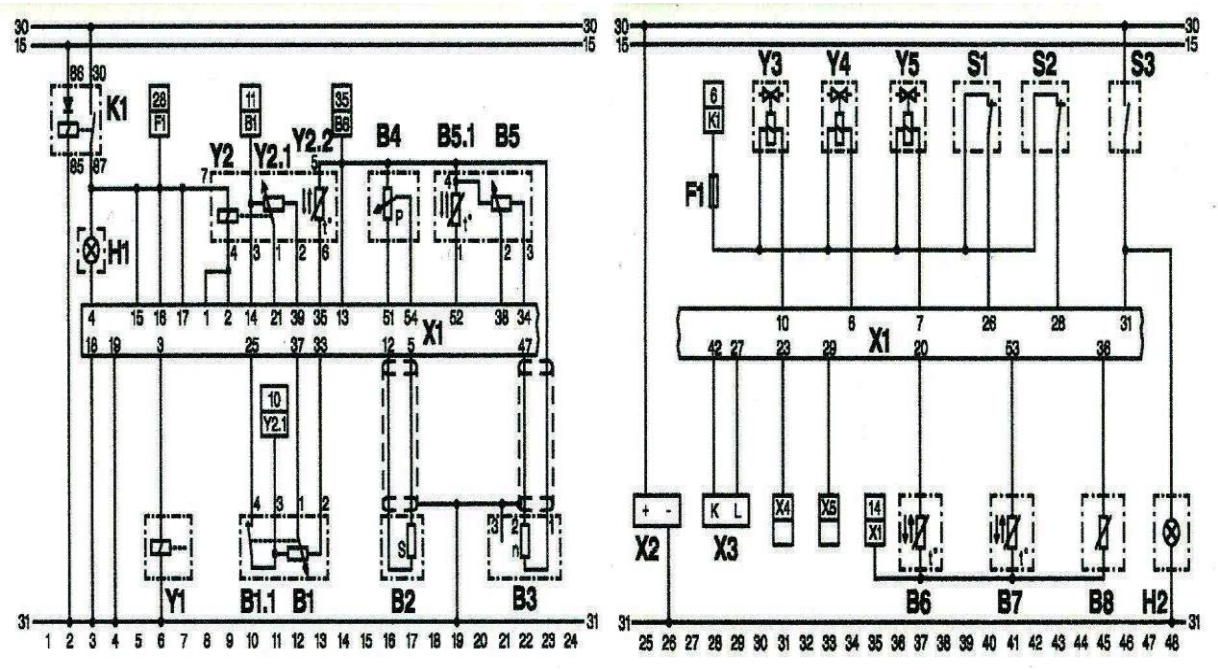


Fig.6. Diagram of the EDC wiring system. K1 - main transmitter, H1 - diagnostic light, Y1 - stop solenoid valve, Y2 - electromagnetic adjuster of fuel dose, Y2.1 – batcher position sensor, Y2.2 - fuel temperature sensor, B1 / B1.1 - accelerator lever position sensor, B2 - needle lift sensor, B3 - speed sensor, B4 - charge pressure sensor, B5 - air flow meter, B.5.1 - air temperature sensor, X1 - engine controller, X2 - power connector of the diagnostic device, X3 - diagnostic connector, F1 - fuse 10A, Y3 – solenoid valve of the injection start, Y4 - EGR solenoid valve, Y5 - solenoid valve of pressure charge regulation, S1 - brake switch 1, S2 - clutch switch, S3 - brake switch 2, B6 - air temperature sensor, B7 - engine temperature sensor, B8 - liquid level sensor, H2 - stop light switch

Table 1. Table presenting measurement results of the fuel system

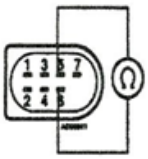
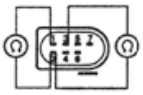
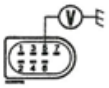
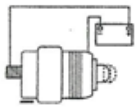
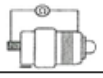
1- fuel system			
Element of the system	No. pin	State check	Measurer indication
Fuel temperature sensor 	5-6	0°C	6kΩ
	5-6	10°C	
	5-6	20°C	
	5-6	40°C	
	5-6	60°C	
	5-6	70°C	
	5-6	80°C	0,25kΩ
Batcher position sensor 	1-3		1.5KΩ
	2-3		2.5KΩ
Fuel batcher 	5-ground	Ignition on	11.5V
Adjuster injection start 	2-3		17Ω
Adjuster injection start 	3-ground	Ignition off	11.5V
Cut off fuel supply solenoid valve 		Battery connected	Open valve
		Battery in-connected	Close valve
Cut off fuel supply solenoid valve 			8Ω

Table 2. Measurement results of the first set of injectors

values	recorded values			
engine rotational speed	800 rpm	1000 rpm	1200 rpm	1600 rpm
drive injection dose	43 mg/H	35 mg/H	35 mg/H	30 mg/H
pre-injection time	0,0 ms	0,0 ms	0,0 ms	0,0 ms
fuel dose in the main injection	431 mg/H	433 mg/H	435 mg/H	440 mg/H
start main injection	0,6 Crank.S	0,7 Crank.S	0,9 Crank.S	1 Crank.S
fuel dose correction, adjustment of idle running, 1 cylinder	-0,3	-0,3	-0,2	-0,3
fuel dose correction, adjustment of idle running, 2 cylinder	0,1	0,1	0,1	0,1
fuel dose correction, adjustment of idle running, 3 cylinder	0,1	0,1	0	0,1
fuel dose correction, adjustment of idle speed running, 4 cylinder	0,1	0,1	0,1	0,1

Fig. 7. Test bench for simulating the work of the EDC system

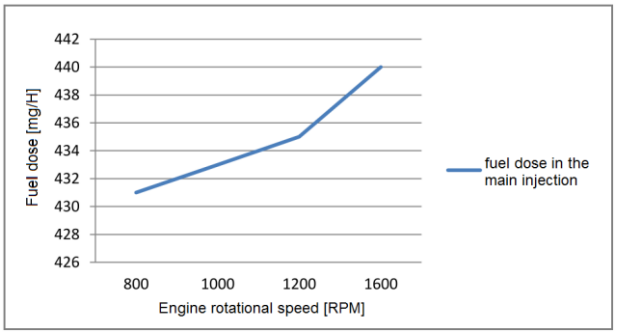


Fig.8. Fuel dose in the main injection

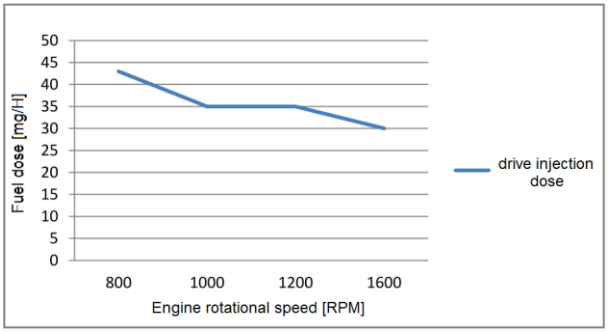


Fig.9. Drive injection dose

CONCLUSIONS

A part of experimental studies of the EDC system was verification analysis carried out by dismounting a high pressure pump which made it possible to identify the most frequent damages to the control system elements shown in the figures.

The proposed diagnostic procedures of the EDC 1.2.9 system made it possible to carry out tests and compare the results with the results provided by the manufacturer. The conducted simulation of damages to electronic elements facilitated the identification of error codes by means of the diagnostic equipment KTS 500 from Bosch. In the course of research, the developed bench enables checking the parameters of other pumps thanks to the universal mounting and the adjustment of the driving engine rotational speed with the use of the inverter.

The conducted study of the Common Rail system made it possible to determine the characteristics of injectors for the fuel dose in the main injection and the drive injection dose at variable engine rotational speed.

The designed and prepared test bench makes it possible to simulate the work of the speed sensor, the camshaft position sensor and determines the impact of particular signals on the parameters of the system. The complete installation of the connections of sensors and driver elements with the controller of the CR system facilitated the observation and simulation of the full diagnostic process and the verification with the theoretical model.

The conducted diagnostic tests preceded by the verification and analysis of the signs of wear after dismantling the pumps and injectors make it possible to predict the state of particular EDC systems reliability.

REFERENCES

1. **Artyukh A., Sidorov M., 2014.** Mathematical modeling and numerical analysis of nonstationary plane-parallel flows of viscous incompressible fluid by R-functions and Galerkin method. ECONTECHMOD. An international quarterly journal, Vol. 3. No. 3, 3-11.

2. **Bochenski C., Mruk R., 2002.** Diagnostic processes of Common Rail unit based on computer systems. Diagnostics, Vol. 27, 11-16.
3. **Bosch R., 2009.** Common Rail injection system, Transport and Communication Publishers. Warsaw.
4. **Bosch R., 2009**. Common Rail accumulated injection system. Transport and Communication Publishers. Warsaw.
5. **Brzezanski M., 2010.** New standards for emissions in self-ignition engine – VW 2.0 TDI engine. Combustion Engines, No. 1, 57-63.
6. **Czech P., 2013.** Application of probabilistic neural network and vibration signals for diesel car engine fuel injectors damage. Scientific Papers of Silesian University of Technology, Transport Series. Vol. 81, 25-30.
7. **Dziubinski M., 2004.** Examination of Automobile Electronic Equipment, Scientific Publishing Gabriel Borkowski, Lublin.
8. **Dziubinski M., 2003.** Automobile electronic systems. Scientific Publishing Gabriel Borkowski, Lublin.
9. **Gautier C., Sename O, Dugard L., Meissonnier G., 2007.** An LFT Approach to H Control Design for Diesel Engine Common Rail Injection System, Oil & Gas Science and Technology, Rev. IFP. Vol. 62, No. 4, 513-522.
10. **Gladysek J., Gladysek M., 2009.** Repair of Common Rail injectors. Auto Moto Service. No. 10.
11. **Gladysek J., Gladysek M., 2009.** Evaluation of Common Rail injector. Auto Moto Service. No. 9.
12. **Gunther H., 2014.** Common Rail injections system in practice workshop. Transport and Communication Publishers, Warsaw.
13. **Gunther H., 2009.** Diagnostics of Diesel engine. Transport and Communication Publishers, Warsaw.
14. **Idzior R., Borowczyk T., Karpiuk W., Stobnicki P., 2011.** The feasibility research of a technical condition of modern injectors for compression ignition. Logistics. N3.
15. **Ignaciuk P., L. Gil., 2014.** Damages to injectors in Diesel engines. Advances in Science and Technology Research Journal. Vol. 21, No. 8, 58-61.
16. **Janiszewski T., 2010.** Mavrantzas Spiros, Electronics injection systems of Diesel engines. Transport and Communication Publishers, Warsaw.
17. **Karczewski M., Koliński K., 2011.** Diagnosis of injectors through common rail pressure measurement, Scientific Papers of the General Tadeusz Kościuszko Military Academy of Land Forces in Wroclaw, No. 1.
18. **Kech J., Willmann M., Gorse P., Boog M., 2011.** Common Rail fuel injection, key technology for clean and economical combustion.
19. **Konieczny Ł., Adamczyk B., Adamczyk G., 2015.** Diagnostics and regeneration of Common Rail injectors. Scientific Papers of Silesian University of Technology, Transport Series, Vol. 86, 65-73.
20. **Kovalyshyn O., Gabriel Yu., 2014.** Development of management systems model of automatic control by using fuzzy logic. ECONTECHMOD. An international quarterly journal, Vol. 3. No. 4, 87-90.
21. **Kuszewski H., Lejda K., Ustrzycki A., 2008.** Determinants of the fuel dosage accuracy in a dispenser supply system Common Rail, Technical magazine. Mechanics, Cracow University of Technology Publisher.
22. **Kuszewski H., Ustrzycki A., 2007.** The influence of working parameters of common rail system on a real fuel injection beginning. Combustion Engines, Vol. 46, 107-117.
23. **Lino P., Maione B., Pizzo A., 2007.** Nonlinear modelling and control of a common rail injection system for diesel engines. Applied Mathematical Modelling, Vol. 31, 1770–1784.
24. **Łukasiewicz M., Dobek S., Kałaczyński T., 2014.** The study of selected elements of the common-rail Diesel engine. Proceedings of Polish Association for Knowledge Management, No. 69, 141-155.
25. **Merkisz J., Mazurek ST., 2004.** Automobile on-board diagnostic systems. Transport and Communication Publishers, Warsaw.
26. **Osipowicz T., Abramek K., 2014.** Catalytic processing of the fuel injectors in the diesel engine, Szczecin.
27. **Osipowicz T., T. Stoeck., 2013.** Repair contemporary Diesel engine injectors. Buses: technology, exploitation, transport systems, Vol. 14, No. 10, 210-212.
28. **Stoeck T., T. Osipowicz, K.F. Abramek., 2014.** Methodology for the repair of Denso common rail solenoid injectors. Maintenance and Reliability, Vol. 16, No. 2.
29. **Ustrzycki A., Kuszewski H., 2006.** The study beginning fuel injection in Common Rail injection system. Materials XVII International Conference on. Computational Methods and Research in Development of Motor Vehicle and Heavy Machinery Self-propelled. Management and Marketing in the Automotive Industry. SAKON 2006, Rzeszow.
30. **Ustrzycki A., Kuszewski H., 2007.** The effect of temperature on fuel injected dose in the tank fuel injection Common Rail. Materials XVIII International Conference on. Computational Methods and Research in Development of Motor Vehicle and Heavy Machinery Self-propelled. Management and Marketing in the Automotive Industry. SAKON 2007, Rzeszow.
31. **Wierzbicki S., Smieja M., 2014.** Controlling the fuel feeding system in a self-ignition engine under fast-prototyping conditions using LabView, Measurement Automation Control, Vol. 60, No. 11, 1049-1052.
32. **Lut M., Mrachkovskyy A., Taran V., 2012.** Measuring equipment and software products in vibrodiagnostics. ECONTECHMOD. An international quarterly journal, Vol. 1, No. 1. 67-70.

ECONTECHMOD. AN INTERNATIONAL QUARTERLY JOURNAL – 2016, Vol.5, No.4, 85-95

BLACK BOX DYNAMIC MODELLING OF PROTON EXCHANGE MEMBRANE FUEL CELLS WITH ARTIFICIAL NEURAL NETWORKS

J. Kapica
Department of Electrical Engineering and Measurement Systems,
University of Life Sciences in Lublin; e-mail: jacek.kapica@up.lublin.pl

Received November 21.2016: accepted December 23.2016

Abstract. The fuel cells are energy sources which can play an important role in transition of the energy sector into broader use of renewable energy. Numerical modelling provides an easy way to investigate properties of the objects modelled. There are various ways to model dynamic behaviour of the PEM fuel cells including methods using artificial neural networks. There are no clear rules of how a neural network should be configured: how many neurons in the hidden layer and which training algorithm should be used. In a time series modelling task additional parameters including sampling frequency, learning data set duration and number of past data points used for training need to be determined. The paper presents results of research on the influence of various model parameters on the PEM fuel cell modelling accuracy.

Key words: PEM fuel cells, neural network model, dynamic behaviour, black box.

INTRODUCTION

The fuel cells (FC) are among the most promising technologies offering transition of the energy sector from the traditional power sources into a new stage. One of the application is energy generation from hydrogen produced by a renewable power source with variable output (mostly wind or photovoltaics). A FC stack supplemented by hydrogen generator creates a power storage unit improving the functionality and usefulness of the renewable energy generation set by load levelling and customer-side peak shaving. In general, storage techniques can be classified into the following four major groups, with FC systems categorized under chemical energy storage technologies [2]:

- electrical energy storage technologies, including: capacitor and supercapacitor storage, superconducting magnetic energy storage,
- mechanical energy storage technologies, including: flywheel energy storage, compressed air energy storage, pumped hydro storage ,
- chemical energy storage technologies, including: battery energy storage, fuel cell systems,
- thermal energy storage technologies, including: aquiferous thermal energy storage, cryogenic energy storage, hot thermal energy storage, pumped heat electrical storage.

Currently there are six types of established fuel cell technologies available on the market [10]:

- Proton Exchange Membrane Fuel Cell (PEMFC),
- Alkaline Fuel Cell (AFC),
- Direct Methanol Fuel Cell (DMFC),
- Phosphoric Acid Fuel Cell (PAFC),
- Molten Carbonate Fuel Cell (MCFC),
- Solid Oxide Fuel Cell (SOFC).

The PEMFCs are considered as having important advantages over other technologies [17]: ability to operate continuously at low temperature and high current densities, long stack life, short start-up time, capability of discontinuous operation, small size and high tolerance to shock and vibration. These features make this technology useful in the creation of electric vehicles, including bicycles, motorcycles, mini-trains, cars and buses[30]. Currently, the main disadvantages are high capital cost and low round trip efficiency of hydrogen fuel cell storage systems, at the level of 40-45 % [3].

Fuel cells, as a relatively new technology, are costly devices which results in high prototyping costs. In order to decrease the expenditure, numerical modelling can be used for general assessment of properties of the system in the preliminary state of the design work. One of the properties of the PEMFCs is scalability, so obtaining a good model of a small (and inexpensive) FC stack allows the creation of a model of a higher power stack, which can be used to test various configurations and layouts of the energy storage or production system.

Another advantage of numerical modelling is that with efficient algorithm and computer system the simulation usually is faster than a real-world experiment.

There can be various approaches to FC modelling. Some authors present models based on the physical and chemical laws that govern inside the cell, thus creating physical or semi-physical models [31, 4, 14, 11]. These models usually provide good accuracy, but they are not convenient to use. The main reason is that they require the cell parameters which are not directly available. Simulating dynamic states is even more challenging as it involves complex phenomena.

The other group is often called “black box”. The name suggests that the model treats the simulated object as a processing unit in which one must find dependency between inputs and outputs without trying to resemble the phenomena taking place in the device. Another name used is data-driven models which suggests that the model is created on the foundation of experiments and

measurements conducted on the object being modelled. The methods used include: Uryson-Model [16], constructing equivalent electrical circuit [22], adaptive spline modelling of observation data [25], non-linear autoregressive moving average model with exogenous inputs [7].

The artificial neural networks (ANNs), as data-driven methods, have also been used to model both static and dynamic performance of the PEMFCs [19, 24, 18, 27]. They are mathematical structures inspired by biological neural systems. The most commonly used architecture is a multilayer perceptron which makes use of weights, usually sigmoid activation functions and a hidden layer consisting of varying number of nodes [23]. There is a number of applications of ANNs which can be divided into the following groups [26]: classification (pattern recognition) [29], density estimation, clustering and regression [20], including modelling of time series [9]. In this case the inputs consists of chosen number of past values of the input variables, enabling forecasting future output of a system. This can be done in one of two ways: open loop, when the output is predicted when past measured values of the output are available and closed loop, when only output of the model for the past points of time is available. The first situation takes place in the one step ahead forecasting, the latter in modelling of a system when only the initial state is known or assumed. A major advantage over other modelling techniques is low computational requirements once the network has been trained.

The selection of the hidden layer size is often regarded as "art" as there are no clear rules of how to approach this problem. Some classical textbooks set the "rule of thumb" that the number of hidden neurons should not be greater than the number of inputs [6, 28]. In practice, the preferable network size depends on the particular case of the object modelled. In case of the times series modelling there are additional questions about how long the training period should be, what sampling frequency will provide best results.

Most, if not all of the publications presenting ANN models are not fully "black box" as they take the inner temperature of the cell as one of the input parameters. The approach presented in this paper can be regarded as a true "black box" as it uses ambient temperature as one of the inputs.

The purpose of the research presented in this paper is to create a simple ANN model driven entirely on data external to the PEM FC, determine which hidden layer size - training set duration - sampling frequency - training function - performance function combinations give better results than other. Also, it was interesting to check whether it is necessary to use temperature and hydrogen pressure as inputs in order to achieve satisfactory modelling accuracy.

ARTIFICIAL NEURAL NETWORKS AND TIME SERIES MODELLING

A feedforward neural network with one input, one hidden and one output layer can be regarded as one of the simplest ANN configurations (Fig. 1). The input layer consists of I nodes, with each node representing one input variable $x_1 \ldots x_I$. The main structure within the network is the hidden layer made of N neurons.

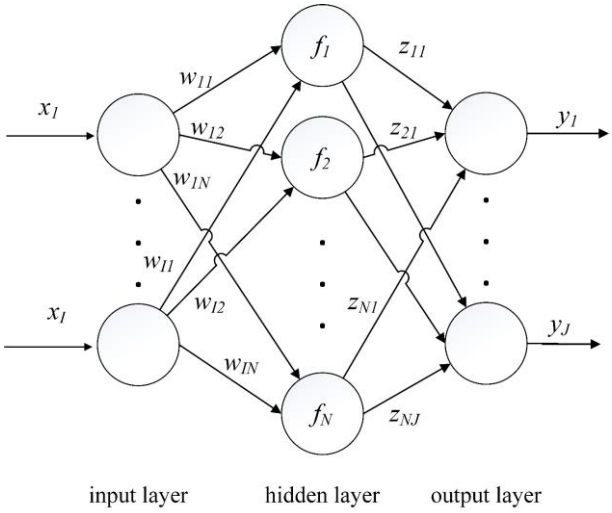


Fig. 1. Structure of a feedforward ANN with one hidden layer

Each of the hidden neurons performs an operation adding products of input variables x, the corresponding weights w and biases b with the use of an activation function g [12]:

$$f_n = g\left[\sum_{i=1}^{I}(w_i + b_n)\right], \quad i = 1,2,...,I\,, \quad n = 1,2,...,N\,. \tag{1}$$

Usually g is a log-sigmoid function defined as:

$$g(x) = \frac{1}{1+e^{-x}}\,, \tag{2}$$

or a tan-sigmoid function [8]:

$$g(x) = \frac{2}{1+e^{-2x}} - 1\,. \tag{3}$$

The output layer performs calculations according to equation:

$$y_j = \sum_{n=1}^{N} f_n z_{nj}, \quad j = 1,2,\ldots,J, n = 1,2,\ldots,N\,. \tag{4}$$

The major step in creation of an ANN model of a given object is training of the network. In this process the information is stored in the weights and biases. It involves minimising a cost function C by finding near-optimal values for the network weights. This is done by feeding the inputs with an example data and calculating partial derivatives of C with respect to all the individual weights within the network [5] with required outputs known from the example data. The cost function can have various forms, one of them is a mean squared error (6).

Time series is a sequence of vectors *X(t)*, with *t* denoting successive points in time, where *t*=1, 2, ... , *n*. *X* is commonly obtained from measurements, in which one of the stages is sampling the continuous signal in order to obtain discrete, usually uniformly spaced in time, data points.

The most common task where ANNs are used is forecasting the *X* values of one or more time steps ahead. This can be achieved by creating a network with input vector *I* consisting of stacked *k* past values of *X*. Assuming that *X* is formed of *m* scalars:

$$I = \begin{bmatrix} X(1,t-k), X(1,t-k+1),..., \\ X(1,t-1), X(2,t-k), X(2;t-k+1),..., X(2,t-1),..., \\ X(m,t-k), X(m,t-k+1),..., X(m,t-1) \end{bmatrix}. \quad (5)$$

Vector *I* is used to model the value of *X* at time point *t*. In addition to the past values of *X*, other variables can be used in creation of *I*. Examples of applications of ANNs to time series modelling include forecasting in the area of finances [1], weather [15], electricity consumption [13], hydrology [21] and other.

EXPERIMENTAL SET-UP AND METHODS USED

Fig. 2 shows the experimental set-up. It consists of a PEM fuel cell, metal hydride hydrogen storage tank and electronic programmable load. In order to measure the parameters current, voltage, temperature and pressure sensors were used. The whole experiment was controlled and the data collected by a computer program developed by the author. The temperature, current and voltage were measured via an analogue to digital converter and the pressure through the RS232 interface. The electronic load was also controlled using the RS232 interface. In order to control the ambient temperature, the fuel cell was placed in a climate chamber. The fuel cell used was a 12 W PEM fuel cell stack, type H-12 manufactured by Horizon. It is cooled by air with an integrated fan. The compact construction makes it suitable for small projects. It is a simple, self-humidified air-fed stack.

The first step was to collect data from an experiment, during which the ambient temperature was changed from 10 to 40 °C. The hydrogen pressure was changing only as a result of the tank discharge and varied between 50 and 56 kPa. During the experiment, the fuel cell was put under load of various current levels changed in a step-like and triangle manner in the range of 0 to 2 A. The data were collected over a period of 28 minutes with the rate of 400 samples per second, which resulted in the raw set of almost 70 000 data points used later to produce input and output sets for the neural network training and testing.

The network was configured as feedforward with backpropagation training algorithm. In order to test the accuracy of the modelling various configurations of the training algorithm and input data set were used which are summarized in Table 1. The two training function algorithms were chosen for the sake of low training computation time as compared to other. In this way 1440 combinations of the parameters were obtained. The following stoppage criteria were defined: the maximum of 400 training epochs or 6 maximum validation failures, whichever comes first.

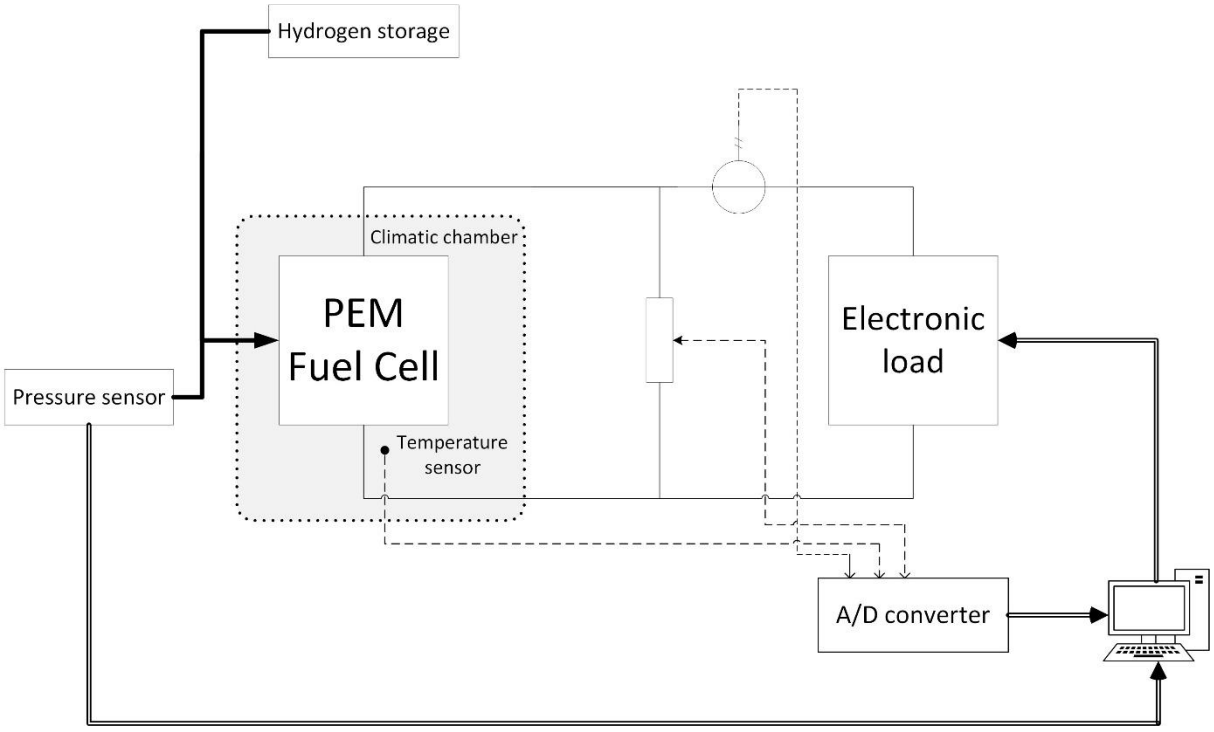


Fig. 2. Diagram of the experimental set-up

Table 1. Values of the parameters changed for modelling accuracy testing

Parameter	Values
Resampling	1, 2, 3
Number of delays	1, 2, 3, 4
Duration of the training and testing set	50, 100, 150, 200, 250 seconds
Training function	Levenberg-Marquardt (trainlm) and Bayesian regulation backpropagation (trainbr),
Performance function	Mean squared normalized (mse), Sum squared error (sse)
Number of neurons in the hidden layer	2, 5, 8, 11, 14, 17, 20, 23

Resampling procedure relates to collecting every i-th data point and deleting other points, thereby reducing the effective sampling frequency. Delay parameter specifies how many past data points are fed into the neural network as input.

Neural network model was implemented in Matlab. A script has been created to handle the parameter variation, training and testing of the network. The procedure involved dividing the original raw set into 30 pieces evenly distributed in time. As the temperature was changing in time, the subsets were also distributed according to temperature. Half of the sets were used for training (with subdivisions into training, testing and validation). The other half was then used to test the modelling accuracy. This was done in order to prevent the effect of over-fitting influencing the results - in our case the learning set and set used for testing and error estimation are completely different. The tests were performed in a closed loop configuration, which means that the past voltage data points were not taken from measurements but as a result of modelling from the previous time points, as it would be in a practical modelling task, where the previous voltage values are not known but have to be calculated. Fig. 3 shows the data flow in the model.

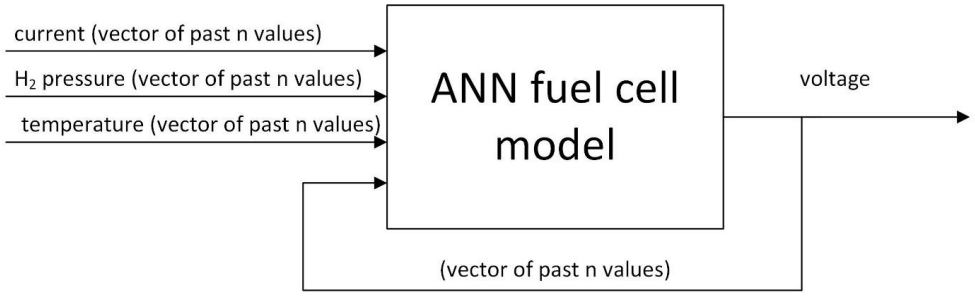


Fig. 3. Data flow in the model

Additionally, three other tests have been performed: modelling without using temperature as input, without pressure and without both of these parameters. This was done to test how the network will perform when limited data is available.

The outcome of the experiments was a 6-dimensional array containing mean square error (MSE) calculated as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - Y_i\right)^2 . \qquad (6)$$

where: X_i is the modelled voltage (output of the ANN) and Y_i is the measured voltage at point i.

RESULTS AND DISCUSSION

In order to evaluate the influence of various factors on the modelling accuracy the results were analysed in two ways. The first method used was to divide the results into two groups: one under a certain MSE level, the other above that point. The MSE threshold values were calculated according to the Matlab quantile function, which returns quantile value for a given probability. Table 2 shows MSE threshold value corresponding to the given quantile threshold probability level, minimum and maximum MSE values. Results are illustrated on Fig. 4 and 5. The 'dot' markers represent configurations for which the MSE was above threshold and the 'x' markers - under the threshold. Therefore, network configurations with better modelling quality are marked with an 'x'.

Table 2. Description of the conditions used in the illustration of the accuracy (Fig. 4 and 5)

Figure	Pressure used as input?	Temperature used as input?	Quantile threshold	MSE threshold (V)	Minimum value of MSE (V)	Maximum value of MSE (V)
Fig. 4a	Yes	Yes	0.003	0.0046	0.0043	143
Fig. 5a	Yes	Yes	0.02	0.0053	0.0043	143
Fig. 4b	Yes	No	0.003	0.0148	0.0137	186
Fig. 5b	No	Yes	0.003	0.0056	0.0053	81
Fig. 4c	No	Yes	0.02	0.006	0.0053	81
Fig. 5c	No	Yes	-	0.0053	0.0053	81

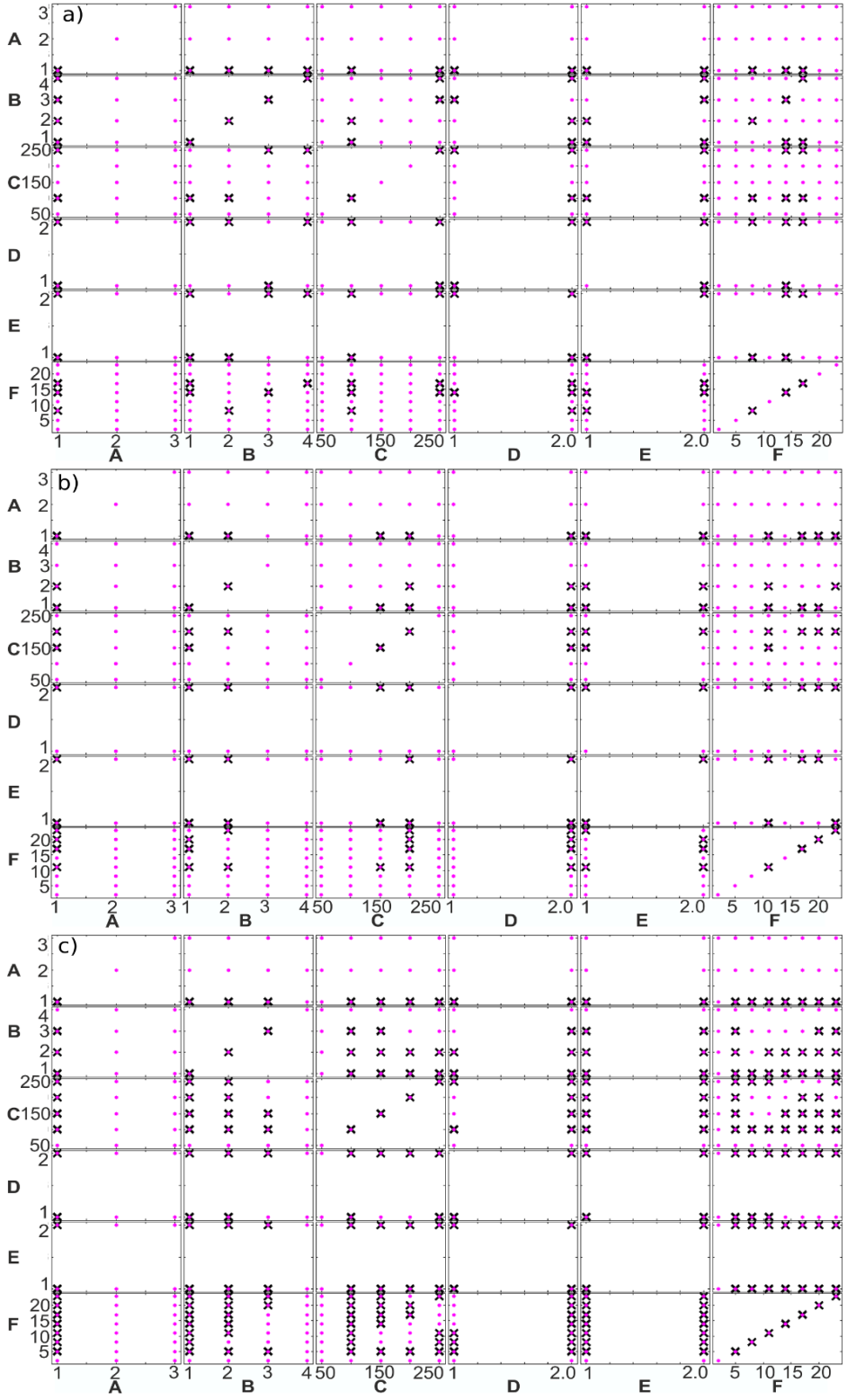


Fig. 4. Accuracy of modelling. The 'x' markers show simulation and learning properties for which MSE is below the threshold. Description of modelling parameters can be found in Table 2. The Levenberg-Marquardt (trainlm) training function is marked as 1 and Bayesian regulation backpropagation (trainbr) - as 2. Mean squared normalized (mse) performance estimator is marked as 1, Sum squared error (sse) - as 2. Axes description: A - Resampling factor, B – Number of delays, C – Training time (s), D- Training function, E – Performance estimator, F – Number of neurons

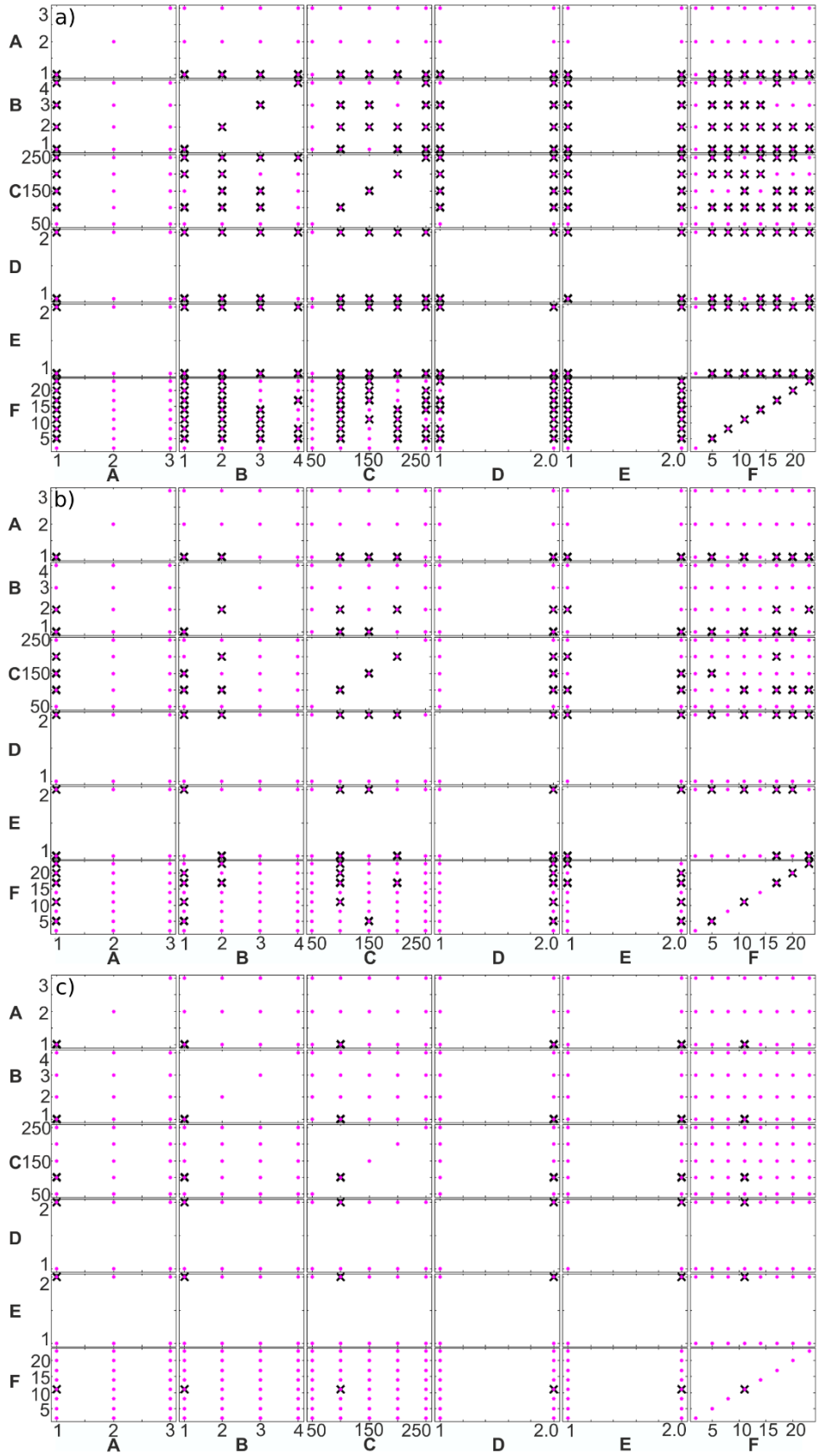


Fig. 5. Accuracy of modelling. Description as in Fig. 4

The table shows that in a case when temperature was not considered, the minimum MSE is much higher than in other cases. Moreover, when both the temperature and pressure were neglected, the accuracy was even worse and therefore not presented in the paper. In the model where only the pressure was ignored the error was slightly higher compared to the case with both temperature and pressure included. This is caused by a low variation of the pressure value during the experiment. All the results show that higher sampling frequency offers the best accuracy of the model. Usually, taking an input vector of just one past value gives best results, except the case shown on Fig. 4a, where longer delays offer good accuracy when combined with longer training period duration. Also increasing the training time period over 100 seconds does not improve results, except the case when the temperature is not taken into account, where at least 150 seconds are needed for better outcome. The Bayesian regulation back-propagation training function usually offers better results. Sum of squared errors serves slightly better as a performance estimator, mean squared error performs in a similar way. Increasing the number of the neurons in the hidden layer over 5 usually improves accuracy. The network can well model the current voltage value with data from just one, past point in time.

In order to achieve similar accuracy in the model without pressure input as in the model with both pressure and temperature (MSE threshold 0.0053 V) the following parameters were needed (Fig. 5c): resampling factor - 1, number of delays - 1, training time - 100 s, training function - trainbr, performance estimator - MSE, number of neurons - 12.

The second way to compare the results was to order them by the MSE, starting with ones with best accuracy (Table 3 and 4 present some of best configurations). Also here it can be seen that shorter example sets in most cases offer best results. Bayesian regulation backpropagation performs better as a training function. It can be seen clearly looking at the tables where this function is placed in most of the cases. To achieve similar results as in the first row of Table 3 with Levenberg-Marquardt backpropagation a vector of 3 past and an example set of 250 s duration is needed and still the MSE is slightly higher. In the case presented in Table 4 with Levenberg-Marquardt backpropagation a network with more neurons in the hidden layer were needed for a similar accuracy as in the first row. Tables 3 and 4 contain a column with time that was needed to train the network. This parameter is highly dependent on the hardware of the computer used so the numbers can be only used to compare various cases. Generally network with more inputs (longer delays) and more nodes in the hidden layer need more time to train, however this is not a strict rule - for example first two rows in both of the tables show that despite larger network size the training time was shorter. The computer used for training was a Windows 7 64 bit - based system with Intel(R) Core(TM) i5-3340M CPU @ 2.70GHz processor with 4 GB of RAM.

Fig. 6 and 7 present time and regression plots of the best performing models from Table 3 and 4. The model output matches the measurement curve well, as can be seen in both cases. In the model without hydrogen pressure considered, the output of the model differs from measurements more significantly especially during fast transitions and floats around the target curve.

The regression plots show very good output approximation by the network. The overwhelming majority of points lay on the $Y=T$ line and the correlation coefficient is well over 0.99 in all of the cases, with small number of outliers.

Table 3. Modelling parameters offering best accuracy - model including both temperature and pressure

Resampling	Delays	Duration of the example set (s)	Training function	Performance estimator	Number of neurons	MSE (V)	Time needed to train (s)
1	1	100	trainbr	sse	14	0.0043	75
1	1	100	trainbr	sse	17	0.0044	55
1	2	100	trainbr	mse	8	0.0045	71
1	3	250	trainlm	sse	14	0.0046	45
1	1	100	trainbr	mse	14	0.0046	99
1	4	250	trainbr	sse	17	0.0046	356
1	2	150	trainbr	sse	20	0.0046	171

Table 4. Modelling parameters offering best accuracy - model including temperature, without pressure

Resampling	Delays	Duration of the example set (s)	Training function	Performance estimator	Number of neurons	MSE (V)	Time needed to train (s)
1	1	100	trainbr	sse	11	0.0053	64
1	1	100	trainlm	sse	20	0.0054	58
1	1	100	trainbr	sse	17	0.0054	29
1	2	200	trainbr	mse	17	0.0055	145
1	1	150	trainbr	sse	5	0.0056	130
1	2	100	trainbr	mse	23	0.0056	38
1	1	250	trainbr	sse	8	0.0056	95
1	1	100	trainbr	mse	8	0.0056	51

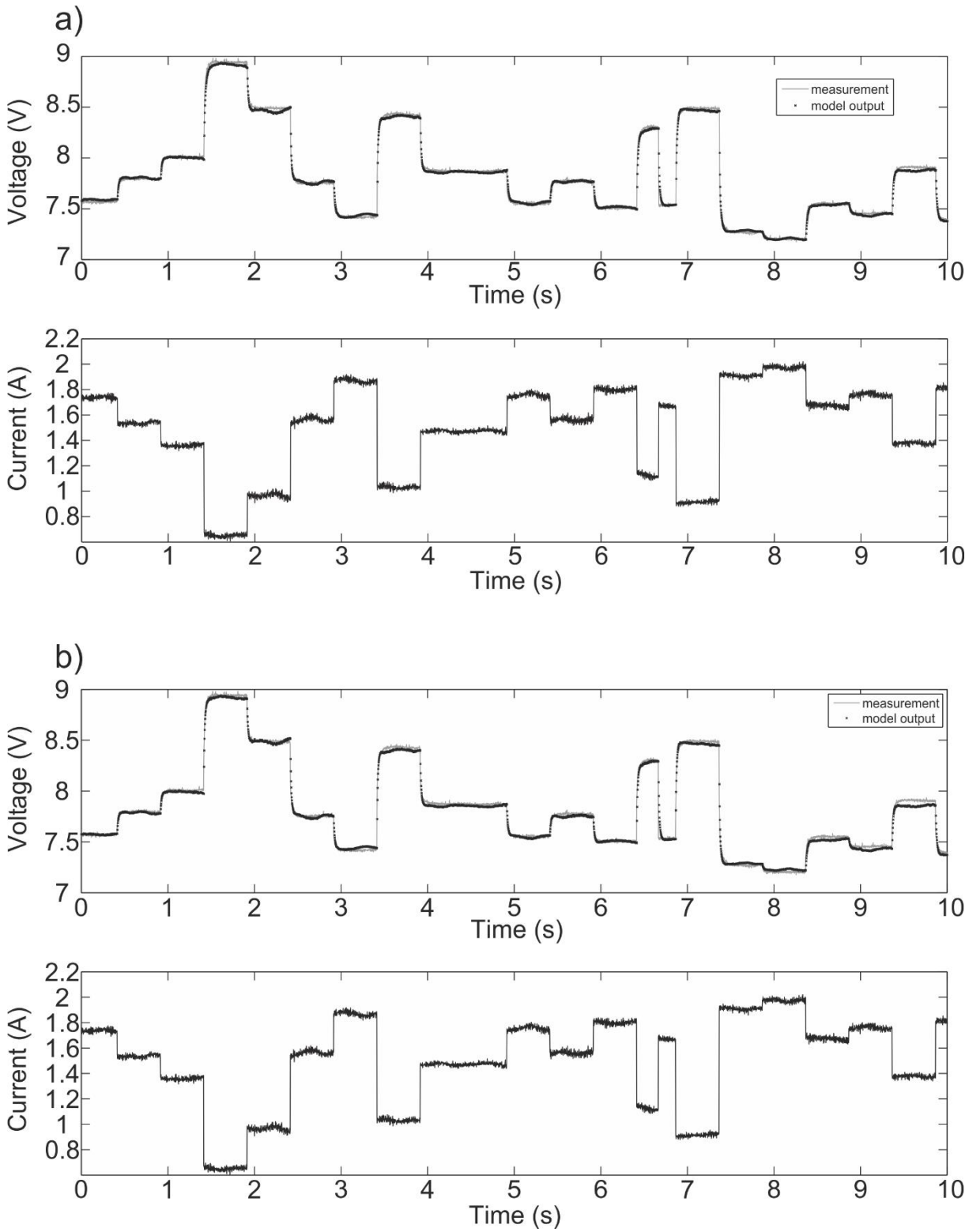


Fig. 6. Time plots for the model configuration providing best accuracy with pressure used as input (a) and without temperature (b)

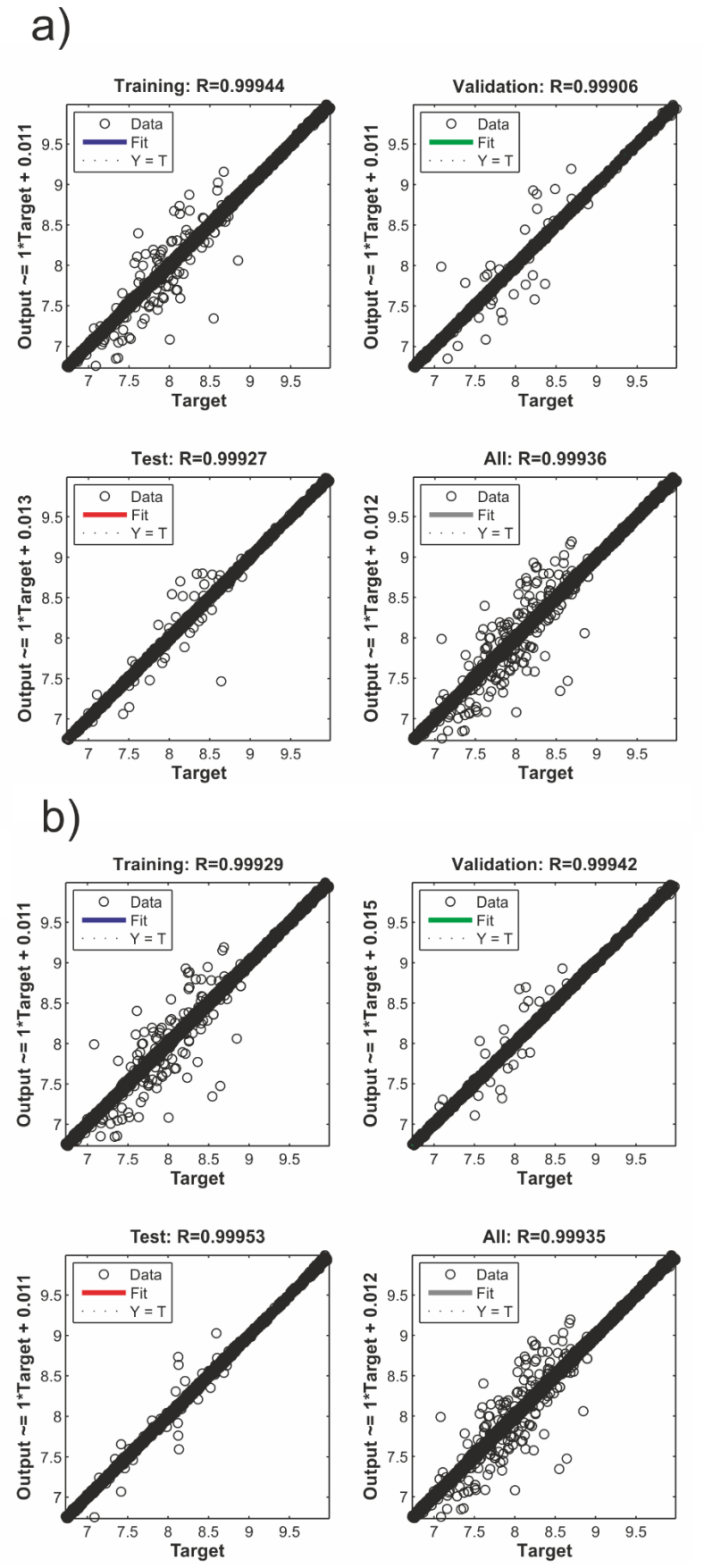


Fig. 7. Regression plots for the model configuration providing best accuracy with pressure used as input (a) and without temperature (b)

CONCLUSIONS

A model of a 12 W PEM fuel cell employing artificial neural networks has been created. The input variables of the model were: electrical current, hydrogen pressure and outside temperature. The output of the model was stack voltage. The model accuracy was tested in various configurations in which the following parameters were changed: sampling frequency, learning dataset duration, training function, performance estimation function, number of neurons, number of delays (past data points used). Altogether 1440 combinations were tested.

The results prove that with proper selection of the parameters it is possible to achieve good simulation accuracy of up to MSE=0.0043 V in modelling of the dynamic state. It has been shown that the high sampling frequency (400 S/s in the analysed case) and one delay point provide best accuracy. There is no serious difference whether MSE or SSE is used as error estimator. Bayesian regulation backpropagation usually gives better training results than Levenberg-Marquardt when used as a training algorithm. In most cases an example data set of a duration of 100 s was enough to provide best results. There is no evidence that the "rule of thumb" (number of neurons in the hidden layer does not need to be higher than number of inputs) mentioned in the Introduction is useful in designing neural network.

It has also been tested if it is necessary to take temperature and hydrogen pressure as inputs of the model. In the analysed case the variation of the hydrogen pressure was relatively small (50 to 56 kPa) and it was possible to achieve good results (up to MSE=0.0053 V) with this variable ignored. Temperature was varied in the range of 10 to 40 °C. Omitting this variable resulted in a considerable decrease of the model accuracy - best results with MSE= 0.0137 V.

The investigation presented in this paper may give guidance to a researcher seeking a simple model, which can be created without detailed physical data of the FC being modelled when sample data can be obtained from an easy to perform experiment. The model can be used to model dynamic behaviour of a PEM fuel cell as a part of energy management system.

REFERENCES

1. **Adhikari R., Agrawal R., 2014.** A combination of artificial neural network and random walk models for financial time series forecasting. Neural Computing and Applications. 24(6). 1441-1449.

2. **Akinyele D., Rayudu R., 2014.** Review of energy storage technologies for sustainable power networks. Sustainable Energy Technologies and Assessments. 8. 74-91.

3. **Andrews J., Mohammadi S., 2014.** Towards a 'proton flow battery': Investigation of a reversible {PEM} fuel cell with integrated metal-hydride hydrogen storage. International Journal of Hydrogen Energy. 39(4). 1740-1751.

4. **Sharifi A., Rowshanzamir S., Eikani M., 2010.** Modelling and simulation of the steady-state and dynamic behaviour of a {PEM} fuel cell. Energy. 35(4). 1633-1646. Demand Response Resources: the {US} and International Experience.

5. **Bengio Y., Goodfellow I., Courville A., 2015.** Deep learning. Book in preparation for MIT Press.

6. **Berry M., Linoff G., 1997.** Data Mining Techniques: For Marketing, Sales, and Customer Support. John Wiley & Sons, Inc. New York, NY, USA.

7. **Cheng S., Liu J., 2015.** Nonlinear modelling and identification of proton exchange membrane fuel cell (pemfc). International Journal of Hydrogen Energy. 40(30). 9452-9461.

8. **Chowdhury S., Das Saha P., 2013.** Artificial neural network modelling of adsorption of methylene blue by naoh-modified rice husk in a fixed-bed column system. Environmental Science and Pollution Research. 20(2).1050-1058.

9. **Cui Y., Shi J., Wang Z., 2015.** Complex rotation quantum dynamic neural networks using complex quantum neuron: Applications to time series prediction. Neural Networks 71. 11-26.

10. **Elmer T., Worall M., Wu S., Riffat S., 2015.** Fuel cell technology for domestic built environment applications: State of-the-art review. Renewable and Sustainable Energy Reviews. 42. 913-931.

11. **Guarnieri M., Alotto P., Moro F., 2015.** Modeling the performance of hydrogen-oxygen unitized regenerative proton exchange membrane fuel cells for energy storage. Journal of Power Sources. 297. 23-32.

12. **Han H., Qiao J., 2013.** A structure optimisation algorithm for feedforward neural network construction. Neurocomputing. 99. 347-357.

13. **Kaytez F., Taplamacioglu M., Cam E., Hardalac F., 2015.** Forecasting electricity consumption: A comparison of regression analysis, neural networks and least squares support vector machines. International Journal of Electrical Power & Energy Systems. 67. 431-438.

14. **Latha K., Vidhya S., Umamaheswari B., Rajalakshmi N., Dhathathreyan K., 2013.** Tuning of {PEM} fuel cell model parameters forediction of steady state and dynamic performance under various operating conditions. International Journal of Hydrogen Energy. 38(5). 2370-2386.

15. **Marohasy J., Abbot J., 2015.** Assessing the quality of eight different maximum temperature time series as inputs when using artificial neural networks to forecast monthly rainfall at cape Otway, Australia. Atmospheric Research. 166. 141-149.

16. **Meiler M., Schmid O., Schudy M., Hofer E. 2008.** Dynamic fuel cell stack model for real-time simulation based on system identification. Journal of Power Sources. 176(2). 523-528. Selected Papers presented at the10th {ULM} ElectroChemical Days.

17. **Moreno N., Molina M., Gervasio D., Robles J., 2015.** Approaches to polymer electrolyte membrane fuel cells (pemfcs) and their cost. Renewable and Sustainable Energy Reviews. 52. 897-906.

18. **Napoli G., Ferraro M., Sergi F., Brunaccini G., Antonucci V., 2013.** Data driven models for a {PEM} fuel cell stack performance prediction. International Journal of Hydrogen Energy. 38(26). 11628-11638.

19. **Ou S., Achenie L., 2005.** A hybrid neural network model for {PEM} fuel cells. Journal of Power Sources. 140(2). 319–330.

20. **Pei J., Mai E., Wright J., Masri S., 2013.** Mapping some basic functions and operations to multilayer feedforward neural networks for modelling nonlinear dynamical systems and beyond. Nonlinear Dynamics. 71(1-2). 371-399.

21. **Piotrowski A., Napiorkowski M., Napiorkowski J., Osuch M., 2015.** Comparing various artificial neural network types for water temperature prediction in rivers. Journal of Hydrology. 529, Part 1. 302-315.

22. **Raga C., Barrado A., Lazaro A., Fernandez C., Valdivia V., Quesada I., Gauchia L., 2014.** Black-box model, identification technique and frequency analysis for pem fuel cell with overshooted transient response. IEEE Transactions on Power Electronics. 29 (10). 5334-5346.

23. **Rocha M., Cortez P., Neves J., 2005.** Evolution of neural networks for classification and regression. Neurocomputing. 70(16-18). 2809-2816, 2007. Neural Network Applications in Electrical Engineering. Selected papers from the 3-rd International Work-Conference on Artificial Neural Networks (IWANN 2005).

24. **Saengrung A., Abtahi A., Zilouchian A,. 2007.** Neural network model for a commercial {PEM} fuel cell system. Journal of Power Sources. 172(2). 749-759.

25. **Schrems A., Pichler K., Steinmaurer G., Wahlmuller E., 2008.** Data based modelling for pem fuel cell monitoring - a test bench application, part i. In Control Applications. CCA 2008. IEEE International Conference. 262-267.

26. **Sodhi S., Chandra P., 2014.** Bi-modal derivative activation function for sigmoidal feedforward networks. Neurocomputing. 143. 182-196.

27. **Sreemathy S., Karthik M., 2015.** Modelling and performance analysis of neural network based fuel cell driven electric traction system. In Innovations in Information, Embedded and Communication Systems (ICIIECS). 2015 International Conference. 1-6.

28. **Swingler K., 1996.** Applying neural networks: A practical guide. London: Academic Press.

29. **van der Velde F., 2015.** Computation and dissipative dynamical systems in neural networks for classification. Pattern Recognition Letters. 64. 44-52.

30. **Wang F., Gao C., Li S., 2014.** Impacts of power management on a {PEMFC} electric vehicle. International Journal of Hydrogen Energy. 39(30). 17336-17346.

31. **Xue X., Cheng K., Sutanto D., 2006.** Unified mathematical modelling of steady-state and dynamic voltage-current characteristics for {PEM} fuel cells. Electrochimica Acta. 52(3). 1135-1144.

Table of contents

List of the Reviewers

1. Yevhenii Putyatin
2. Vladimir Beskorovainyi
3. Valeriy Semenets
4. Volodymyr Timofeyev
5. Lyudmyla Kirichenko
6. Manap Hazhmuradov
7. Volodymyr Kartashov
8. Valentin Filatov
9. Andriy Yerokhin
10. Janusz Myslowski
11. Grzegorz Wcisło
12. Stanisław Sosnowski
13. Yevgeniy Bodyanskiy

Editors of the "Econtechmod" magazine of the Commission of Motorization and Energetics in Agriculture would like to inform both the authors and readers that an agreement was signed with the Interdisciplinary Centre for Mathematical and Computational Modelling at the Warsaw University referred to as "ICM". Therefore, ICM is the owner and operator of the IT system needed to conduct and support a digital scientific library accessible to users via the Internet called the "ICM Internet Platform", which ensures the safety of development, storage and retrieval of published materials provided to users. ICM is obliged to put all the articles printed in the "Econtechmod" on the ICM Internet Platform. ICM develops metadata, which are then indexed in the "BazTech" database.

We are pleased to announce that the magazine "ECONTECHMOD" (ISSN 2084-5715) has undergone a positive evaluation of the IC Journals Master List 2013, the result of which is granting the ICV Index (Index Copernicus Value) 6.52 pts., ICV 2014 – 75,50 pts., ICV 2015 – 80,46 pts.. The resulting score was calculated on the basis of a survey submitted by the Editorial Team as well as assessments made by the professionals from Index Copernicus. We invite you to familiarize yourself with the methodology of IC Journals Master List evaluation:

http://journals.indexcopernicus.com/masterlist.php?q=econtechmod

Impact factor of the "ECONTECHMOD" quarterly journal according to the Commission of Motorization and Energetics in Agriculture is 2,00 (December 2016).

GUIDELINES FOR AUTHORS (2016)

The journal publishes the original research papers. The papers (min. 8 pages) should not exceed 12 pages including tables and figures. Acceptance of papers for publication is based on two independent reviews commissioned by the Editor.

Authors are asked to transfer to the Publisher the copyright of their articles as well as written permissions for reproduction of figures and tables from unpublished or copyrighted materials.

Articles should be submitted electronically to the Editor and fulfill the following formal requirements:
- Clear and grammatically correct script in English,
- Format of popular Windows text editors (A4 size, 12 points Times New Roman font, single interline, left and right margin of 2,5 cm),
- Every page of the paper including the title page, text, references, tables and figures should be numbered,
- SI units should be used.

Please organize the script in the following order (without subtitles):
Title, Author(s) name (s), Affiliations, Full postal addresses, Corresponding author's e-mail
Abstract (up to 200 words), Keywords (up to 5 words), Introduction, Materials and Methods, Results, Discussion (a combined Results and Discussion section can also be appropriate), Conclusions (numbered), References, Tables, Figures and their captions

Note that the following should be observed:
An informative and concise title; Abstract without any undefined abbreviations or unspecified references; No nomenclature (all explanations placed in the text); References cited by the numbered system (max 5 items in one place); Tables and figures (without frames) placed out of the text (after References) and figures additionally prepared in the graphical file format jpg or cdr.

Make sure that the tables do not exceed the printed area of the page. Number them according to their sequence in the text. References to all the tables must be in the text. Do not use vertical lines to separate columns. Capitalize the word 'table' when used with a number, e.g. (Table1).

Number the figures according to their sequence in the text. Identify them at the bottom of line drawings by their number and the name of the author. Special attention should be paid to the lettering of figures – the size of lettering must be big enough to allow reduction (even 10 times). Begin the description of figures with a capital letter and observe the following order, e.g. Time(s), Moisture (%, vol), (%, $m^3m^{-3)}$ or (%, gg^{-1}), Thermal conductivity ($W\ m^{-1}K^{-1}$).

Type the captions to all figures on a separate sheet at the end of the manuscript.

Give all the explanations in the figure caption. Drawn text in the figures should be kept to a minimum. Capitalize and abbreviate 'figure' when it is used with a number, e.g. (Fig. 1).

Colour figures will not be printed.

Make sure that the reference list contains about 30 items. It should be numbered serially and arranged alphabetically by the name of the first author and then others, e.g.
7. Kasaja O., Azarevich G. and Bannel A.N. 2009. Econometric Analysis of Banking Financial Results in Poland. Journal of Academy of Business and Economics (JABE), Vol. IV. Nr 1, 202–210.

References cited in the text should be given in parentheses and include a number e.g. [7].

Any item in the References list that is not in English, French or German should be marked, e.g. (in Italian), (in Polish).

Leave ample space around equations. Subscripts and superscripts have to be clear. Equations should be numbered serially on the right-hand side in parentheses. Capitalize and abbreviate 'equation' when it is used with a number, e.g. Eq. (1). Spell out when it begins a sentence. Symbols for physical quantities in formulae and in the text must be in italics. Algebraic symbols are printed in upright type.

Acknowledgements will be printed after a written permission is sent (by the regular post, on paper) from persons or heads of institutions mentioned by name.