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MATHEMATICAL MODELING AND NUMERICAL ANALYSIS OF NONSTATIONARY PLANE-PARALLEL FLOWS OF VISCOUS INCOMPRESSIBLE FLUID BY R-FUNCTIONS AND GALERKIN METHOD

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Abstract. This paper is dedicated to nonstationary plane-parallel flows of viscous incompressible fluid in finite simply connected domains. Theorem of the solution uniqueness is presented. The method of successive approximation, the Galerkin method and the R-functions method are used to obtain the numerical solution, which was tested on the problem with known solution.

Key words: nonstationary flow, incompressible fluid, stream function, method of successive approximation, R-functions method, Galerkin method.

INTRODUCTION

It is known that nonstationary plane-parallel flows computations are used for mathematical modeling in hydrodynamics, aerodynamics, heat-power engineering, biomedicine and etc. That's why such problems are relevant nowadays [2–6, 25, 29].

These problems are mainly solved using the finite difference and finite element methods [1,7–9, 11,12,24,30]. They are easy to program, but new grid generation and boundary simplification are required every time a transition to a new area is made. The R-functions method developed by the academician of the Ukrainian Academy of Sciences V.L. Rvachev is free of these issues [14,21–23, 26]. This method allows us to consider the geometry of the problem accurately.

The aim of this work is the mathematical simulation of nonstationary plane-parallel flows of viscous incompressible in finite simply connected domains by means of the R-functions method, the Galerkin method and the method of successive approximation.

PROBLEM STATEMENT

Let's consider simply connected area Ω bounded by piecewise smooth bound $\partial\Omega$. Also consider the stream function $\psi(x, y, t)$ connected with the vector $\mathbf{v} = (v_x, v_y)$ of fluid velocity by the equations below:

$$v_x = \frac{\partial\psi}{\partial y},\ v_y = -\frac{\partial\psi}{\partial x}.$$

The mathematical model using stream function and dimensionless variables in area Ω takes the following form [16–18]:

$$-\frac{\partial\Delta\psi}{\partial t} + \nu\,\Delta^2\psi = \frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}, \quad (1)$$

where: x and y are dimensionless coordinates, $t > 0$ – dimensionless time, ν – kinematic coefficient of viscosity, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ – Laplace operator.

Based on the statement of $\mathbf{v}|_{\partial\Omega}$ and $\mathbf{v}|_{t=0}$ we can complete the equation (1) with boundary and initial conditions:

$$\psi|_{\partial\Omega} = f_0(s,t)\,, \tag{2}$$

$$\left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = g_0(s,t)\,, \quad s\in\partial\Omega,\ t\ge 0\,, \tag{3}$$

$$\psi|_{t=0} = \psi_0(x,y)\,, \quad (x,y)\in\bar{\Omega}\,, \tag{4}$$

where: $\frac{\partial f_0}{\partial s}$, g_0 – some distributions of the velocity normal and tangential components, $\mathbf{n}$ – outer normal vector to the boundary.

SOLUTION METHOD

The Galerkin method, the R-functions method and the method of successive approximation are used for the initial-boundary problem (1) – (4) solving.

Let's consider an area Ω in space $\mathbb{R}^2$ with a piecewise smooth bound $\partial\Omega$. It is required to construct a function $\omega(x,y)$ that would be positive inside Ω, negative outside of Ω, equal to zero at $\partial\Omega$ and $\frac{\partial\omega}{\partial\mathbf{n}} = -1$. The equation $\omega(x,y)=0$ determines an implicit form of the locus for the points that belong to the boundary $\partial\Omega$ of the region Ω.

The works [13,27,28] showed that the following bundle of functions satisfies the boundary conditions (2), (3):

$$\psi = f - \omega(D_1 f + g) + \omega^2\Phi\,,$$

where: $f = ECf_0$, $g = ECg_0$ – extensions of f_0 and g_0 to Ω respectively, $\Phi = \Phi(x,y,t)$ – unknown structure component,

$$D_1 v = (\nabla\omega, \nabla v) = \frac{\partial\omega}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial\omega}{\partial y}\frac{\partial v}{\partial y}\,.$$

Let

$$J(u,v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\,.$$

Let u_0 is the solution of the following problem:

$$\frac{\partial(-\Delta u_0)}{\partial t} + \nu\Delta^2 u_0 = 0\,,$$

$$u_0|_{\partial\Omega} = f_0(s,t)\,, \quad \left.\frac{\partial u_0}{\partial\mathbf{n}}\right|_{\partial\Omega} = g_0(s,t)\,,$$

$$u_0|_{t=0} = \psi_0(x,y)\,.$$

Let's make a change in the problem (1) – (4):

$$\psi = u_0 + u\,,$$

where u – new unknown function. The solution of the problem for u_0 can be obtained using algorithm for the linear problem [3].

In order to achieve this, the initial-boundary problem (1) – (4) can be written as:

$$\frac{\partial(-\Delta u)}{\partial t} + \nu\Delta^2 u = J(\Delta(u_0+u), u_0+u)\,, \tag{5}$$

$$u|_{\partial\Omega} = 0\,, \quad \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0\,, \tag{6}$$

$$u|_{t=0} = 0\,. \tag{7}$$

Let's consider operators A, B and J with their domains and energy norms respectively:

$$Au = \Delta^2 u\,, \quad Bu = -\Delta u\,,$$

$$J = J(\Delta(u_0+u), u_0+u)\,,$$

$$D_A = \left\{ u \,\middle|\, u\in C^4(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$D_B = \left\{ u \,\middle|\, u\in C^2(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$D_J = \left\{ u \,\middle|\, u\in C^3(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$\| u\|_A^2 = \iint_\Omega (\Delta u)^2\, dxdy\,, \quad \| u\|_B^2 = \iint_\Omega |\nabla u|^2\, dxdy\,.$$

Thus, (1) – (4) can be written in the operator form:

$$\frac{d}{dt}Bu + \nu Au = Ju\,,\ (x,y)\in\Omega,\ t>0\,, \tag{8}$$

$$u|_{t=0} = 0\,. \tag{9}$$

Let's denote the classical solution of the problem (8), (9) as $u(t)$, i.e. for any $t\ge 0$ $u(t)\in D_A$ and $u(t)$ is continuously differentiable and satisfies (8) and (9).

Also let us assume $v(t)$ denotes the smooth function in $\bar{\Omega}\times[0,+\infty)$, which satisfies the boundary conditions (6) and at some value $T>0$ $v(T)=0$. Multiply (8) in $L_2(\Omega)$ by the arbitrary function $v(t)$ and integrate it from 0 to T:

$$-\int_0^T \left[u, \frac{\partial v}{\partial t} \right]_B dt + \nu\int_0^T [u,v]_A\, dt =$$

$$= [u_0, v(0)]_B + \int_0^T (Ju, v)_{L_2(\Omega)} dt\,. \tag{10}$$

Last equation is assumed to be a generalized (weak) solution of (8), (9).

Let's denote:

$$W_T = \{u \mid u \in L_2(0,T;H_A),$$

$$u' \in L_2(0,T;L_2(\Omega)),\ u(T)=0\},$$

as some set of functions.

Function $u(t)$ is called a generalized (weak) solution of (8), (9) if the following:

a) $u(t) \in L_2(0,T;\overset{\circ}{W}{}_2^2(\Omega))$,

b) for any $v(t) \in W_T$ the equation (10) is true.

Consider the method of successive approximation to solve the problem (8), (9) (therefore, problem the (1) – (4)). Assume that an initial approximation $u^{(0)}$ is set. Then one can find next the $(k+1)$ approximation using known the k approximation as a linear problem solution:

$$\frac{\partial(-\Delta u^{(k+1)})}{\partial t} + \nu\Delta^2 u^{(k+1)} =$$

$$= J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) \text{ in } \Omega,\ t>0, \qquad (11)$$

$$u^{(k+1)}\Big|_{\partial\Omega} = 0, \quad \frac{\partial u^{(k+1)}}{\partial \mathbf{n}}\Big|_{\partial\Omega} = 0, \qquad (12)$$

$$u^{(k+1)}\Big|_{t=0} = 0, \quad k = 0,1,2,... \qquad (13)$$

The variational formulation of the (11) – (13) can be written as follows:

$$\frac{1}{2}\frac{d}{dt}|u^{(k+1)}|_B^2 + \nu|u^{(k+1)}|_A^2 =$$

$$= (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}), u^{(k+1)})_{L_2(\Omega)}, \quad (14)$$

$$\left\|u^{(k+1)}\right\|_{L_2(\Omega)}^2 = 0, \quad t=0. \qquad (15)$$

Let's integrate (14) from 0 to t and using some equalities and inequalities listed below [15]:

$$(J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}), u^{(k+1)})_{L_2(\Omega)} =$$

$$= (J(u_0+u^{(k)}, u^{(k+1)}), \Delta(u_0+u^{(k)}))_{L_2(\Omega)},$$

$$\left|(u,v)_H\right| \le \|u\|_H \|v\|_H,$$

$$\left|(J(u,v),\Delta u)_{L_2(\Omega)}\right| \le$$

$$\le c_0 \|\Delta v\|_{L_2(\Omega)} \|\nabla u\|_{L_2(\Omega)} \|\Delta u\|_{L_2(\Omega)},$$

$$u,v \in W_2^2(\Omega);$$

$$\|\nabla u\|_{L_4(\Omega)}^2 \le c\|\nabla u\|_{L_2(\Omega)} \|\Delta u\|_{L_2(\Omega)},$$

$$u \in \overset{\circ}{W}{}_2^1(\Omega) \cap W_2^2(\Omega),$$

we are able to estimate (14) as follows:

$$\left\|u^{(k+1)}(t)\right\|_{L_2(\Omega)}^2 + \int_0^t |u^{(k+1)}|_A^2 d\tau \le \frac{c_1}{\nu}T +$$

$$+\frac{c_2}{\nu}\left(\operatorname*{ess\,sup}_{0\le t\le T}\left\|u^{(k)}\right\|_{L_2(\Omega)}^2 + \int_0^T |u^{(k)}|_A^2\, d\tau\right)^2, \qquad (16)$$

where: c_1 and c_2 are known constants, which depend only on the area geometry.

Therefore, we can say that the boundedness of our solution is proved in the space:

$$V = L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;H_A).$$

Further, let's prove the iterative (11) – (13) convergence. Consider differences $\delta u^{(k+1)} = u^{(k+1)} - u^{(k)}$, which satisfy the following equation and the boundary and initial conditions:

$$\frac{\partial(-\Delta\delta u^{(k+1)})}{\partial t} + \nu\Delta^2\delta u^{(k+1)} =$$

$$= J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), \qquad (17)$$

$$\delta u^{(k+1)}\Big|_{\partial\Omega} = 0, \quad \frac{\partial \delta u^{(k+1)}}{\partial \mathbf{n}}\Big|_{\partial\Omega} = 0, \qquad (18)$$

$$\delta u^{(k+1)}\Big|_{t=0} = 0. \qquad (19)$$

The variational formulation of the (17) – (19) can be written as follows:

$$[\delta u'^{(k+1)}, v]_B + \nu[\delta u^{(k+1)}, v]_A =$$

$$= (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), v)_{L_2(\Omega)}, \quad (20)$$

$$(u^{(k+1)}, v)_{L_2(\Omega)} = 0,\ t=0.$$

Let's integrate (20) from 0 to t and substitute $u^{(k+1)}$ instead of v:

$$\frac{1}{2}|\delta u^{(k+1)}(t)|_B^2 + \nu\int_0^t |\delta u^{(k+1)}|_A^2 d\tau =$$

$$= \int_0^t (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), \delta u^{(k+1)})_{L_2(\Omega)} d\tau,$$

$$\left\|\delta u^{(k+1)}\right\|_{L_2(\Omega)}^2 = 0.$$

One can estimate the last equation using the previous equalities and inequalities and the next ones:

$$J(u_1,v_1) - J(u_2,v_2) =$$

$$= J(u_2, v_1-v_2) + J(u_1-u_2, v_2),$$

$$\left|(J(u,v),w)_{L_2(\Omega)}\right| \le$$

$$\le c_1 \|\Delta u\|_{L_2(\Omega)} \|\Delta v\|_{L_2(\Omega)} \|w\|_{L_2(\Omega)},$$

$$u, v \in \overset{\circ}{W}{}_2^2(\Omega), \quad w \in L_2(\Omega).$$

Therefore:

$$\operatorname*{ess\,sup}_{0\le t\le T} \left\|\delta u^{(k+1)}\right\|^2_{L_2(\Omega)} + \int_0^T |\, \delta u^{(k+1)}|^2_A d\tau \le$$

$$\le \frac{c_3}{\nu}\left(\operatorname*{ess\,sup}_{0\le t\le T} \left\|\delta u^{(k)}\right\|^2_{L_2(\Omega)} + \int_0^T |\, \delta u^{(k)}|^2_A \, d\tau \right),$$

where $t \in (0;T]$.

Hence we can say that the boundedness of $\delta u^{(k+1)}$ is proved in the metric space V.

Therefore, if $\frac{c_3}{\nu} \le \alpha < 1$ then:

$$\left\|\delta u^{(k+1)}\right\|_V \le \alpha \left\|\delta u^{(k)}\right\|_V \le \ldots \le \alpha^k \left\|\delta u^*\right\|_V,$$

i.e. the limit below exists:

$$\lim_{k\to\infty} u^{(k)} = u^*.$$

One can prove the following theorem.

Theorem. Let function $u_0 \in L_2(0,T;L_2(\Omega))$. Therefore the variational problem (14), (15) has a unique solution:

$$u \in L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;H_A).$$

COMPUTATION SCHEME

According to the R-functions method the solution structure of (11) – (13) is:

$$u^{(k+1)}(x,y,t) = \omega^2(x,y)\Phi^{(k+1)}(x,y,t).$$

Next, let's approximate an undefined component:

$$\Phi^{(k+1)}(x,y,t) \approx \Phi_N^{(k+1)}(x,y,t) =$$

$$= \sum_{j=1}^{N} c_j^{(k+1)}(t)\tau_j(x,y),$$

where: $\{\tau_j\}$ – some complete system of functions in the space $L_2(\Omega)$ (trigonometric or algebraic polynomial, B-splines and etc.). Then an approximation for $u^{(k+1)}(x,y,t)$ has the following form:

$$u_N^{(k+1)}(x,y,t) = \sum_{j=1}^{N} c_j^{(k+1)}(t)\varphi_j(x,y),$$

where: $\varphi_j = \omega^2\tau_j$.

According to the Galerkin method [19] for the nonstationary problems one can find functions $c_j^{(k+1)}(t)$, $j = 1,\ldots,N$, using the following ordinary differential equation system:

$$\left(\frac{d}{dt} B u_N^{(k+1)} + \nu A u_N^{(k+1)} - \right.$$

$$\left. - C(\varphi + u_N^{(k)}) - F, \varphi_j\right)_{L_2(\Omega)} = 0,$$

$$(u_N^{(k+1)}\Big|_{t=0} - u_0, \varphi_j)_{L_2(\Omega)} = 0, \; j = 1,2,\ldots,N,$$

or in expended form:

$$\sum_{j=1}^{N} \dot{c}_j^{(k+1)}(t)[\varphi_j,\varphi_i]_B + \nu\sum_{j=1}^{N} c_j^{(k+1)}(t)[\varphi_j,\varphi_i]_A =$$

$$= \left(C(\varphi + u^{(k)}) + F, \varphi_i\right)_{L_2(\Omega)}, \tag{21}$$

$$\sum_{j=1}^{N} c_j^{(k+1)}(0)(\varphi_j,\varphi_i)_{L_2(\Omega)} =$$

$$= (u_0,\varphi_i)_{L_2(\Omega)}, \; i = 1,2,\ldots,N, \tag{22}$$

where the dot denotes the time derivative.

Let's consider the matrices and vectors:

$$\Xi = \left\|[\varphi_j,\varphi_i]_B\right\|_{i,j=\overline{1,N}}, \quad \Upsilon = \left\|[\varphi_j,\varphi_i]_A\right\|_{i,j=\overline{1,N}},$$

$$\Gamma = \left\|(\varphi_j,\varphi_i)_{L_2(\Omega)}\right\|_{i,j=\overline{1,N}},$$

$$\xi(t) = \left\|\left(C(\varphi + u^{(k)}) + F, \varphi_i\right)_{L_2(\Omega)}\right\|_{i=\overline{1,N}},$$

$$\gamma = \left\|(u_0,\varphi_i)_{L_2(\Omega)}\right\|_{i=\overline{1,N}}.$$

We note that matrices Ξ, Υ, Γ are symmetric and invertable.

Denote:

$$c^{(k+1)}(t) = (c_1^{(k+1)}(t),\ldots,c_N^{(k+1)}(t)),$$

$$\dot{c}^{(k+1)}(t) = (\dot{c}_1^{(k+1)}(t),\ldots,\dot{c}_N^{(k+1)}(t)),$$

therefore, a Cauchy problem (21), (22) can be written as:

$$\Xi\dot{c}^{(k+1)}(t) + \nu\Upsilon c^{(k+1)}(t) = \xi(t), \tag{23}$$

$$\Gamma c(0) = \gamma. \tag{24}$$

We can use the Runge–Kutta method to solve (23), (24).

NUMERICAL RESULTS

Problem 1. Let's consider a test problem [21] to validate the proposed method. It consists of the equation (1) and boundary and initial conditions listed below:

$$\psi|_{\partial\Omega} = f_0(s,t) = \begin{cases} e^{-2\pi^2 t}\cos\pi y, x = 0, \\ e^{-2\pi^2 t}\cos\pi x, y = 0, \\ 0,\ x = \frac{1}{2}, y = \frac{1}{2}, \end{cases}$$

$$\left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = g_0(s,t) = \begin{cases} -\pi e^{-2\pi^2 t}\cos\pi x, y = \frac{1}{2}, \\ -\pi e^{-2\pi^2 t}\cos\pi y, x = \frac{1}{2}, \\ 0,\ x = 0 \text{ or } y = 0, \end{cases}$$

$$\psi|_{t=0} = \psi_0(x,y) = \cos\pi x\cos\pi y .$$

Assume that $\nu = 1$, Ω – square $0 < x < \frac{1}{2}$, $0 < y < \frac{1}{2}$, $t \in [0,1]$.

Function $\omega(x,y)$ have the below form:

$$\omega(x,y) = x(1-2x) \wedge_0 y(1-2y),$$

where $\wedge_0$ - R-conjunction:

$$u \wedge_0 v = u + v - \sqrt{u^2 + v^2} .$$

Functions $f(x,y,t) = EC f_0(s,t)$ and $g(x,y,t) = EC g_0(s,t)$ are set as follows:

$$f(x,y,t) = \\ = \frac{e^{-2\pi^2 t}\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right)(y\cos\pi y + x\cos\pi x)}{y\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + x\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + xy},$$

$$g(x,y,t) = -\pi e^{-2\pi^2 t} xy \times \\ \times\frac{e^{-2\pi^2 t}\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right)(y\cos\pi y + x\cos\pi x)}{y\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + x\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + xy} .$$

The exact solution of this problem is:

$$\psi(x,y,t) = e^{-2\pi^2 t}\cos\pi x\cos\pi y .$$

We used the Runge–Kutta method to solve (23), (24) and B-splines [10] as τ. The Gauss formula with 16 knots was used for evaluation of integrals in the Galerkin method.

Now let’s have a look at the results of this numerical experiment.

The differences between the exact and approximated solution in 3D are presented below on Fig. 1 and Fig. 2. The difference reduces with time.

The stream lines and stream function in 3D are given in figures 3 and 4. They are similar to the exact solution.

The error norm in $L_2(\Omega)$ is shown on Fig. 5 with dependency from time. Fig. 5 shows method convergence.

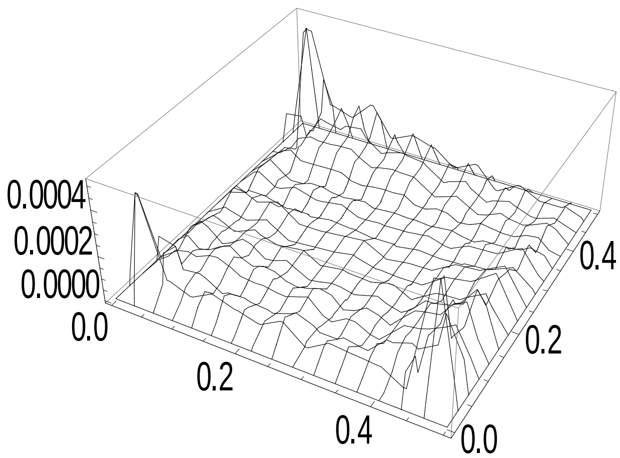

Fig. 1. The difference between the exact and approximated solution, $t = 0.1$

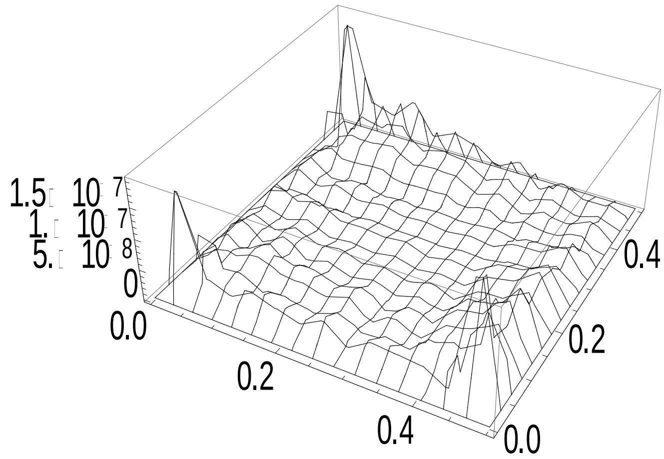

Fig. 2. The difference between the exact and approximated solution, $t = 0.5$

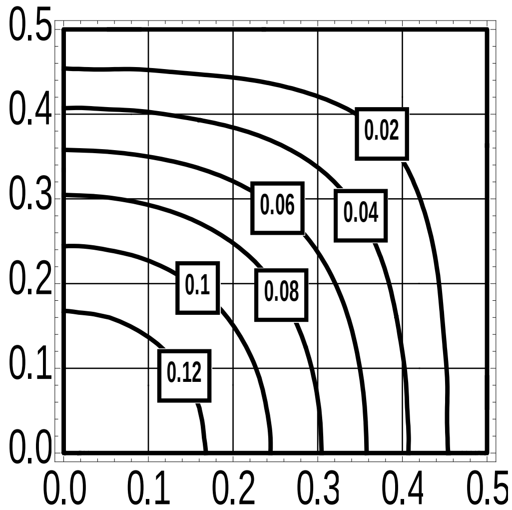

Fig. 3. The stream lines, $t = 0.1$

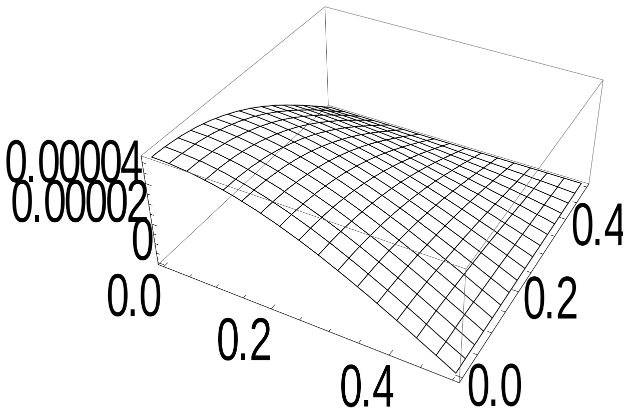

Fig. 4. The stream function, $t = 0.1$

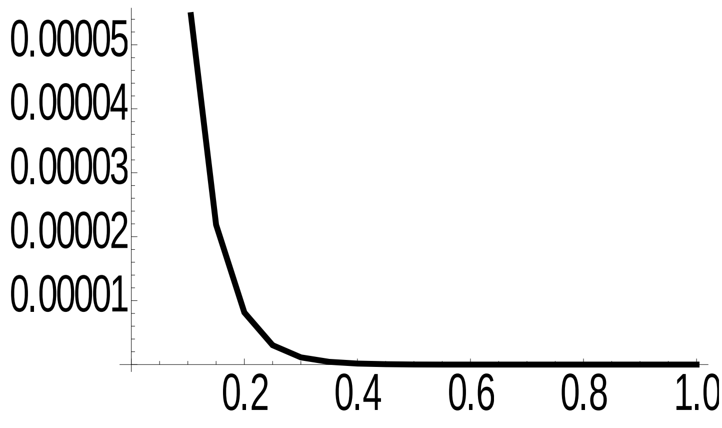

Fig. 5. The error norm in $L_2(\Omega)$

Problem 2. Let's consider the equation (1) and next boundary and initial conditions:

$$\psi\big|_{\partial\Omega} = 0,$$

$$\left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = \begin{cases} e^{-t} - 1, & y = 1, \\ 0, & x = 0,\ y = 0,\ x = 1, \end{cases}$$

$$\psi\big|_{t=0} = 0,$$

where: $\Omega = \{(x, y) \,|\, 0 < x < 1,\ 0 < y < 1\}$, $\nu = 1$, $t \in (0;5]$. Function $g(x, y, t) = ECg_0(s, t)$ is set as follows:

$$g(x,y,t)=\frac{\dfrac{e^{-t}-1}{\omega_1(x,y)}}{\dfrac{1}{\omega_1(x,y)}+\dfrac{1}{\omega_2(x,y)}}=$$

$$=\frac{(e^{-t}-1)(y-4(x-0,5)^2)}{y-4(x-0,5)^2+\sqrt{64(x-0,5)^2+1}},$$

where:

$$\omega_1(x,y)=1-y,$$

$$\omega_2(x,y)=\frac{y-4(x-0,5)^2}{\sqrt{64(x-0,5)^2+1}}.$$

Therefore, the problem structure is

$$\psi(x,y,t)=$$

$$=-\omega(x,y)\frac{(e^{-t}-1)(y-4(x-0,5)^2)}{y-4(x-0,5)^2+\sqrt{64(x-0,5)^2+1}}+$$

$$+\omega^2(x,y)\Phi(x,y,t).$$

We also used the Runge–Kutta method to solve (23), (24) and B-splines as τ. The Gauss formula with 16 knots was used for evaluation of integrals in the Galerkin method.

The stream lines and stream function in 3D are given in figures 6, 7. The vorticity lines and vorticity function in 3D are given in figures 8, 9.

Fig. 6 – 9 showed that the achieved numerical results are consistent with other results [7].

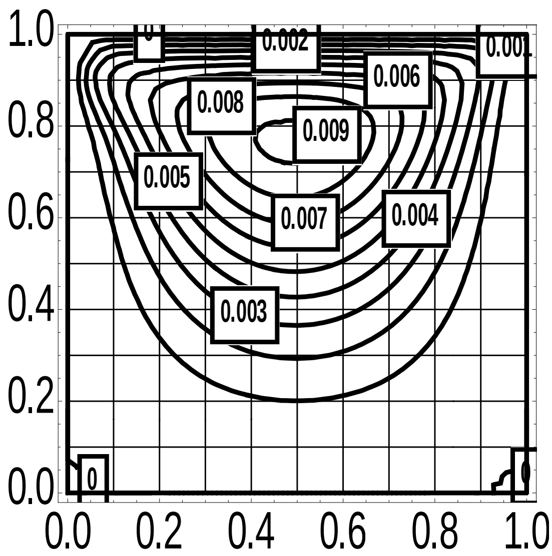

Fig. 6. The stream lines, $t=0.1$

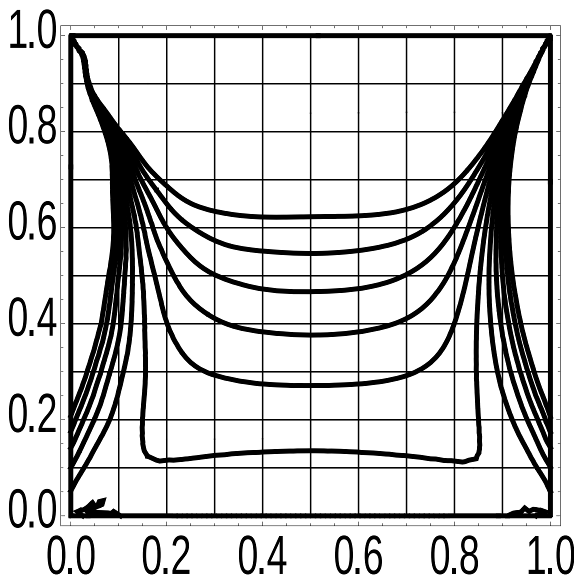

Fig. 8. The vorticity lines, $t=0.1$

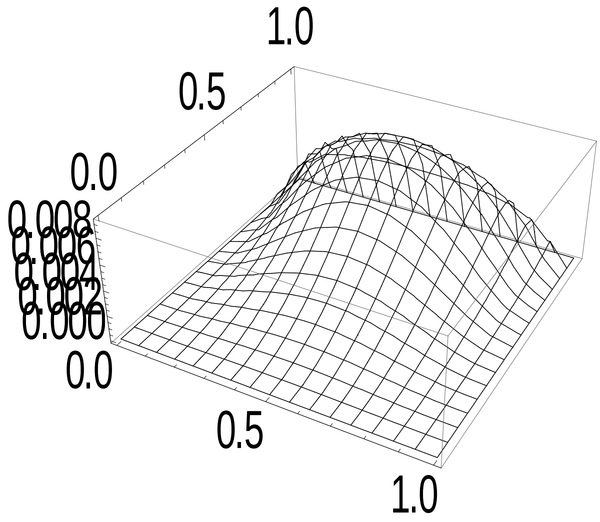

Fig. 7. The stream function, $t=0.1$

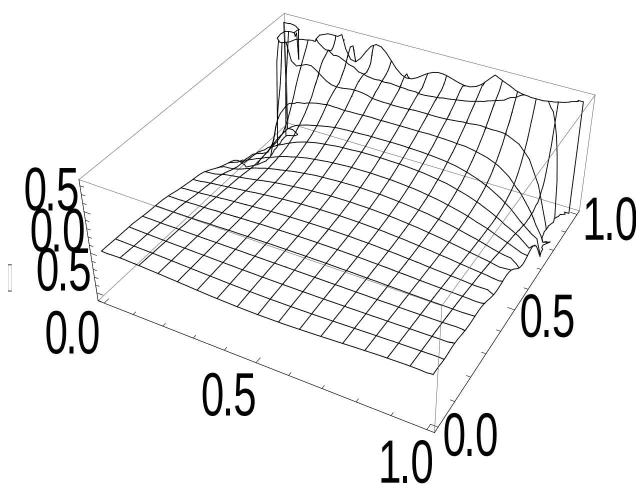

Fig. 9 The vorticity function, $t=0.1$

CONCLUSIONS

The nonstationary plane-parallel flow of viscous incompressible fluid is investigated. The algorithm for solving the problem based on the R-functions method and the Galerkin method is used. The solution structures of unknown function were built by means of the R-functions method, and the Galerkin method was used for the approximate undefined components. Thus, the stream function was represented in an analytical way.

The advantage of the suggested algorithm is that it does not have to be modified for different geometries of the regions being reviewed, which illustrates the scientific innovation of the results obtained. As a result, the approximate solution for such streams investigation problems is obtained in the non-classic geometry field.

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FORMATION OF A REPRESENTATIVE SELECTION OF IT-INDICATORS TO ASSESS THE QUALITY INFORMATION SYSTEMS AT ENTERPRISE

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Abstract. For the effective use of the acquired or existing information system (IS) at the enterprise it is necessary to solve the problems related to an assessment of the IS quality. The information system is represented as a set of the functional components therefore it is necessary at first to evaluate quality of each of the components, and then get an integral assessment of the entire system. Quite tricky in the processes of IS quality assessment is the ambiguity of existing methods of formation the set of IT-indicators, most adequately characterizing the current state of functioning of the IS. It is necessary to define quantity and the list of IT-indicators (representative selection) which can be used to evaluate the quality of information systems in the enterprise to ensure the accuracy and veracity of quality assessment of IS. For formation of such selection it is offered to use an expert method. Expert's task is to select the criteria for the assessment quality of IS components, the formation of a representative selection of its indicators that best describe the real state of the information system at the enterprise, establishment of weights for IT-indicators, i.e. the importance of each IT-indicator to assess the quality of entire IS. The implementation of such actions, taking into account the recommendations of existing standards and using the method hierarchies' analysis, are considered.

Key words: information system, quality of the IS, criteria of an assessment of quality, IS status, IT-indicators, selection, representativeness.

INTRODUCTION

There are many methods, criteria and indicators for assessing the economic status of the enterprise [4,9,10,14] and decision-making about the ways of its development in order to achieve a more profitable and sustainable position in the market economy [2,5,11-13, 17]. All are based on the use of a plurality of enterprise information systems, which collect and systematizes a variety of information about its activities. Big enterprises have a large number of information systems for different purposes.

Practice of implementation and maintenance of the modern information systems testifies that the enterprises very often don't receive expected results from their investment in the system of computer data processing and control [20]. This is mainly connected with the increase and complexities of the functions and features of modern information systems, an increase in the volume of processed and stored information, with widespread use of network technologies of distributed information processing, with increasing their vulnerability, with the need to adapt methodologies the exploitation of information systems to rapid progress and perfection in information and communications technologies, etc. On the other hand, the use of information systems is one of the main instruments for the control and management of the business activity of any enterprise this entails the need to address the problems of efficiency and quality of IS. In spite of the fact that information systems provide mass of opportunities for the enterprises, they often are a source of new problems and tasks which require a professional IT-solution. Therefore for effective use of the acquired or already existing automation systems of information processing it is necessary to solve the problems connected to an assessment of quality of information systems that is inseparably linked with the solution of problems of objectivity and reliability of the received estimates. Suggested methods of selection criteria are justified by quality assessment and the formation of a representative sample of IT-indicators that assess the quality of IS ventures. For the entire set of IT-indicators, which determines state and operation of IS, a quantitative and qualitative assessment measures are offered. They may characterize positive and negative trend changes. Veracity and objectivity of selection process the criteria quality, IT-indicators, as well as the experts, who make decisions about the quality operation of the IS at the enterprise, are confirmed by the use the method of analysis of hierarchies.

MATERIALS AND METHODS

Effective position of the enterprise in the market is determined its ability by adaptation to permanently changing living conditions of economic systems [3]. For steady functioning of the enterprise on each of stages of its life cycle it is necessary to provide execution of a purposeful package of measures and mechanisms of transition to a new level. Rather essential role is played by the automation systems of information processing and control, allowing most quickly take appropriate decisions based on reliable and adequate information. The understanding of dependence of profit and economic productivity of activities of the enterprise from implementation and use of information systems, demands from a management the ability to evaluate benefits which systems of information processing automatization and control can bring, and also to understand how it is possible to check and measure results from investment into the IS. To evaluate benefits means, first of all, the ability to objectively evaluate the quality of information systems. However the analysis of publications on a problem of quality of the IS shows that they belong, generally to the software of information systems. Now there are many different, formal approaches to an assessment and quality control of the IS. However, all of them cover only some specific aspects, such as: determination of financial indicators, formation of quality or heuristic indicators, safety and protection, risk control, probable estimates of quality, control of creation and implementation processes of IS, control of IT expenses, etc.

The analysis of existing methods of IS quality assessment confirmed that the development of an adequate system of evaluation and quality management of IS in the enterprise continue to be actual. Unfortunately, many enterprises are forced to work still at legacy hardware and software based on MS DOS. On the other hand, the most advanced enterprises prefer to conduct preventive measures to avoid problems and accidents in the IT infrastructure, using the recommendations of standards such as ITIL, thus maintaining the required quality of the IS at the enterprise [7].

The most effectively resolved issues at the formal level are questions of quality management of a software which are formulated in the DSTU [16] standards. Essential attention is paid to valuation methods and quality control in the determination of complex quality characteristics that reflect some measurable factors and influence the quality of a software system in general. One of the methods of complex evaluation of the quality of the IS is the Cobit standard which is widely spread around the world and is actively promoting by ISACA association [6]. The processes of operation and functioning of the IS according to the Cobit standard operate with IT indicators many of them defines an IS status at the enterprise.

As demonstrated by the analysis of the IT indicators characterizing an IS status, instruments of their measurement, i.e. the procedure for enforcement measures IT assessment indicator to a normalized scale, for example, an interval scale. On such a scale change of an assessment from 0 to 1 means quality improving process, i.e., than the closer the score is to 1 – the subjects "better, above" quality, respectively, the closer the score is to zero – the subjects "worse, below" quality of an IT indicator.

Such change of a measure represents the positive tendency in a quality assessment. However there is the significant amount of the IT indicators a measure of them has the negative tendency which is characterized by that growth of quality of an IT indicator is defined by the measure which is coming nearer to zero as, for example, an indicator "turnover rate in the IT organizational unit".

For the entire set of the IT indicators defining a status and processes of functioning of the IS, a measure of an assessment can be as the quantitative, and qualitative [16], for example, "What is the percentage of certified IT staff? (2%; 10% or 0,02; 0,1 etc.)", or "What is the level of satisfaction of management and users by experience and skills of IT staff? (satisfactorily – 1, unsatisfactory – 0, the middle level – 0,5, etc.)". The most difficult for formalization are assessment procedures such components of IS as an informational support because of the basic criterion for their evaluation is the training level or professionality of IT staff.

Thus, the measure of an assessment of quality is characterized by the positive (+) or the negative (-) an upward trend of quality and for interval scale from 0 to 1 is defined as:

$$M_{(+)i} = P_i/P_b\ ;\ \ M_{(-)i} = 1- P_i/P_b\ ,$$

where: $M_{(+)i}$ – is the value of a measure of P_i quality score with the positive trend of growth quality, $i=1,\dots n$, $M_{(-)i}$ – is the value of a measure of P_i quality score with the negative trend of growth quality, $i=1,\dots n$, P_b – basic value P_i of a quality score.

For example, the measure "Number of employees IT subdivisions, completed advanced training courses, training, training or certification in relation to all employees" is the measure $M_{(+)i}$, for which P_i is equal to "number of the employees who completed increase courses …", P_b is equal to "all employees in IT subdivision". The measure "Churn rate of frames (T) in IT subdivision for the researched period" is a measure $M_{(-)i}$, for which P_i is equal to "number of the dismissed employees for the period …".

The assessment of quality of each IS component is defined as an arithmetic average of measures. Thus P_i and P_b values are defined by experts as a result of interviews with relevant staff of IT departments (selected by a method of the analysis of hierarchies).

Quality of the received information products and services also depends of quality of use and processing of information resources by staff of the enterprise [15]. The quality assessment each component is defined by means of the appropriate indicators and measures.

In general case, the measure of quality assessment of components can be represented as the following model:

$$M_i = \langle p_{i\,j}, \mu_{p_j}, k_{p_j} \rangle,\ \ i=1,\dots, n,$$

where: M_i – measure of an quality assessment of i component, $p_{i\,j}$ – indicator (metrics) of quality for IS valuation, $j \in J$, J – set of all quality indicators (metrics), μ_{p_j} – measure of an quality assessment of indicator p_j, k_{p_j} – criterion of an assessment of indicator p_j which is defined on an interval scale from 0 to 1 ([0,1] or [1,0]).

The veracity of this quality assessment is based on a representativeness of selection indicators, which is made from general set of the IT indicators offered in the Cobit 4.1 [18] and ITIL [7] standard. For this purpose is used all set of the IT indicators, offered in [18] (340 indicators for monitoring and management of information systems and technologies).

Let a set J – general set of all IT-indicators that describe the state of information systems and technologies in the enterprise, then subset p_j is a part of objects of general set IT-indicators which correctly reflects general set for enterprise and may be called representational. Research of an assessment of quality of the IS on such selection IT-indicators has probable character and the accuracy of results depends from the size of this selection. Probability with which it is possible to claim that the error of selection of IT-indicators won't exceed some given value, is defined as confidential probability. Assuming that characteristics of general set IT-indicators can be in selection with probability of 95% it is possible to determine the volume of representative selection [1, 19]:

$$n = \frac{z^2 s^2 N}{\Delta^2 N + z^2 s^2},$$

where: z – trust coefficient (z=1,96 for 95% reliability,

n – selection volume,

s^2 – selective dispersion, $s^2 = pq$,

N – general set volume

p – sign share, q =(1-p)

Δ^2 – the selection error, for example, let will be ≤ 4% or ≤ 0,04.

For a representativeness of selection it is desirable that dispersion was maximum that is reached in case of p = 0,5, selection volume depending on error amount of selection is given in the Table 1.

Thus, using these tables it is possible to determine the volume of selection of IT indicators in case of the given error of selection provided that the volume of population of indicators is equal 340.

However, the volume of selection is a necessary but not sufficient condition for quality research. For support of accuracy and reliability of results of estimation of quality of the IS it is necessary to define composition and selection structure, i.e. to make a choice of those IT indicators which can be used for an assessment of quality of the IS at the enterprise as the method of formation of selection is more significant characteristic, than selection volume.

As such method of formation of selection the method of expert formation is used. And the task of the expert will consist in:

– choice of criteria of an assessment of quality of component ISs and IS in general,

– selection of IT indicators which the best method describe a real status of functioning of the IS at this enterprise and can be used for an assessment of quality of the IS,

– establishment of scales of W IT-indicators, i.e. importance of an IT-indicator for an assessment of quality of the IS at this enterprise (a simple choice of points on the given scale).

Known methods of determination of criteria [6] are finished taking into account features of systems of information processing automatization and control at the domestic enterprises. The following requirements to a choice of criteria of an assessment of quality of IS component for the domestic enterprises are as a result offered:

objectivity, which means independence from bias and subjective opinions, which may adversely affect the results of professional evaluation of a quality index. The criterion of objectivity completely depends on responsibility of the expert holding a certain position and performing their duties,

measurability which means that the criterion should be able to be measured adequately reality so that the results of measurements of various experts were identical,

clarity, which means that the criteria must be clear and unambiguous and did not differ in the interpretation of the various users and experts,

completeness, which means the criteria should be comprehensive enough to include all the conclusions about the process, which may affect the assessment of the IS quality,

relevance that means compliance to criteria which are related to a subject of estimation and respond the estimation purposes.

For an objective choice of criteria of an assessment of quality of IS component and the IS in general the method of the hierarchies analysis (MAI) is used.

The received results of a choice show that the IT indicators responding to the following criteria most adequately describe a status of quality of functioning of the IS: reliability, practicality, reliability which also can be taken for conditions of a choice of a set of figures of merit of the IS at a stage of formation of representative selection of IT figures of merit.

Using a method of the hierarchies analysis, from the population, offered $P_{i,}$ quality IT-indicators are selected from the Cobit standard [18], by i=1 … n, n – sample volume.

1. Thus the purpose of expert selection of IT indicators is the choice of such IT indicators which the best way to describe a real status of IS functioning at this enterprise and can be used for an assessment of IS quality.

2. As criteria or characteristics of selection are used:

a) possibility of measurement of an IT-indicator,

b) usefulness in achievement of business purposes,

c) importance for an assessment of quality of the IS,

d) reliability (criterion of maintenance),

e) practicality (constructive design criterion of the IS),

f) veracity (information criterion).

The first three characteristics define the general semantic approach of experts in case when specifying data clusters, i.e. groups of nodes of one level subordinated to some node of other level or the dominating node or peak of a cluster.

3. As alternatives IT-indicators of the Cobit 4.1 standard [18] are used.

Formation composition of IT-indicators was carried out taking into account that the total quantity of the selected indicators shall be at least Selection Volume value according to the selection error selected by value (see Table 1).

For each indicator selected by the MAI method its importance (or W weight) in an assessment of quality of functioning of the IS at the researched enterprise (Table 2) is set.

Table 1. Scoping of representative selection in case of the given error

Selection error, Δ2	Trust coefficient, z	Sign share, p	q =(1-p)	General set volume, N	Selection volume, n
0,04	1,96	0,5	0,5	340	217
0,05	1,96	0,5	0,5	340	180
0,06	1,96	0,5	0,5	340	149
0,07	1,96	0,5	0,5	340	124
0,08	1,96	0,5	0,5	340	104
0,09	1,96	0,5	0,5	340	88
0,1	1,96	0,5	0,5	340	75

Table 2. Determination of weight W (importance) of an IT-indicator (fragment)

IT indicator name	Weight W	
	0-10	0-1 (normalized value)
Share of employees of the IT division which were trained or finished the advanced training courses	8	0,8
The current ratio of workers of the IT division employed on the contract and staff of the IT division, in comparison with a plan ratio	4	0,4
Share of the employees of the IT division who underwent testing regarding the tolerance to operation	7	0,7
Share of positions in the IT division, provided with the qualified duty regulations	6	0,6
Share of defects in the acquired IS, found prior to commercial operation	9	0,9
Share of abbreviation of serious operational incidents in a month	9	0,9
Quantity of technological software platforms at the enterprise	8	0,8

It is necessary to mark that objectivity of the received results in the course of formation of representative selection of IT indicators is provided with experts of the IT division of this enterprise who it was made by method of a multicriteria choice with the elementary hierarchy consisting of three levels: purpose, criteria and alternatives.

1. The purpose is a choice of the expert which can give the most objective estimates of quality of functioning of the IS, owning the general understanding of quality of functioning of the IS and methods of management of quality
2. Criteria: position, length of service, experience in IT
3. Alternatives: all employees of the IT division.

Procedure of rating of alternative versions of decisions on a method of the analysis of hierarchies is the cornerstone of making decision on a choice of the expert. Peak of a cluster is the Expert's Choice node.

For carrying out rating for a cluster "A choice of the expert" the scale of the relative importance of criteria a position, length of service the general, experience in IT (Table 3-5) is used. As a result for criteria "Position", "Standing" and "Experience" procedure of a choice of the expert is rather objective and the transparent.

It is necessary to mark that at the enterprises the chief and basic conductor of systems of automatization information processing are departments of ACS which traditionally exists in an organization structure of enterprise management. Applying the offered cluster "Position", practically the head of department of ACS or the IT manager of the enterprise will be always selected as the expert that doesn't give full objectivity in a quality assessment, in connection with not possible to use other employees of the IT division as experts.

Therefore it is expedient that employees of different levels of control appeared as experts. For this purpose potential experts it is possible to break on 2, 3, 4, etc. groups and from each group select the expert by method of the rating analysis of hierarchies. It is possible to break, for example, on three groups, then a cluster "Position" will consist of three parts: "The top management of the IT division", "Middle administrative staff of the IT division", "Service personnel IT division" (Table 6).

Table 3. Scale of the relative importance of criterion "Position" (According to The National classifier of Ukraine “Classifier of professions-2007”[8])

Relative importance	Position
1	Head of Enterprise Automatic Control Systems Department
2	IT-manager
3	Head of Computer Center (Data-processing center)
4	Head of Information Technologies Department
5	The chief specialist (or the Head) on information security
6	Head of department of ADP equipment
7	Head of department of information
8	Head of testing department
9	Head of department of technological development and implementation of computer center (data-processing center)
10	Head of department of automation and mechanization of productions
11	Database manager
12	Network administrator
13	System administrator
14	Software engineer
15	Specialist of department of the IT service and the material security of control of information technologies
16	Engineer on service of computer systems of technical department
17	Expert of information and technical department

Table 4. Scale of the relative importance of criterion "Length of service (general)"

Relative importance	Length of service
1	1-5
2	6-10
3	11-15
4	16-20
5	21-25
6	26-30
7	31-35
8	36-40
9	41-45

Table 5. Scale of the relative importance of criterion "Experience in IT"

Relative importance	Experience in IT
1	1-5
2	6-10
3	11-15
4	16-20
5	21-25
6	26-30

Table 6. Scale of the relative importance of criterion "Position" on groups of control According to The National classifier of Ukraine “Classifier of professions-2007”[6]).

<table>
<tr><th>Relative importance</th><th>Group of control</th><th>Position</th></tr>
<tr><td>1</td><td rowspan="4">The top management of the IT division</td><td>Head of Enterprise Automatic Control Systems Department</td></tr>
<tr><td>2</td><td>IT-manager</td></tr>
<tr><td>3</td><td>Head of Computer Center (Data-processing center)</td></tr>
<tr><td>4</td><td>Head of Information Technologies Department</td></tr>
<tr><td>1</td><td rowspan="6">Average administrative staff of the IT division</td><td>The chief specialist (or the Head) on information security</td></tr>
<tr><td>2</td><td>Head of department of ADP equipment</td></tr>
<tr><td>3</td><td>Head of department of information</td></tr>
<tr><td>4</td><td>Head of testing department</td></tr>
<tr><td>5</td><td>Head of department of technological development and implementation of computer center (data-processing center)</td></tr>
<tr><td>6</td><td>Head of department of automation and mechanization of productions</td></tr>
<tr><td>1</td><td rowspan="7">Service personnel of the IT division</td><td>Database manager</td></tr>
<tr><td>2</td><td>Network administrator</td></tr>
<tr><td>3</td><td>System administrator</td></tr>
<tr><td>4</td><td>Software engineer</td></tr>
<tr><td>5</td><td>Specialist of department of the IT service and the material security of control of information technologies</td></tr>
<tr><td>6</td><td>Engineer on service of computer systems of technical department</td></tr>
<tr><td>7</td><td>Expert of information and technical department</td></tr>
</table>

Applying a method of the analysis of hierarchies to each group, it is possible to select the appropriate experts for an assessment of quality of the IS.

The experts selected thus can participate in formation of representative selection of IT-indicators of all set of indicators of the IT provided by the Cobit 4.1 standard [18].

Thus, the created set of IT-indicators for use in case of an assessment of quality of component ISs and IS in general, is representative selection and therefore it is possible to speak about veracity of results of an assessment of quality of the IS of the enterprises.

CONCLUSION

1. Procedure of formation of representative selection of IT-indicators from general set by means of which it is possible most veracity and objectively to estimate quality of functioning of the IS at the enterprise is offered.

2. Veracity and objectivity of selection processes of quality criteria, selection IT-indicators and also experts, who make decisions on quality of functioning of the IS at the enterprise, are confirmed by use method analysis of hierarchies.

3. For all set of the indicators defining a status and processes of functioning of the IS are offered the quantitative and qualitative measures of an assessment which are characterized by the positive and negative tendency of change.

4. One of the principal conditions of support of objectivity and reliability of an assessment of quality of functioning of the IS is the correct formation of a set of IT indicators which will describe most adequately an IS status at the enterprise. Such method of formation of representative selection of a set of IT indicators was offered in this paper.

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TOPOLOGICAL PROPERTIES AND METHODOLOGY OF RESEARCH OF COMPLEX LOGISTIC SYSTEMS EFFICIENCY

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A b s t r a c t . It is indicated that sets of elements of complex logistic system, their parameters, numerical values, properties and finally – structures of complex logistic system, are topological spaces between which properties of homeomorphism can exist, that allows to apply mathematical apparatus of the theory of topology at the solution of a number of tasks. The general principles of an estimation of efficiency of functioning of logistical complexes on the basis of modeling with the account of hierarchical construction and influence of the human factor are considered.

K e y w o r d s : logistic system, logistical complexes, topological space, set members, structure.

INTRODUCTION

At present we observe significant interest increase in development of logistics and the data systems that provide effective integration and interaction of logistic processes in the world community. The development of this trend is very important for Republic of Belarus, as it carry out search of new ways of accelerated development of economy. As the world experience shows, one of such ways is the use of instruments of logistics and creation of logistic systems at micro and macroeconomic level. The development of logistic production, trade, transport and data systems has paramount importance, because it will allow us to accelerate integration of our country into outer economic and information space.

The solution of logistic tasks in the modern world is impossible without active use of information technologies. It is impossible to imagine formation and the organization of work of a logistic chain without intensive exchange of information real-time, without opportunities and means of supply of fast dynamics response of the market requirements. At present it is almost impossible to provide quality of goods and services, demanded by consumers, without use of information systems and software packages for the analysis, planning and commercial decision-making support in a logistic chain. Moreover, because of the development of information systems and technologies, that provided possibility of automation of standard technological operations, logistics became a dominating form of the organization of goods movement in highly competitive markets of economically developed countries. And the most perspective trend is adoption of information systems and technologies into logistic integration processes.

The most important feature of the functioning of logistics information systems (LIS) is associated with the participation of people in data reduction process and decision-making control. Man's role in the management process is manifested in all elements of the logistical system.

Another significant feature of the functioning of LIS is their random nature, which is caused not by full the conditions difinedness, in which these processes occur, and also various random deviations and errors encountered when collecting information, generating control signals and their performance.

Thus, the result of the functioning of the LIS is random and characterized, on the quantitative side, by laws distribution parameters expressing this result.

MODELS AND OBJECTS OF LOGISTIC SYSTEMS

Mathematical models of logistic systems can have various forms depending on a stated task and research methods. The economic-mathematical methods applied in logistics, include [4,13,14,18-20]:

- statistical analysis methods;
- methods of mathematical economy and econometrics,
- operations research methods,
- methods of economical cybernetics.

Each of the methods is based on the use of the appropriate mathematical apparatus.

In most cases the logistic system represents the set of elements (producers, mediators, consumers), united by logistic streams [2,18]. There are material, financial, service and accompanying them, data streams. Thus a logistic stream is the set of the objects (sets) united on a certain sign, moved in space and in time and adapted to quantitative and qualitative transformations according to the influence of the agent of management of logistic system [3,15].

Each of logistic streams is characterized by a set of the objects. In a material stream objects are material resources (raw materials, materials, products, etc.), subjects of production and complete product. In a financial stream objects are financial assets in the cash or non-cash form, providing effective functioning of logistic system and its parts in the conditions of the commodity-money relations. The certain set of the nonmaterial values received by clients according to their demands acts as the set of objects of a service stream.

Data streams accompany all the other types of streams and represent certain finished messages generally – in the electronic form, intended for decision-making and implementation of administrative decisions. Data streams form the logistic data systems, classified by scale, range of application and a way of the organization [7-11].

According to tasks and methods of research the following types of models of logistic systems can be considered [1,3-6, 12,16]:

- models covering separate logistic operations or functions,
- models covering several logistic operations or functions,
- models of logistic systems (channels, networks).

According to [2,16], at mathematical formalization it is convenient to describe complex logistic system as a certain set of the interconnected and interacted subsystems on the basis of graph theory. That gives a graphical idea of structure of system and functional relations between its elements. Thus the specific properties become apparent. That, at research of technical system, allows us to use ample opportunities for mathematical apparatus of the set theory and topology.

We will consider properties of the graph $G(V,E)$ that corresponds to the complex logistic system. This graph is characterized by two finite sets:

$V=\{v_1,v_2,...,v_{nv}\}$ - vertex set,

$E=\{e_1,e_2,...,e_{ne}\}$ - set of graph edges,

nv, ne - number of vertices and number of edges of a system graph, respectively.

As each edge is defined by two vertices, the set of edges E represents the system (family) of subsets from V, meeting the following conditions:

1) empty set Ø and V belong to E,

2) if the sets belong to E, than the crossing also belongs to E,

3) combination of any family of sets from E belongs to E.

Therefore, E is a topology in V and the pair is a topological space [17].

TOPOLOGICAL PROPERTIES OF LOGISTIC STREAMS

For real logistic system at elementary level of the description the set $V = \{v_1, v_2 ... v_{nv}\}$ represents a set of elements, which form the given system $N = \{N_1, N_2 ... N_{nv}\}$. For material streams, for example, it can be points of delivery and consumption of materials, production, for financial streams – points of implementation of financial operations. By transition to higher level of the description (abstraction) of systems, the whole systems (subsystems) can be represented as set members N.

There is certain basic material or financial parameter, corresponding with each element of $N_i \in N$, that characterize it. For example, for material or financial streams, mentioned above, it can be various goods, currency, etc. For the systems with high level of abstraction, parameters of elements of the system (subsystems) accept as a set of the parameters reduced to one generalizing. Therefore, a certain set of parameters of $\theta = \{\theta_1, \theta_2 ... \theta_{n\theta}\}$ of these elements corresponds with a set of elements of system N. If each element of the system is characterized by only one parameter, than $nv = n\theta$ and sets of N and θ are equipotent.

Each parameter $\theta_i \in \theta$ can take various numerical values $\varphi_i \in \Phi_i$, and $\Phi_i \in \Phi$, where $\Phi = \{\Phi_1, \Phi_2 ... \Phi_{n\Phi}\}$ - a set of numerical values of parameters of material or financial streams.

In one's turn the data characteristics of N_i - x elements which in their turn represent a set of objects of logistic data streams $\Pi = \{\Pi_1, \Pi_2 ... \Pi_{n\Pi}\}$, depend on numerical values of parameters $\varphi_i \in \Phi_i$.

On its own the logistic data stream is rather complex system and it is divided into number of components: accessory, indicator, document and array [2].

Accessory is an elementary unit of the message. The accessory characterizes a quantitative or qualitative component of information set. For example, accessories — the organization name, the description of goods, the goods price, etc. Each accessory can be represented by a set of symbols: digital, alphabetic, special.

The documents used in control process, can include one or several indicators with obligatory certificate (the signature or the seal) of the person responsible for information containing in documents. As the organization of basic data is a field of activity of the person, the majority of documents is created at a stage of collecting and logging of data, though the considerable share of documents enter the system from external (higher, etc.) organizations. For example, in accounting the indicator, its basis is a result of the account, weighing, etc. It forms a basis for organization of summary accounting and statistical data which in their turn will be incoming information by drawing up statistical reports in a perspective of the organization, branch, region, etc.

The array represents a set of homogeneous data that have a single technological base and it is united by single meaning content. Data (processes, phenomena, facts, etc.) represented in the formalized condition suitable for transfer by communication channels and for processing on the computer. Records are the basic elements of arrays defining their contents.

Records are the elements of arrays which the users use when information processing. Information fields are the elements of the records having single meaning content.

The data belonging to one array, record by the general rules (according to technology of accumulation, storage and processing of data adopted in the organization). The type of the array is defined by its content (for example, an array of material standards, the array of materials suppliers), functions in data processing (input, output, intermediate arrays). The information array supplied with a symbolical name, unambiguously defining it in information system, is called the file.

If information streams represent relational databases as Microsoft Access, than the tables, inquiries, forms, reports, pages of access to data, macros, moduli are the objects Π. Sets Φ and Π are not equipotent because between their elements there is no one-one imaging $(n_\Phi \neq n_\Pi)$, the quantity of their elements is various.

Between sets N and θ, θ and Φ, Φ and Π there are connections and dependences which are generally defined by the binary relations r that establish compliance between elements of one and the other set:

$$N \overset{r_1}{\sim} \theta,\ \theta \overset{r_2}{\sim} \Phi,\ \Phi \overset{r_3}{\sim} \Pi.$$

Let us consider a N×θ - a set of the ordered pairs of elements (N_i, θ_j), from which $N_i \in N$, $\theta_j \in \theta$. As the binary relation $r \subset N \times \theta$ is defined everywhere on N, that is its range of definition *dom r* coincides with a set N, than it is imaging of a set N in a set θ and writes down as $\phi_1 : N \to \theta$. Therefore, the set $\{N{:}|\exists\theta((N,\theta)\in r)\}$ is a prototype of the relation r1, and the set $\{\theta{:}|\exists N((N,\theta)\in r)\}$ is an image of the relation of r1. For any two various elements N1 and N2 from N their images $\theta_1 = \theta(N_1)$ and $\theta_2 = \theta(N_2)$ are also various. At the same time, being image for N, the set θ is a prototype for a set Φ which in its turn is an image for θ and a prototype for a set Π, i.e. $\phi_2 : \theta \to \Phi$, $\phi_3 : \Phi \to \Pi$. In practice, taking into account properties of real logistic system, images ϕ_1 and ϕ_3 are single-valued, and the relation ϕ_2 is multiple-valued.

As the result, the vertex set $V = \{v_1, v_2 ... v_{nv}\}$ can characterize different physical notions, depending on what is the purpose of the researcher, and the use of the homomorphism –a mappingof set of elements of one model in a set of elements of another model system - allows for the same technical system to create and explore various types of models: physical, abstract, information, conceptual and other.

The graph edge set of a complex system $E = \{e_1, e_2 ... e_{ne}\}$ characterizes the relationship between the elements (the topology into V). The pair (N, E) is a topological space of the system elements.

Under the homomorphism into another set (to the other models), corresponding connection (topology) between the parameters θ, and their numerical values Φ and properties Π are obtained. Thus, a transition to a new topological spaces is carried: (θ, E), (Φ, E), (Π, E).

Mapping of a topological space (N, E) into a topological space (θ, E) is continuous at each N point. Here we have the mutual inverse mapping $\varphi_1^{-1} : \theta \to N$, it follows that we are dealing with homeomorphic topological spaces (N, E) and (θ, E). In general, this is not true of topological spaces (Φ, E) and (Π, E), which do not have a one-to- one direct and inverse mapping. However, for a number of practical problems, in applying additional restrictions, homeomorphism between all of the above topological spaces can be achieved: (N, E), (θ, E), (Φ, E), (Π, E).

In real complex logistics system some connections between system elements (subsystems) for various technical or subjective reasons can die out or change at random times. This means that there is a weakening of the topology - instead of E, we have a set $E^{(s)} = \{..., e_{ns}\}$, $(s = \overline{1, ns})$. If there are no other sudden changes in the system, except the changing relations between the subsystems, *ns*–is a number of possible system states (structures) $E^{(s)} \subset E$, as a result $E^{(s)}$ is the weaker topology compared to *E*. Among all topologies on *V*, zero *(V, Ø)* – is the weakest, and the so-called discrete *(V, E)* is the strongest, as it consists of all subsets (edges). Both these topologies are extreme in the scale of comparison of topologies.

In general, under the condition (structure) of a complex logistics system means not only the presence or absence of appropriate links between subsystems, but also the subsystems' state, which is characterized by a significant difference between their properties. A complex of possible structures with the logistics system is a set of

$S = \{s_1, s_2 ... s_{ns}\}$. This set depends on many system properties $\Pi = \{\Pi_1, \Pi_2 ... \Pi_{n\Pi}\}$.

There bijection between the sets Π and S, since they have different potencies (cardinals). However, due to the fact that the elements of S (subsystems structure) are defined by the elements of Π (subsystems properties), for the i-th subsystem s_i is the image for Π_i what it is $\phi_4 : \Pi_i \to s_i$. Consequently, there is a map $\phi_5 : \Pi \to S$.

The S is finite and for each s_i an explicit mathematical model is valid, which in a state of spaceis characterized by a variety of phase coordinate system $X^{(s)} = \{X_1^{(s)}, X_2^{(s)} ... X_{nx}^{(s)}\}$. $X_i^{(s)}$ - is the set of phase coordinates of *i*-th subsystem - a subset of X; $X_i^{(s)} \subset X_i^{(s)}$.

The X = X (t) and S = S (t) setsare the basis for the use of the mathematical apparatus of the theory of dynamical systems with random structure and the use of methods of the topology theory allows to take into account the specific properties of complex multistructural systems under their analysis and synthesis.

In summary, sets of elements of a complex logistics multistructural system, their parameters, numerical values, properties, and finally, logistics system structures are topological spaces that have the properties of the homeomorphism that allows using the mathematical apparatus of topologyin solving logistics management systemstasks.

Formation of information systems is impossible without streams research in section of certain indicators. For example, it is impossible to complete the task of equipping a certain workplace computer technology without knowledge of the information content passing through this workplace, and without determining the required speed processing.

It is possible to manage the information flow quickly and efficiently through the organization of information system, performing the following operations:

- Redirection of the information flow,
- Limiting the transmission rate to the corresponding reception rate,
- Increasing or decreasing information content in certain slots of information transmission,
- Limiting the stream capacity to the amount of data trough put of separate unit or route section.
- Information systems in logistics allow managing of material and financial flows at the enterprise level, and can contribute to the organization of logistical processes in the regions, countries and groups of countries.

MULTILEVEL SIMULATION OF LOGISTICAL COMPLEXES

The main method of studying the functioning laws of logistical complexes, which include LIS, is the simulation of logistical processes. In a broad sense, modeling is an imitation of studied processes no matter by what means it is secured. In practice, the modeling of complex systems that have to deal with the evaluation of the effectiveness of logistical complexes, there are two different approaches to the construction of models.

The first approach, which is essentially a simulation, involves modeling states of each element of the system from beginning to end the process. Let’s conventionally call it "moving" modeling. Consolidation of information in such models occurs only at the output of the model.

The advantage of "moving" models is in principle possibility to use all the information for research and for the organization of rational (optimal) operation of involved system. Disadvantages of "moving" modeling are associated with practical difficulties of detailed modeling of complex systems, which leads to the need for introduce a number of significant assumptions and, consequently, the loss of some information.

The second approach is that, in accordance with the hierarchical structure of involved system, where information of varying degrees of detail at different levels of information is used, the model of such a system is a hierarchical system models. Each model of a lower-level unit is the block of highest level model and linked it with a limited number of channels through which circulates already partially generalized information. This simulation is conventionally called "hierarchical".

The advantage of these models is that the information used in them for quite a detailed analysis of logistical information of complexes at the lower levels of the model is applied to the highest levels in the form of summarized indexes. This scythes the information content circulated in the model, and therefore, simplifies it. Under the same level, details of background information "hierarchical" model are then much easier to "moving". Disadvantages of "hierarchical" models are associated with the loss of information, which occurs during its partial generalization at the lower levels of the model, and as a result, the inability to use the full information content to optimize the functioning of the whole system.

Despite its disadvantages, the "hierarchical" modeling usually is a more effective method for complex systems studying, as it allows analyze research in a number of relatively particular problems, which are united by limited number of connections.

Let's dwell on the structure of "hierarchical" models and introduce the following terminology. We will name “level model” a number of stages of partial consolidation of information in it. Thus, the I level model is a "moving" model. It summarizes all the information only on the output. II level model consists of I level models and the block of II level model, that processes the information received from the outputs of the I level models and maybe a limited portion of the original information. N-level model consists of (N-1) level and the block of N level models. And, the (N-1) level model is a model of the final control element and control element of the N- level complex. Thus, the N-level model is obtained by adding to it on a limited small number of links of the block N-th level. The problem consists in determining the number of required connections, the nature and information content on the inputs and outputs of the units withdrawn from the model. The values or functions taken from the "outputs" of each model should characterize the functioning of an appropriate set of results and allows us to calculate the criteria (indicators) its effectiveness. Resolving this issue is determined by the structure and objectives of the system being simulated, as well as the objectives of this particular study.

During each research as well appears the following question: “A model of which scale is needed to solve the problem of the current research?” The matter is that the effectiveness of functioning of any complex of the lower level has an impact on effectiveness of complexes on higher levels. It’s clear that the effectiveness on the economics on the whole depends on the effectiveness of the enterprise logistics. Thus the following question arises: “The phenomenon of what scale should be viewed in order to estimate the impact of the given set of parameters of the complex on its effectiveness?” To answer the question let’s take the notions of “necessity” and of “sufficiency” of the level of model for the purpose of the given research. Say that for the purpose of the research of the impact of the given set of parameters on the effectiveness of the system is needed the model of the level N, if at least one of the researched parameters is directly used in the N level model (is given on the input of N level block). The N-level model is sufficient for the research of the given set of parameters if none of the researched parameters is used in higher level models (not given on the input of higher level models). This means that all the researched parameters appear indirectly with the help of composite index in the N+1 model and higher level models. For example, “necessary” and “sufficient” model of the research of the influence of sales of products of enterprises on the effectiveness of the sector is the functioning enterprise model in the production and sales product process. The model is “necessary” because the demand is used during the realization modeling as a parameter describing the terms of raw materials purchase, components and production. The model is “sufficient” because during the sector working modeling (higher level complex) the raw materials purchase and components don’t appear in it. In this case enterprise is the final control element in the sector and is characterized by generalized parameters, which is, for example, the possibility of successful sales of products. Production value has an indirect impact on the sector functioning, through the possibility of its realization.

Hierarchical model systems give us the opportunity to study each of the models separately, and each of the models is built by one and the same way. It consists of 3 major elements which are the block that models input information: control block and block modeling final control element.

Thus functioning logistic complex model of any rank supposes the necessity of solving 3 major problems:

1) Modeling of input information according to the properties of the information complex element. It consists of realization on the distributional law in the models which are measured (observed) by the information element of the given set of parameters (elements) of the economic situation.

2) Modeling of the control element functioning. It consists of realization of "control law" complex. What is meant by the notion "control law" is the set of rules (algorithm) according to which control commands are worked out depending on the existing economic information.

3) Modeling of the final control element functioning. It consists of realization of functioning complex result in the model depending on control commands.

Creation of functioning model of each of the logistic complex in general assumes the formulation and realization of "control law" in the model, during forming of which human activity plays a huge role, taking into account that human activity hardly can be exactly described and optimized. In these cases the satisfying modeling of the working complex can be reached either by including a person or a group of persons into modeling process who will imitate human activity in the studying process or by creating specific subprograms which will model the actions of people, the se-called heuristic programs, which may possess a certain property of teaching or self-teaching. Such modeling could provide the fullest description of the real processes. However theory and practice of the creation of such models currently aren't perfect and can't be considered as completed. Thus when modeling of functioning of logistic complex the "control law" is usually set on the basis of the study of the economic methods of its use in the form of its functions or algorithms which set into accordance the meaning of parameters of control commands meanings of a limited number of basic parameters which characterize the information about the economic situation.

Modeling of the work of the information and control final elements of the logistic complex is not a difficult task, but the reception of the distribution parameter law, which characterizes the result of functioning analytically, in most cases, has many difficulties because of complicatedness of the researched objects. Thus it's appropriate to consider the problem of modeling of functioning of logistic complex to be solved if the block-diagram of the model functioning is built, which enables us to receive the realization of parameters which characterize the result of functioning (output) when the realization of parameters is put on "input". Such decision presupposes the indication of input and output parameters of the midship section blocks and the formulating of transformation law of input parameters into output parameters. If such model is built the reception of the necessary distribution laws with any given accuracy can be reached by using the statistical test method after processing of a special output realizations set which appear as a result of multiple modeling processes when the distribution laws of input parameters are given.

When using the method you may not put unnecessary limits on the ongoing processes and consider the task to be done. Alongside with it we will examine the analytical methods mainly based on the linear theory of transformation of random variables and random functions. At the same time random variables and functions will be characterized by their numerical characteristics. In some cases the knowledge of numerical characteristics of random variables and random functions enables us to define the distribution laws of particular random variables, if the conclusions based on physical considerations and limiting probability theorems about the kind of distribution law can be made. The set of approximate analytical methods can be viewed as approximate economic efficiency theory.

Thus the primal problem of the economic efficiency research of the logistic complexes is receiving distribution laws of parameters characterized the result of their functioning possessing a goal of calculating the criteria of efficiency.

The basic research logistic complex method is the modeling of economic processes which consist of:

- Hierarchical model systems of certain complexes which have similar structure and linked to each other by a limited number of inputs and outputs.
- Modeling the functioning of each complex.

Calculating the performance of logistics centers is not an end in itself. Practical value of cost-effectiveness studies is the ability to analyze the impact of various factors on the cost-effectiveness of systems. The results of cost-effectiveness studies are used in two main ways:

- Study of the influence of technical parameters of logistics complexes on their cost-effectiveness with a view to bringing legitimate claims to the parameters of complexes and their choice in the design and modernization.
- Quantitative research methods of using logistic complexes with a view to making recommendations to improve the application, to develop rational (optimal) "control laws" complexes.

Methods for solving these two major problems are not fundamentally different from each other. The only difference is in the degree of detail of the accounting or other factors. Thus, when solving the first problem is basic. Attention to detail the technical aspects of accounting, while the "control law" is usually given with a sufficient degree of conditionality, not distorting, however, the nature of the influence of technical parameters on the result of functioning systems. In solving this problem more important to obtain comparative estimates. In solving the second problem focuses on "the laws of the control" of the complex, while the technical parameters can appear in the form of a relatively generalized averages. To solve this problem relatively greater importance is attached to the possibility of obtaining absolute estimates.

MODELING OF EACH COMPLEX FUNCTIONING

The calculation of indicators of effectiveness of logistic complexes is not an end in itself. The practical value of economic efficiency research result is lying in the possibility of analyzing of the influence of different factors onto economic efficiency of the complexes. The result of economic efficiency research is used in two major directions:

The research of the logistic complex technical parameters influence of their economic efficiency for the purpose of presentation of grounded requirements to the complex parameters and their choice during designing and modernization;

The quantitative research of the ways of logistic complex use for the purpose of generating recommendations in order to increase effectiveness of use i.e. generating optimal “control law” complexes.

The methods of solving these two major problems don’t differ. They differ only in the level of detail of taken factors. Thus while solving the first problem details of technical aspects are in the center of attention; and the “control law” as a rule is set with sufficient level of conditionality without deceiving the character of technical parameter impact onto the complex functioning result. While solving this problem more attention is given to the receiving of comparative evaluations. While solving the second problem “law control” complex is under center attention and the technical parameters may appear as relatively generalized mean value. While solving the problem more attention is given to the opportunity of receiving absolute estimates.

LOGISTIC COMPLEX EFFECTIVENESS

When designing logistic information systems as the key elements of logistic complexes or when taking them into exploitation we have to solve the problem concerning the alternative rational choice which could be effective enough in the given conditions of application. Suppose the existence of different ways of solving the problem (for example different types of production):

$$B_1, B_2, ..., B_m$$

and the set of conditions of its distribution:

$$A_1, A_2, ..., A_n.$$

Let W_{ij} is the indicator of effectiveness (probability of receiving economic effectiveness) when applying i-type decision (i-type production) and applying j conditions. Let's make up the matrix of effectiveness. (table 1).

Table 1. Matrix of effectiveness

Alternate solutions	Alternate conditions			
	A_1	A_2	…	A_n
B_1	W_{11}	W_{12}	…	W_{1n}
B_2	W_{21}	W_{22}	…	W_{2n}
…	…	…	…	…
B_m	W_{m1}	W_{m2}	…	W_{mn}

To illustrate the example we can built a diagram of dependency between indicators of effectiveness and alternate conditions for each alternative solution. Let's put the effectiveness W on the ordinate axis and alternate conditions on the abscissa axis put in a certain order. Let us connect W_{ij} by appropriate lines. In the result we will get a typical case when in some cases one solution gives more effectiveness, and in the other cases the other solution gives is more effective. However it's impossible from economic point of view to have many technical solutions and use each of them in a certain case. Thus you have to choose one or sometimes it's possible to choose few solutions which would be most effective for the whole range of use.

In some cases a problem of stating the probability or frequency of condition appearing:

$$p_1, p_2, \ldots, p_n,$$

arises and it's necessary to choose the solution when averaged indicator and W_i will be greatest:

$$W_i = p_1W_{i1} + p_2W_{i2} + ... + p_nW_{in}.$$

If the probabilities p_j (j = 1, …, n) are given and don't change in time it should be taken as that. However it's impossible to get the p_j value. Besides, this alternative may turn to be effective only in certain conditions of stable operation of economy. In the result the solution will be effective for a short period of time. Thus it makes no sense to choose the solution accordingly to the given formula and to lose. Therefore it's better to find a compromise which would suit for all the range of conditions even if it's not optimal in certain range of conditions. The best solution is based on the tutor's analysis of the matrix of effectiveness taking into account the forecast in economy environment change.

CONCLUSIONS

1. A method for determining the effectiveness of the main tasks of LIS is researched.
2. There is recommended decisions making method for use under uncertainty information conditions.

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MATHEMATICAL SIMULATION OF INFOCOMMUNICATION NETWORKS APPLYING CHAOS THEORY

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Abstract. The advent and wide usage of computer networks as well as an increasing number of various network services have resulted in the fact that the network traffic has become more complex and unpredictable. These properties have become especially apparent with the appearance of high-speed data transmission technology. It is connected with the fact that one of the main quality indeces (QoS) of operation of packet transmission networks is the number of lost packets. The loss of packets results to additional network traffic and, finally, “congestions”. At high speeds of data transmission packet losses, expressed in portions of a per-cent, lead to considerable information losses. It has been shown in numerous papers devoted to the research of network traffic that the abovementioned phenomena are related to the properties of the traffic self–similarity which is mainly caused by the TCP protocol behaviour. However to date models have not been offered which adequately describe the behaviour of the communication networks of information systems and which allow to apply the whole arsenal of classic methods of analyzing nonlinear dynamic systems.

Key words: chaos, dynamic systems, self-similarity, TCP/IP.

INTRODUCTION

Presently TCP protocol is the basic Internet transport protocol, which ensures reliable delivery of a byte flow with connection establishment and it is applied in those cases when guaranteed messages delivery to be required. Also it includes the flow control mechanism which ensures that a sender does not overfill the recipient buffer and congestion control mechanism which tries to prevent the oversized data volume injection in a network, which leads to the loss of packets.

The numerous studies of processes in the Internet network have shown that statistical characteristics traffic have the temporal scalable invariance property [13,18,25] (self-similarity) and TCP protocol being the basic data transmission protocol is the principal reason of traffic self-similarity. Applying the concept of self-similarity to telecommunication systems has been for the first time suggested by B.B. Mandelbrot [2].

Besides the Internet architecture of protocols places certain limitations. For example, competitive TCP flows [28,29] and all plurality of transport protocols used in computer networks “don’t know anything about each other”, i.e. data flow to be managed with various schemes (or it doesn’t occur at all) [11].

In addition different models of the congestion control in TCP and other protocols should coexist in harmony. If one of the TCP implementations turns out to be more aggressive than the others, then it will use the major part of the bandwidth, which will hinder the data transmission in “adjacent” connections. At the same time the unnecessary conservatism of an algorithm will have a negative impact on the general protocol performance [1,9].

In recent years several new modifications of TCP protocol have been proposed. They include Binary Increase Control TCP (BIC-TCP), CUBIC TCP, Westwood TCP (TCPW), Parallel TCP Reno (P-TCP), Scalable TCP (S-TCP), Fast TCP, HighSpeed TCP(HS-TCP), HighSpeed TCP Low

Priority (HSTCP-LP), Hamilton TCP (H-TCP), Yet Another Highspeed TCP(YeAH-TCP), Africa TCP, Compound TCP etc.

For instance TCP Vegas [12] introduces a new congestion control mechanism which tries to prevent them and it doesn't react to the appeared congestions. Almost all of them are based on the well-known old TCP versions and they differ from each other by various congestion avoidance means. To be more precise, they are based on different ways of detecting the packets loss occurrence fact, which means the appearance of congestion. Various formulas for *cwnd* (congestion window) calculation are used in different modifications.

A new approach to analyze the communication network behaviour of information systems with TCP protocol has been suggested in this paper – to consider these systems as nonlinear dynamic systems [6,21] manifesting chaotic properties at certain values of parameters.

TEST BED

One of disadvantages in research of infocommunication systems is a high cost of equipment. Not nearly every laboratory (and even an Internet provider) can afford buying several routers, hardware protocol analyzers, wireless access point suites and other equipment for testing new protocols, architecture optimization solutions or certain topology selection. This is exactly why program products were created which allow info-communication systems to be simulated [20]. Developing such software make it possible to perform required research and experiments much less expensive and to obtain practically the same results as using real hardware (OPNET [4], NS [22]). Apart from apparent saving the approach of simulator using allow to conduct experiments without a real network designing.

The congestion state occurring in a computer network has been modelled by means of a discrete-time simulator with an open source code ns-3 (Network Simulator 3) in which numerous real protocols and various types of environment data transmission to be implemented. NS-3 is able to simulate a wide range of protocols and processes in real time and to integrate the simulator with a real network as well. The NS-3 is used as a tool by a large amount of researchers during quite long time and it includes numerous tests, which guarantees validity of the obtained results. The NS-3 simulator includes a great number of tests for all the components, which ensures the reliability of the results obtained.

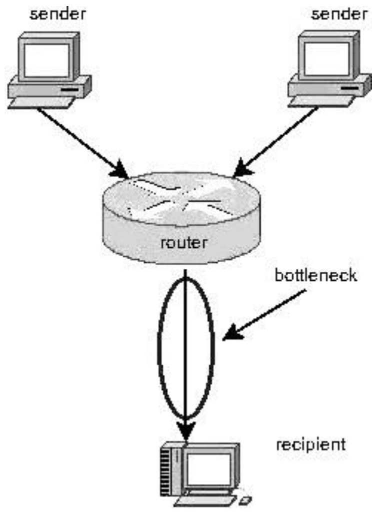


Fig. 1. Topology of a model network

The TCP/IP network model has been created by means of the NS-3 simulator where all the hosts are connected with a router by the point-to-point connection type (Fig.1). The operation of the applications has been simulated in the sender-hosts sending data with a permanent bitrate to the recipient host where the application operated and received data from the both types of hosts [8,26]. The numerical experiments have been conducted in which the sender-hosts transmitted data with different bitrate exceeding the capacity of the recipient channel, which resulted in overfilling the router buffer and unnecessary packets were rejected. The data generation rate senders (C_f), the delay (d_b) and capacity (C_b) of the channels in a bottleneck as well as the delay and capacity of the channels in the sending hosts could be varied specifying particular parameters in the given model. The receiving host window was deliberately made very large to make the value of the congestion window ($cwnd$) the only limiting factor. The value of congestion window ($cwnd$) has been chosen as the observed parameter and the most informative one as it directly affects the transmitted data size. The TCP-Tahoe mechanism of congestion control has been used for the network operation simulation [24].

The *cwnd* value has been monitored for each TCP-connection throughout modeling of the congestion state in the network. The paper [27] has suggested using the values of the time sequence $[x_t, x_{t-\sigma}, x_{t-2\sigma}, \ldots]$ averaged over N as an easily measured characteristic of complex systems, and it has been shown that it is applicable to the recovery of the hidden multidimensional trajectories. The given method applied to the *cwnd* values leads to the relations [28]:

$$x[i] = \frac{1}{N}\sum_{j=1}^{N} cwnd_x[i-j]$$
$$y[i] = \frac{1}{N}\sum_{j=1}^{N} cwnd_y[i-j] \quad (1)$$

Here x and y denote two TCP flows. The N quantity is responsible for an averaging scale and the more N is the more hidden dimensions of the system can be recovered.

In this case $cwnd(t)$ functions are different for each of the hosts and only moments of changing the values of this function have been fixed. Therefore applying the abovementioned method and plotting a phase portrait require taking $cwnd$ values at the same moment of time.

For the further analysis of the data obtained one should plot a phase portrait – dependence $cwnd1(cwnd2)$. In order to get that done one should take $cwnd$ values at the same moment of time.

NUMERICAL RESULTS

Under certain parameters of the test bed such a system that seems to be rather simple, manifests quite a complicated behaviour. In particular, below are given the graphs of $cwnd(t)$ dependence at C_f =5Mbps, d_b =10ms, C_b =5Mbps, Q_s =20 packets (1 packet = 536 bytes, for all the numerical experiments).

The phase portraits corresponding to Fig. 2 and obtained by data processing, according to the algorithm described in the previous section, at N =2000 and dt =10 ms, are shown in Fig. 3.

As it can be seen the phase trajectories shape a limit cycle which has quite a delicate structure. At that this trajectory is rather steady an image point after a little roaming begins describing the same closed trajectory at changing the start time of TCP flows relative to each other. Such trajectories are called attractors. More technical definition is proposed in [16].

For comparison, below are given data and the phase portrait for the case when the host-senders don’t overfill the recipient bandwidth i.e. there is no congestion (Fig. 4 and Fig.5). In this case, as one would expect, the $cwnd$ value of the both hosts indefinitely increases and there are no anomalies in the phase portrait.

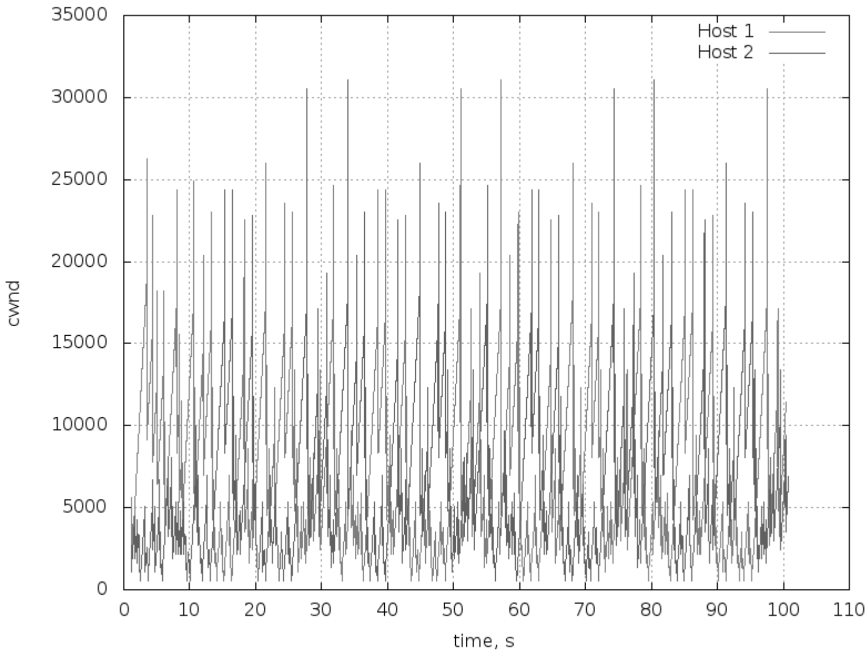


Fig. 2. Congestion window dependence on time under C_f =5Mbps, d_b =10ms, C_b =5Mbps, Q_s =20

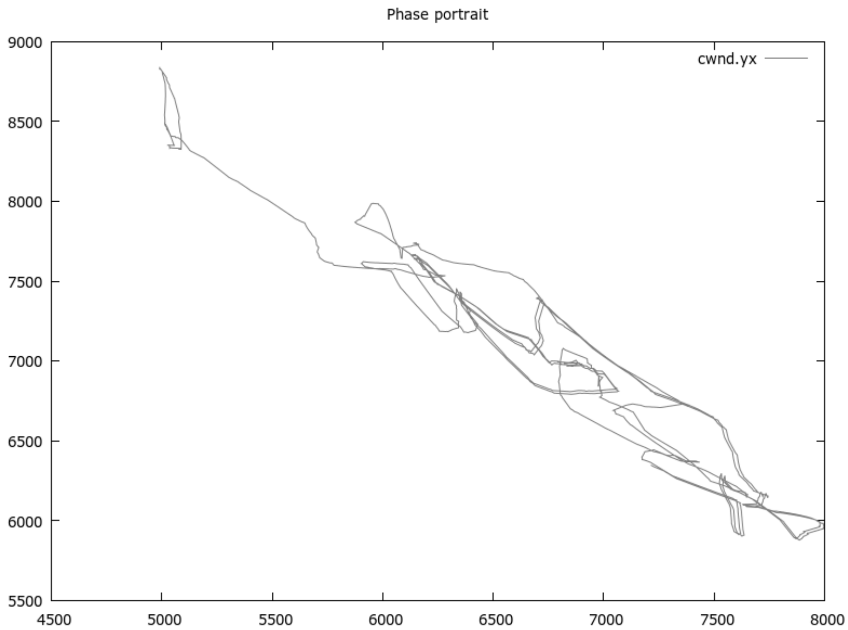


Fig. 3. Phase portrait under C_f =5Mbps, d_b =10ms, C_b =5Mbps, Q_s =2

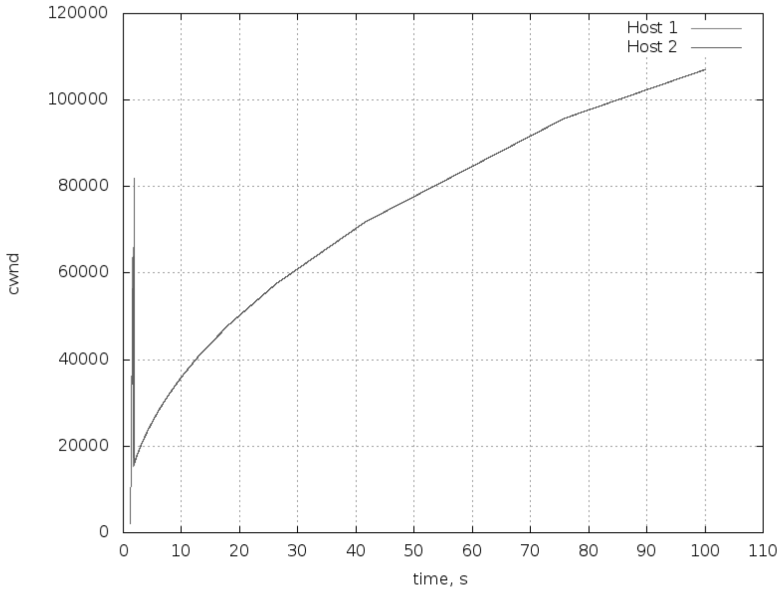


Fig. 4. Congestion window dependence on time under C_f =2Mbps, d_b =10ms, C_b =5Mbps, Q_s =100

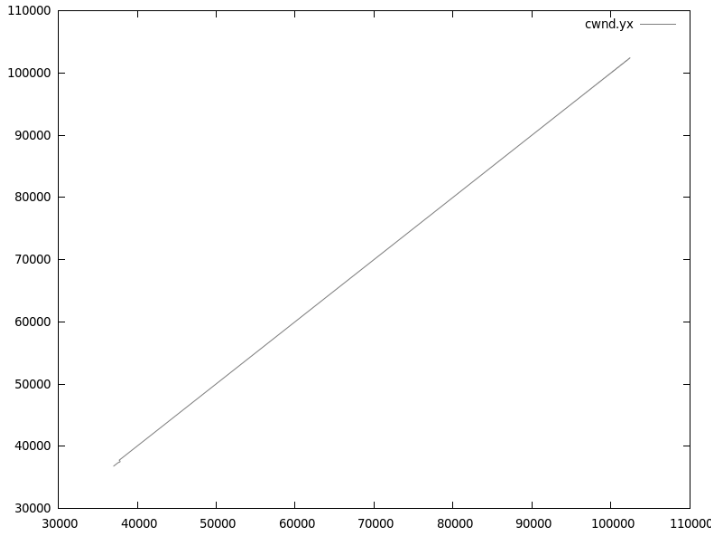


Fig. 5. Phase portrait under C_f =2Mbps, d_b =10ms, C_b =5Mbps, Q_s =100

MAXIMUM LYAPUNOV EXPONENT

Phase portraits are suitable as they are able to visualize the state of a dynamic system. Having a phase portrait one can calculate a maximum Lyapunov exponent (MLE), for example, using Benettin's algorithm [3], – the quantity which is characteristic of the recession rate of close trajectories whose positive value is considered to be an indication of the system chaotic behaviour [3, 17]. The utility package TISEAN has been chosen as an instrument which would allow the obtained data to be analyzed independently of the number of available TCP-sessions and it is intended for analysis of time series and which is based on the theory of nonlinear deterministic systems or chaos theory [7].

The utility lyap_k from TISEAN package has been used for calculating the maximum Lyapunov exponent. The result of its work is a set of data which represents the dependence of the logarithm of trajectories recession on time $S(\varepsilon, m, \Delta n)$[19], which is calculated in the following way:

$$S(\varepsilon, m, \Delta n) = \frac{1}{N}\sum_{n_0=1}^{N} \ln\left(\frac{1}{\left|U\left(S_{n_0}\right)\right|} \times \sum_{S_n \subset U\left(S_{n_0}\right)} \left|S_{n_0+\Delta n} - S_{n+\Delta n}\right|\right) \quad (2)$$

Where ε is the neighbourhood of point S_{n_0}, m is dimensionality of the phase space, Δn is time, and $U\left(S_{n_0}\right)$ is the neighbourhood of the point S_{n_0} of diameter ε.

In the given algorithm the point S_{n_0} is chosen in the phase space and its "neighbours" are marked which are remote from S_{n_0} within distance ε.

If quantity $S(\varepsilon, m, \Delta n)$ shows a linear increase with the same slope in the reasonable range of values ε, then the slope ratio approximating this part of the line can be considered to be roughly equal to the maximum Lyapunov exponent.

The results obtained with the help of the lyap_k utility after processing and visualizing $cwnd(t)$ time series, corresponding to Fig.6, Fig.7 are represented here. The curves $S(\varepsilon, m, \Delta n)$ for five different values ε and the straight line $y = a + bx$ approximating the linear section of these curves are shown in the figures. Thus, value b equals the maximum Lyapunov exponent numerically.

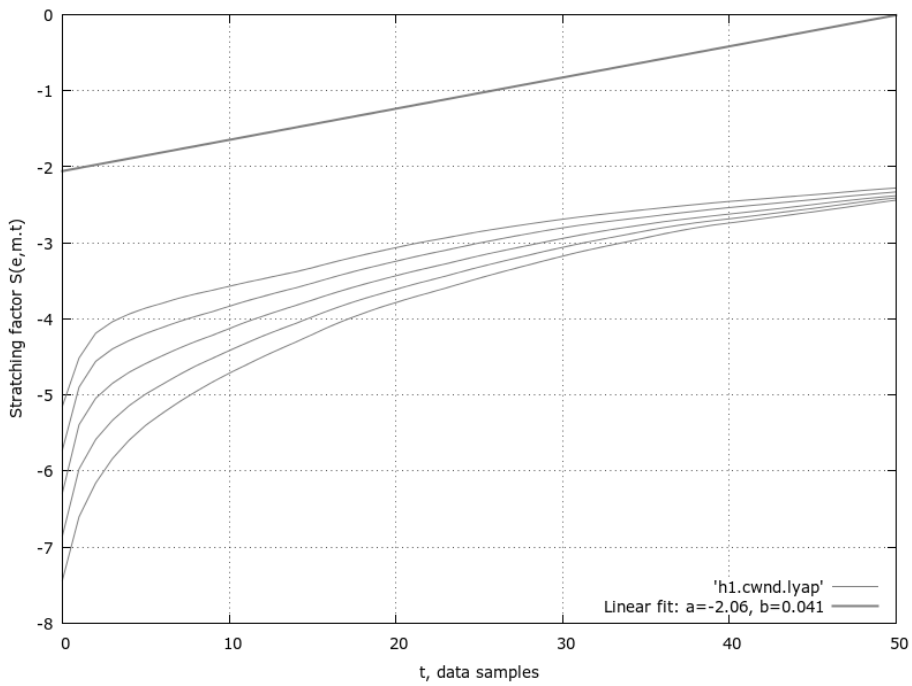


Fig. 6. The Lyapunov exponent calculation C_f =5Mbps, d_b =10ms, C_b =5Mbps, Q_s =2: λ ~ 0,041

The data obtained in the process of the network work simulation in the absence of congestion have been analyzed. As it can be seen from the graph (Fig. 7), in this case $\lambda < 0$ and this means that such a system does not show a chaotic behaviour [5].

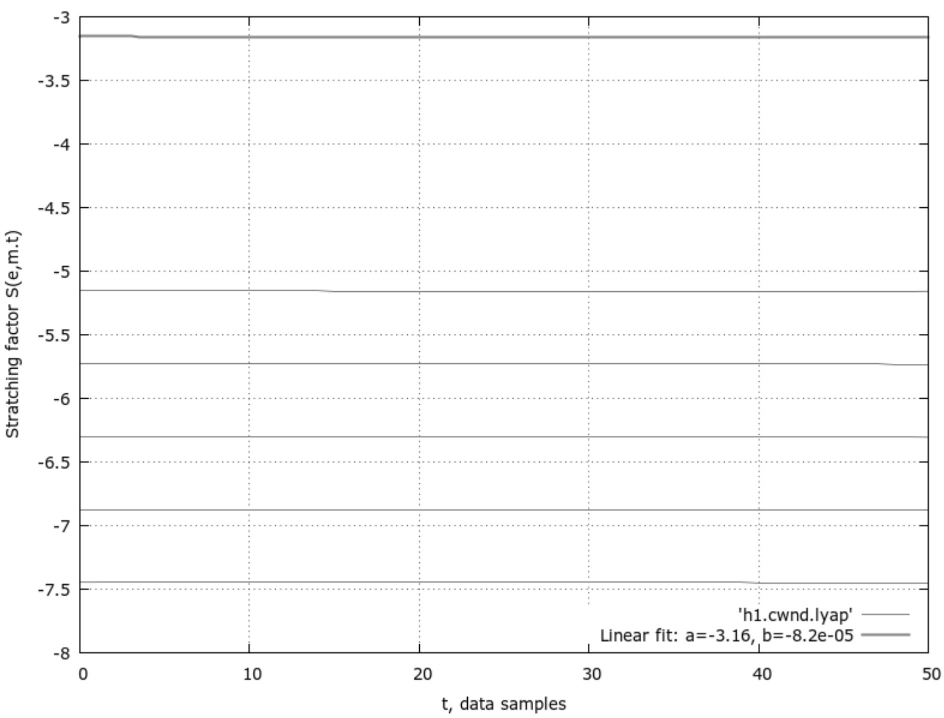


Fig. 7. The Lyapunov exponent calculation C_f =2Mbps, d_b =10ms, C_b =5Mbps, Q_s =100: λ ~ -0,00008 – absence of chaos

A number of numerical experiments have been conducted in order to analyze all the possible states of the model network under various parameters. The data obtained from NS-3 simulator have been processed by TISEAN package [23] for calculating maximum Lyapunov exponent [14]. The graph can be plotted by means of the data obtained, which demonstrates possible dynamic operation modes of the system. Thus one is able to define the parameters of the computer network affect the occurrence of chaotic phenomena and the parameters don’t. The router buffer size, the data generation rate senders and maximum Lypunov exponent as an indicator of chaotic processes in the system have been chosen as the coordinate axis. Such a choice of the axis allows to plot the surface which the peaks and the hills on it to be considered as occurrence of chaos (MLE>0) and the cavities and the hollows (MLE<0) – absence of chaos. The grid with the chosen step has been applied to the graph and therefore the values in the grid nodes provide information about the current state of the computer network. At the same time a few parameters hasn’t been changed in the process of calculation: the delay (10ms) and the recipient bandwidth (5MB/s). When one adds a new sender into the simulated network, then the analyzed phase space dimensionality will also increase by one and to analyze obtained data will be more complicated, not to mention the fact that the phase space visualization is possible, only if its dimensionality is smaller than 4. During carrying out of the calculation series the value of the data generation rate senders was changing in a range of 1 to 10 MB/s with the step 1 MB/s and the router buffer size – from 2 to 100 packets with the step 10 packets. The simulation time for each experiment was 100 seconds.

The result of the conducted calculation series is the graph of maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size (Fig.8,9) for each TCP-connection of the model network, which obtained after modeling of the operation computer network by means of NS-3 and the further processing by the lyap_k utility from TISEAN package.

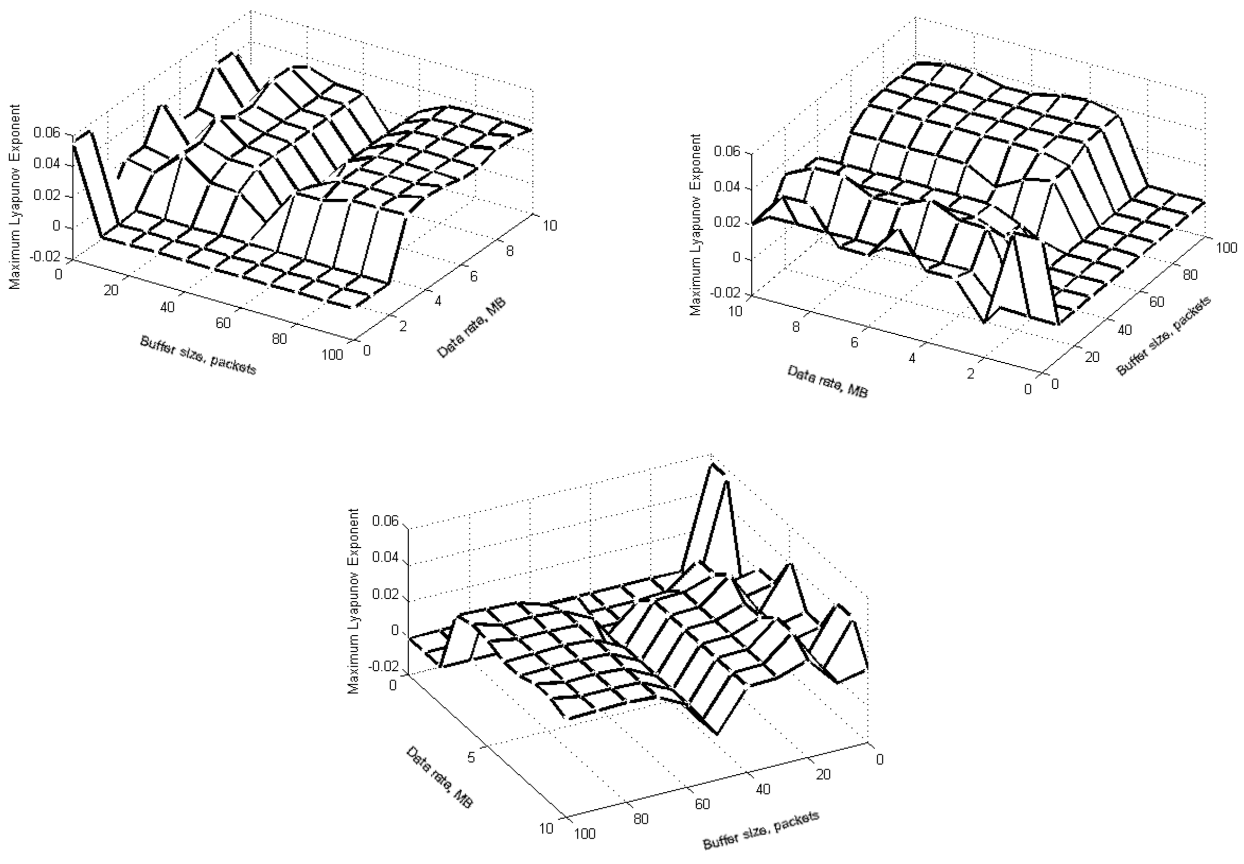
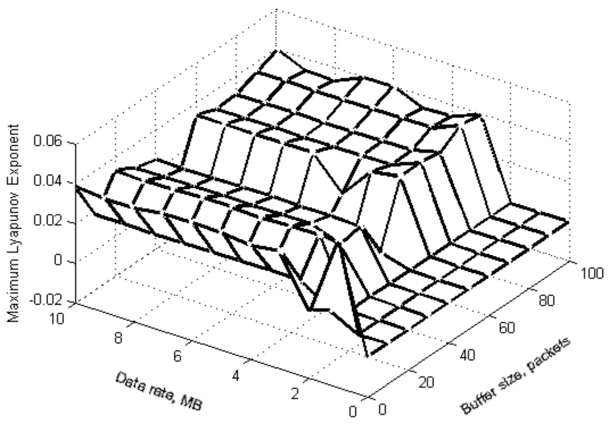
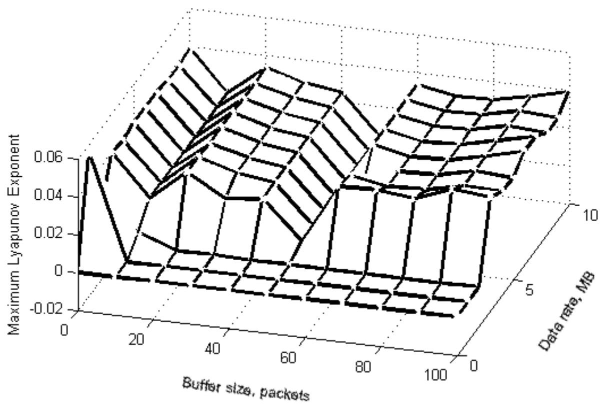


Fig. 8. Maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size for the 1st TCP connection under C_f =1-10Mbps, d_b =10ms, C_b =5Mbps, Q_s =2-100





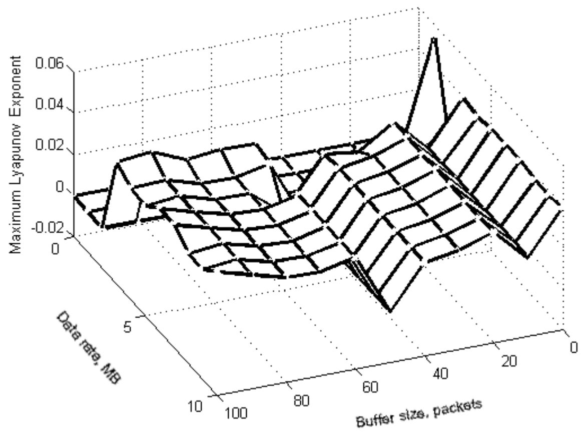


Fig. 9. Maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size for the 2nd TCP connection under C_f =1-10Mbps, d_b =10ms, C_b =5Mbps, Q_s =2-100

The considered system doesn't manifest any chaotic behaviour (MLE<0) in case the host-senders transmits data with a rate not exceeding the recipient bandwidth and overfilling the router buffer. At the same time MLE>0 corresponds the congestion state of the network. Consequently the router buffer size and the pipe bandwidth are the key parameters affect the congestion state in the considered network. Hence the behaviour of such systems can be predicted with a percent of probability having such a set of the instruments for analysis of dynamic systems.

CONCLUSIONS

Even such a simple system consisted of two competitive TCP flows shows very complex chaotic behaviour under certain conditions. Obviously, to solve the problem of congestions and the loss of packets is not possible on a global scale of the whole Internet network as one can't reorganize the entire network in virtue of technical and economical reasons. However it is possible to give some recommendations on design and further operation of the networks limited in scale (even large enough) that will allow negative phenomena of chaotization to be minimized. The advent of congestion in networks especially strongly affects its working efficiency in some areas of science and technology [10,15,30].

The diagrams plotted in the Fig. 8,9 allow estimating the values of the parameters of infocommunication networks in terms of availability (or absence) undesirable chaotic phenomena. By proper choosing of these parameters it is possible to make sure stable operation of infocommunication networks in the modes without chaos, which accords with non-occurrence of congestions.

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NUMERICAL ANALYSIS OF THE PROBLEM OF FLOW PAST A CYLINDRICAL BODY APPLYING THE R-FUNCTIONS METHOD AND THE GALERKIN METHOD

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Abstract. The article considers the stationary problem of viscous incompressible fluid flow past a cylindrical body. For solving the problem it is proposed a numerical method, based on the joint use of R-functions method and the Galerkin method. The computational experiment has been conducted for the task of flow past square cylinder for different Reynolds numbers.

Key words: viscous incompressible fluid, flow problem, the R-functions method, the Galerkin method.

INTRODUCTION

Recently, mathematical modeling and numerical analysis are increasingly used to study the dynamics of a viscous fluid. The necessity to simulate viscous flows occurs, for example, in fluid dynamics, thermal power, chemical kinetics, biomedicine, radio electronics, etc. [2-4, 11, 20].

The Navier-Stokes equations [12, 16, 17], describing such problems, have a number of specific features, such as nonlinearity and the presence of a small parameter at the highest derivative (the value is inverse to the Reynolds number). Furthermore, they often have to be solved in the areas of complicated geometry. When solving the exterior problems, the area under consideration may also be unlimited, but in the numerical solution it is modeled as a finite domain. It complicates the implementation of boundary conditions at infinity. Moreover, conditions at infinity are being demolished on certain contour, located far enough away from the streamlined body, which leads to additional errors in the approximate solution.

There is an extensive class of flows in which the nonlinear terms can be neglected to obtain the linear problem. The complete neglect of the inertial terms leads to the so-called equations of creeping flows or Stokes equations [5,11,13]. But there is no solution of the Stokes equations (Stokes paradox [5,11,25,26]) for the problem of unbounded viscous incompressible fluid flow past a cylindrical body. In this case, the Oseen approximation should be used [1,11,25].

When solving the hydrodynamics problems, the adequate consideration of the area geometry is important. It is implemented in a variety of computational methods with different degrees of effectiveness. The constructive apparatus of the R-functions theory of V.L. Rvachev [9,22-24], Ukraine National Academy of Science academician, allows to take into account the geometry of the area accurately and satisfy the boundary conditions of the problem precisely. The R-functions method in hydrodynamics problems was used by Kolosova S.V., Suvorova I.G., Maksimenko-Sheiko K.V., Sidorov M.V., but the problems of calculating the ideal fluid flows [6] and the viscous fluid flows in limited areas [8,28-30] or in the presence of helical symmetry [18] were considered.

The problems of external viscous fluid flows around bodies using the R-functions method were discussed in [7,13-15]. In [13] the problem of calculating the external slow viscous incompressible fluid flows past bodies in spherical and cylindrical coordinate systems was studied. For solving the nonlinear stationary problem of the viscous incompressible fluid flow past the cylindrical body in a cylindrical coordinate system, the paper [14] proposed a numerical method, based on the joint use of the R-functions method, the successive approximation method and the Galerkin-Petrov method. In [7] the application of the R-functions method, the successive approximation and the Galerkin-Petrov methods to the calculation of axisymmetric viscous incompressible fluid flows (the flowing around the finite bodies of rotation) was considered. In [15] the problem of mass transfer of the body of revolution with uniform translational flow was considered.

The purpose of this research is to apply the R-functions method and the Galerkin method for mathematical modeling of the linear and nonlinear stationary problem of viscous incompressible fluid flow past the cylindrical body in a rectangular coordinate system.

PROBLEM STATEMENT

Problem 1. Let us consider the problem of slow uniform viscous incompressible fluid flow with velocity U_∞ past a cylindrical body, the cross-section of which is a finite region Ω with piecewise continuous boundary $\partial\Omega$ [1,11,19]:

$$\Delta^2\psi + \mathrm{Re}\cdot A(\Delta\psi) = 0 \text{ outside } \overline{\Omega}, \quad (1)$$

$$\psi\big|_{\partial\Omega} = 0, \ \left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0, \quad (2)$$

$$\psi \sim U_\infty y \text{ as } \sqrt{x^2+y^2} \to \infty, \quad (3)$$

where:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \qquad \Delta^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4},$$

$A(\Delta\psi) = -\dfrac{\partial\Delta\psi}{\partial x}$, Re is the Reynolds number, $\psi = \psi(x,y)$ is the stream function, associated with the velocity vector components by means of these relations:

$$v_x = \frac{\partial\psi}{\partial y}, \ v_y = -\frac{\partial\psi}{\partial x},$$

$\mathbf{n}$ is external normal to the $\partial\Omega$.

Problem 2. Consider the nonlinear stationary problem of viscous incompressible fluid flow past a cylindrical body [21]. In this case, the stream function satisfies the equation:

$$\Delta^2\psi = \mathrm{Re}\left(\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}\right) \text{ outside } \overline{\Omega}. \quad (4)$$

Eq. (4) is supplemented by the boundary conditions (2) and the condition at infinity (3).

NUMERICAL METHOD

The R-functions method [9,22-24] of V.L. Rvachev, Ukraine National Academy of Science academician, is proposed for solving the problem 1 and 2.

Let outside of $\overline{\Omega}$ a sufficiently smooth function $\omega(x,y)$, that satisfies the following properties, is known:

$$\begin{gathered} 1)\ \omega(x,y) > 0 \text{ outside } \overline{\Omega}, \\ 2)\ \omega(x,y)\big|_{\partial\Omega} = 0, \\ 3)\ \left.\frac{\partial\omega}{\partial\mathbf{n}}\right|_{\partial\Omega} = -1, \end{gathered} \quad (5)$$

where: $\mathbf{n}$ is external normal to the $\partial\Omega$.

Let us introduce a sufficiently smooth function $y = f_M(x)$ from [27], which satisfies the following requirements:

$$\begin{gathered} \text{a) } f_M(0) = 0, \quad \text{b) } f_M'(0) = 1, \\ \text{c) } f_M'(0) \ge 0 \ \forall x \ge 0, \\ \text{d) } f_M(x) \equiv 1 \ \forall x \ge M \ (M = const > 0). \end{gathered} \quad (6)$$

The conditions (7) are satisfied, for example, by means of the function [27]:

$$f_M(x) = \begin{cases} 1 - \exp\dfrac{Mx}{x-M}, & 0 \le x < M; \\ 1, & x \ge M. \end{cases} \quad (7)$$

Obviously, $f_M(x) \in C^\infty[0,\infty)$.

Let us denote $\omega_M(x,y) = f_M[\omega(x,y)]$.

It is easy to verify that the function $\omega_M(x,y)$ satisfies the conditions 1) – 3) of (5). Besides, $\omega_M(x,y) \equiv 1$, if $\omega(x,y) \geq M$.

Note that this condition means: if the function $\omega(x,y)$ is increasing monotonically while removing from the contour $\partial\Omega$, then the function $\omega_M(x,y)$ is different from unity only in some finite annular region $\{0 \leq \omega(x,y) < M\}$, that is lying outside $\bar{\Omega}$ and is adjacent to the $\partial\Omega$.

The following theorem has been proved.

Theorem. For any choice of sufficiently smooth functions Φ_1 and Φ_2

$$\left(\frac{\Phi_1}{\sqrt{x^2+y^2}} \to 0 \text{ as } \sqrt{x^2+y^2} \to +\infty\right)$$

the function

$$\psi = \omega_M^2(\psi_0 + \Phi_1) + \omega_M^2(1-\omega_M)\Phi_2,$$

exactly satisfies the boundary conditions (2) and the condition at infinity (3). Here $\psi_0 = U_\infty y\left(1 - \frac{R^2}{x^2+y^2}\right)$ is the solution of the ideal fluid flow past circular cylinder of radius R (the cylinder of radius R entirely lies inside the streamlined body), $\omega_M = f_M(\omega)$, $f_M(\omega)$ has the form of (7), and ω is function with the properties of (5).

Problem 1. For approximating the indefinite components Φ_1 and Φ_2 it is proposed to use the Galerkin method [10]. The functions Φ_1 and Φ_2 will be presented as follows:

$$\Phi_1 \approx \Phi_1^{m_1} = \sum_{k=1}^{m_1} \alpha_k \cdot \varphi_k, \qquad \Phi_2 \approx \Phi_2^{m_2} = \sum_{j=1}^{m_2} \beta_j \cdot \tau_j, \tag{8}$$

where: $\{\varphi_k(\rho,\varphi)\} = \left\{\rho^{2-k} \begin{matrix}\cos k\varphi \\ \sin k\varphi\end{matrix}, k = 3,4,...; \right.$ $\left. \rho^{-k} \begin{matrix}\cos k\varphi \\ \sin k\varphi\end{matrix}, k = 1,2,... \right\}$ is the complete system of partial solutions of the equation $\Delta^2\psi = 0$ relative to the exterior of the cylinder of finite radius,

$$\{\tau_j(\rho,\varphi)\} = \left\{\cos 2\varphi, \sin 2\varphi, \rho^{j+2} \begin{matrix}\cos j\varphi \\ \sin j\varphi\end{matrix}, \right.$$

$\left. \rho^{j} \begin{matrix}\cos j\varphi \\ \sin j\varphi\end{matrix}, j = 1,2,... \right\}$ is the complete system of partial solutions of the equation $\Delta^2\psi = 0$ for the region $\{\omega(x,y) < M\}$. Here $\rho = \sqrt{x^2+y^2}$ and φ is defined by the relations:

$$\cos\varphi = \frac{x}{\sqrt{x^2+y^2}}, \ \sin\varphi = \frac{y}{\sqrt{x^2+y^2}}.$$

Thus, the approximate solution of the problem (1) – (3) is sought in the following form:

$$\psi \approx \psi_N = \omega_M^2(\psi_0 + \Phi_1^{m_1}) + \omega_M^2(1-\omega_M)\Phi_2^{m_2}. \tag{9}$$

Let us determine the complete sequence of functions relatively to the whole plain:

$$\{f_i(\rho,\varphi)\} = \left\{\omega_M^2(\rho,\varphi)\rho^{2-k} \begin{matrix}\cos k\varphi \\ \sin k\varphi\end{matrix}, k = 3,4,...; \right.$$
$$\omega_M^2(\rho,\varphi)\rho^{-k} \begin{matrix}\cos k\varphi \\ \sin k\varphi\end{matrix}, k = 1,2,...;$$
$$\omega_M^2(\rho,\varphi) \begin{matrix}\cos 2\varphi \\ \sin 2\varphi\end{matrix}, \omega_M^2(\rho,\varphi)\rho^{j+2} \begin{matrix}\cos j\varphi \\ \sin j\varphi\end{matrix},$$
$$\left. \omega_M^2(\rho,\varphi)\rho^{j} \begin{matrix}\cos j\varphi \\ \sin j\varphi\end{matrix}, j = 1,2,... \right\}. \tag{10}$$

The values of the coefficients α_k $(k = 1,2,...,m_1)$ and β_j $(j = 1,2,...,m_2)$ in accordance with the Galerkin method will be found from the condition of the residual $R_N = \Delta^2\psi_N + \mathrm{Re}\cdot A(\Delta\psi_N)$ orthogonality to the first N $(N = m_1 + m_2)$ elements of the sequence (10):

$$(R_N, f_i) = 0, \ i = 1,2,...,N. \tag{11}$$

Besides, by the properties of ω_M and coordinate functions, the integration in (11) can be done only over a finite region $\{0 \leq \omega(x,y) < M\}$ when calculating the scalar products.

Problem 2. For solving the task (4), (2), (3) we propose to use a nonlinear Galerkin method. Approximate solution of problem (4), (2), (3) will be sought in the form (9), where Φ_1 and Φ_2 have the form (8). The values of the coefficients α_k and β_j will be found from the condition of the residual

Q_N orthogonality to the elements $f_1, \ldots, f_N$ of the sequence (10):

$$(Q_N,\ f_i) = 0,\ i = \overline{1,\ N},\ N = m_1 + m_2, \quad (12)$$

where:

$$Q_N = \Delta^2\psi_N - \mathrm{Re}\left(\frac{\partial\psi_N}{\partial y}\frac{\partial\Delta\psi_N}{\partial x} - \frac{\partial\psi_N}{\partial x}\frac{\partial\Delta\psi_N}{\partial y}\right).$$

Besides, by the properties of ω_M and coordinate functions, the integration in (12) can be done only over a finite region $\{0 \le \omega(x, y) < M\}$ when calculating the scalar products.

As a result, we obtain a system of nonlinear equations, each of which is a quadratic function with respect to α_k and β_j. The resulting system can be solved by Newton's method. As an initial approximation, the set of α_k and β_j, corresponding to the solution of the Oseen problem or, at high Reynolds numbers, to the solution obtained at lower Reynolds numbers, can be chosen.

RESULTS

The computational experiment has been conducted for the problem of flow past the cylindrical body $x^8 + y^8 = 1$ at $U_\infty = 1$ and Reynolds numbers $\mathrm{Re} = 0,01;\ 5;\ 10;\ 15;\ 20$.

Fig. 1 – 5 shows the streamline contours of the obtained approximate solution for $M = 10$, $m_1 = 48$, $m_2 = 35$.

Detailed pictures of the streamline contours and vector velocity fields are shown in Fig. 6 – 10.

As can be seen from the figures, at low Reynolds numbers the flow is symmetric, has no separation zone in the aft area of the body. As Reynolds number is increased, the flow character is changed: the secondary vortices occur behind the body, their size and intensity grows, what coincides with physical experiments.

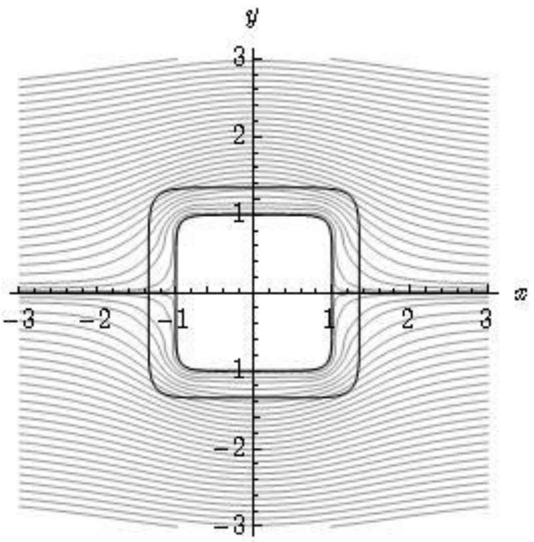

Fig. 1. The streamline contours at $\mathrm{Re} = 0,01$

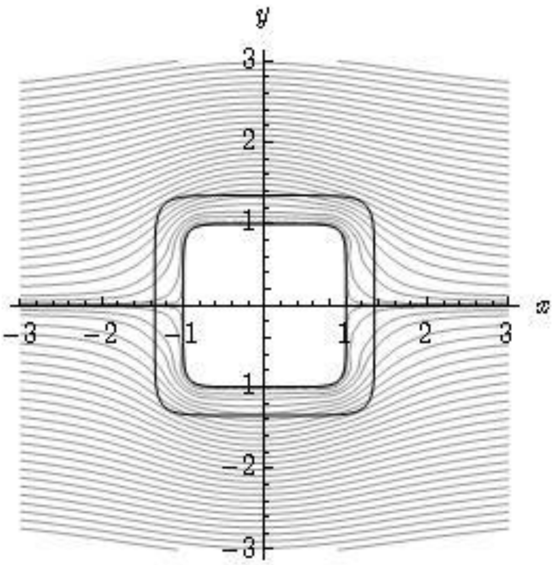

Fig. 2. The streamline contours at $\mathrm{Re} = 5$

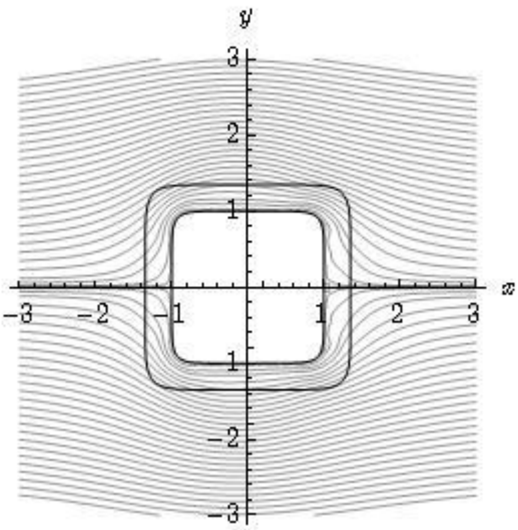

Fig. 3. The streamline contours at $\mathrm{Re} = 10$

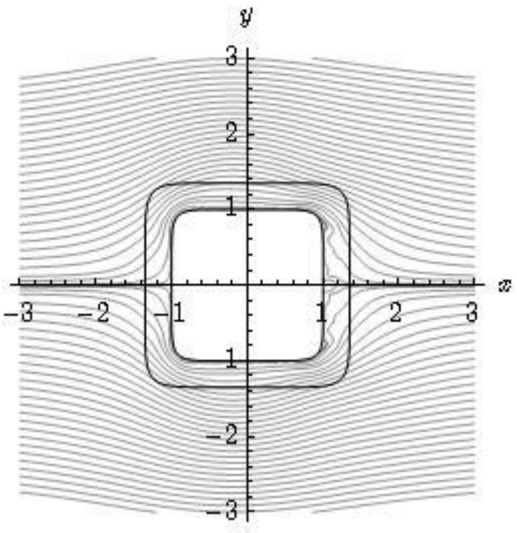

Fig. 4. The streamline contours at $\mathrm{Re} = 15$

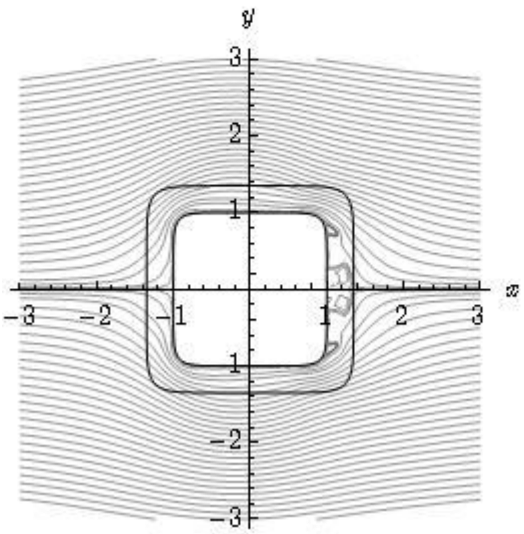

Fig. 5. The streamline contours at $\mathrm{Re} = 20$

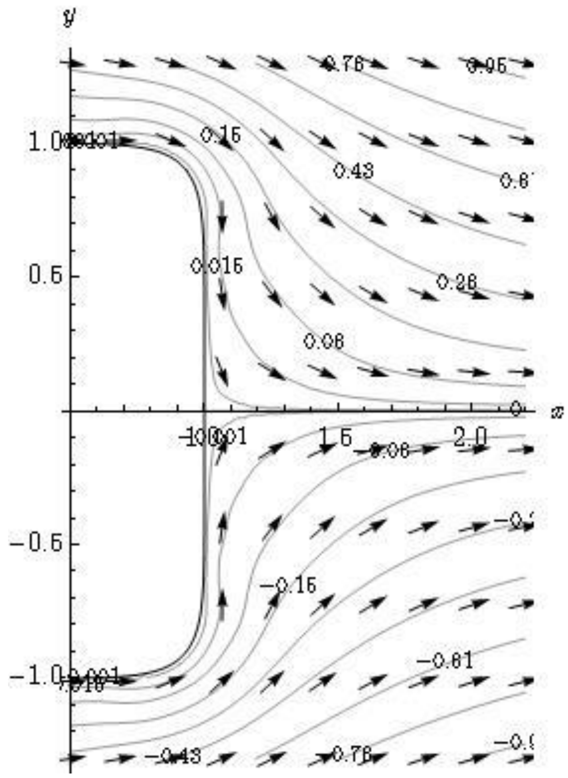

Fig. 6. Detailed pictures of the streamline contours and velocity vector fields at $\mathrm{Re} = 0,01$

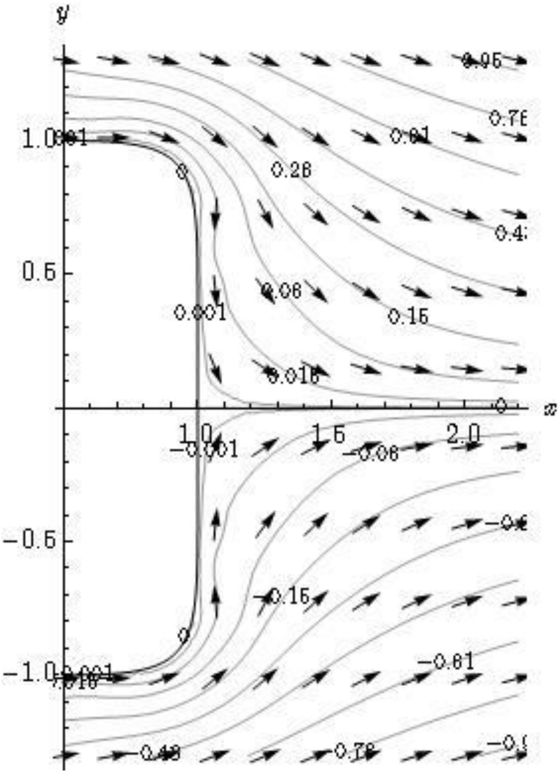

Fig. 7. Detailed pictures of the streamline contours and velocity vector fields at $\mathrm{Re} = 5$

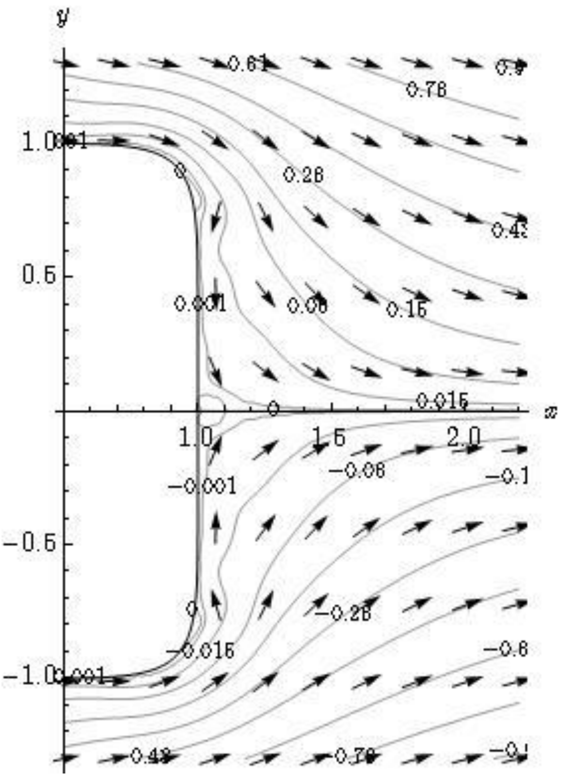

Fig. 8. Detailed pictures of the streamline contours and velocity vector fields at $\mathrm{Re} = 10$

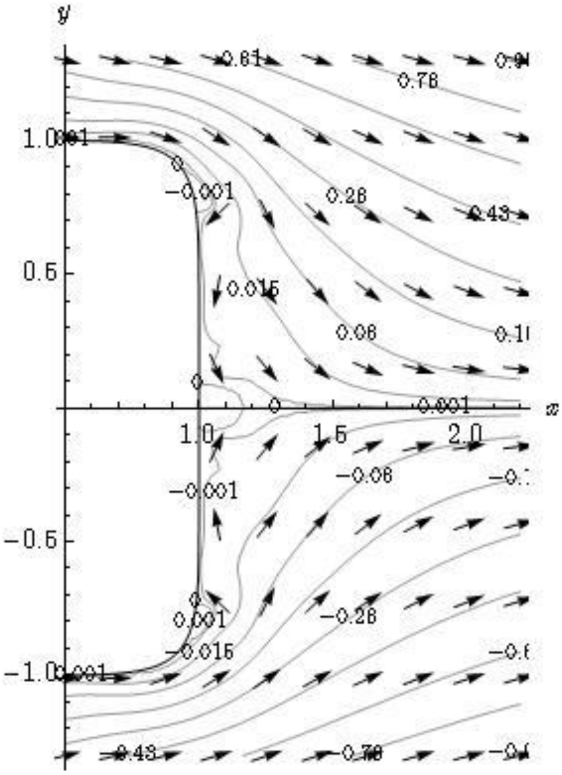

Fig. 9. Detailed pictures of the streamline contours and velocity vector fields at $\mathrm{Re} = 15$

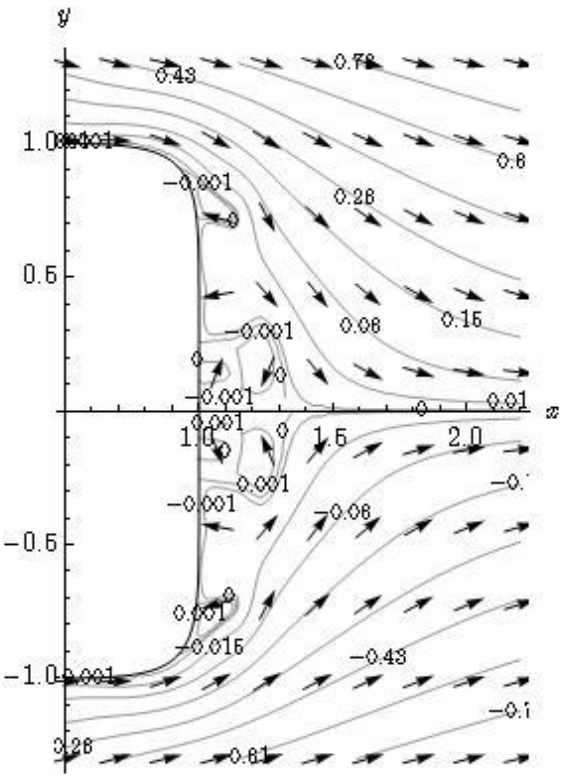

Fig. 10. Detailed pictures of the streamline contours and velocity vector fields at $\mathrm{Re} = 20$

CONCLUSIONS

The method for calculating the external flow of a viscous incompressible fluid, based on the joint use of the R-functions structural method and the Galerkin projection method, which differs from the known methods of universality (the algorithm does not change with changes in the geometry of the field) and the fact that the structure of the solution exactly takes into account the boundary conditions at the boundary of the body and the condition at infinity, has been proposed. For different Reynolds numbers the stationary problem of viscous incompressible fluid flow past the cylindrical body in a rectangular coordinate system has been solved numerically. The method developed allows to conduct the mathematical modeling of various biological, physical and mechanical flows.

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ABOUT ONE METHOD OF MATHEMATICAL MODELLING OF HUMAN VISION FUNCTIONS

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Abstract. The comparison method, widely used in the colorimetric experiments, is applied to simulation of human vision functions. To solve such class of problems the mathematical model is obtained with the help of special type predicates.

Key words: visual system, colorimetric experiment, null-organ method, mathematical model of the vision organ.

INTRODUCTION

The visual system of a man is one of the most perfect systems created by evolution in the process of a long phylogenesis. Visual perception is a complex neurophysiological process formed under the action of photic stimuli on the retina photoreceptors. The spectral sensitivity of an eye coincides with the solar spectrum distribution curve maximum. The maximal sensitivity of an eye at light is observed to more long wavelength beams (the area of the green-orange part of the spectrum), the maximal sensitivity of an eye at half-lights is observed to more short-wave beams (the area of the blue-violet part of the spectrum). In the genegal understanding the form of an eye resembles an incorrect sphere because of some prolated fore-part, fig. 1.

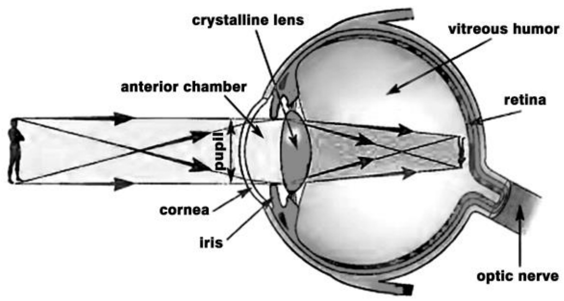

Fig. 1. Human eye

There is the pupil inside this sphere directly in the iris. As it is, as a matter of fact, a hole, it seems black, as behind it there is a dark interior of the eye. The iris has a form of the disk with a hole (pupil) at the centre. The iris of each man is pigmented in a particular colour: shades of grey or brown. The crystalline lens is located behind the iris. It has a fotm of a convexo-convex lens. The crystalline lens actively participates in accommodating the eye to the external conditions. The outside shell of the eye consists of sclera (protein) and cornea. Sclera envelops the whole eyeball and is a peculiar housing, fulfilling the function of protection and providing persistence of the form of the eye. The spherical form of a human eyeball reminds by its functional capabilities the camera with a field of view of 160^{o} across the width and 135^{o} throughout the height. The front convex and transparent part of the outside shell is named a a cornea, which represents an objective lens. Between the iridescent shell and cornea there is “ a chamber fluid”, which is the lens, as well as the crystalline lens. The back internal surface of the eye is"lined” with a retina, which is formed of millions of photosensitive cells. The retina is a receiver of light pulses, due to its functioning we can see this or that object

THE HUMAN EYE FUNCTIONS

At bright rays the iris is expanding, and the pupil is narrowing**.** In the dark everything happens vice versa. Having passed through the pupil, the rays are refracted by the crystalline lens, the the crystalline lens form can vary depending on the distance between a subject and the human eye. If the subject is located close to the eye, the crystalline lens becomes thicker, and if the subject is located far then the crystalline lens becomes thinner. Then the light stikes the retina, where the photosensitive cells convert it to nervous impulse through the composite chemical processes. This impulse is transmitted by the optica nerve in that part of the brain which is responsible for vision, where it is processed, then a visual pattern of the considered subject is reconstructed. A part of nerve fibers of the optic nerve catches red light, another one catches green light and the third optic nerve catches blue light. The brain processes all information, and as a result the man sees colour.

According to anatomy the eyeball in the section on the vertical plane of symmetry looks [9] like as it is shown in fig. 2

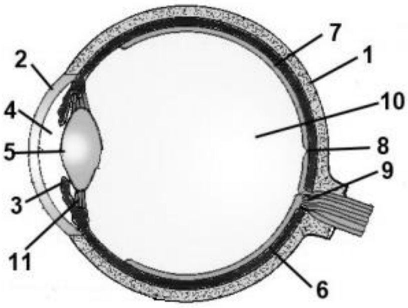

Fig. 2 Structure of the human eye

1 – is the sclera –the dense white color shell of the eye, which protects it against damages; 2 – is the cornea – the front, transparent, strongly curved window, refracting light. Light through the cornea strikes the eye before it is partially overlapped by the color and opaque surfaces of the iris; 3 – is the iridescent shell (or iris), a thin opaque shell in the form of the disk with a hole in the center (pupil), which contains colorants (add color to eyes) and muscle making it possible to change a size of the pupil; 4 – is the pupil, a hole in the center of the iridescent shell, which diameter varies from 1 up to 8 mms, depending on lighting. At poor lighting the pupil is expending to increase energy, which strikes the retina. At normal lighting the pupil is narrowing to resist spherical aberrations in the eyeball and to receive sharper image. The refracting force (analogue of the focal length) of the eye generally depends on refraction at the interface and is will regulated by the change of the crystalline lens form, and also holds the object strictly in focal point. An adult normal person has a focal length variation in the limits of 15 – 17mm; 5 – is the crystalline lens, a biological lens, which is located behind the pupil (iris) and performs an important role of light refraction and accommodation; 6 – is the choroid shell, which enriches the retina with nutrient materials; 7 – is the retina, highly differentiated nervous tissue with photoreceptors perceiving adequate light rays with a wavelength from 380 - 770 nm, i.e. from violet up to red. There are approximately one hundred twenty million rods and seven million cones in the human eye. The rods are sensitive photoreceptors, which are capable to react to one photon, though they give not enough information on a spatial arrangement of the object, as they are bound to the same neuron on the retina. The cones become active at more high level of illumination and the signal, given by every cone, is decrypted by several neurons, that has an effect of a high-resolution in this area; 8 – is the yellow spot (or macula), the point of the highest visual acuity on the retina, which is located opposite to the pupil (there are only cones in the macula). 9 – is the area of the disk of a visual nerve, which is a “blind zone” of the fundus of the eye, as it contains not photoreceptors but the nervous fibers, which depart from the cells of the retina. This is the point of the visual nerve exit from the eyeball; 10 – is the vitreous humor, which fills 2/3 of the eye sizes and provides shape and density for it; 11 – is the cylindrical body, which is, on the one hand, the gland for ultrafiltration of the intraocular fluid, on the other hand it is an accommodation muscle providing conditions for near and far clear vision.

Perception of the color spectrum diversity of the surrounding world is realized by the cones – the retina cells. Three types of color-perceiving elements (photosensitive colorants) are included in them, each of them perceives only one of three primary colors - red, green or violet. All remaining colors and shades can be obtained using different versions of these colors mixing**.** In the process of color perception it happens due to that the visible part of the spectrum of luminous radiation includes waves of different length. The long-wave radiations (552-557 nm) act on the red color- perceiving element, MF band radiation (530 nm) acts on green color- perceiving element, short-wave radiation (426 nm) acts on violet color- perceiving element. (Fig.3).

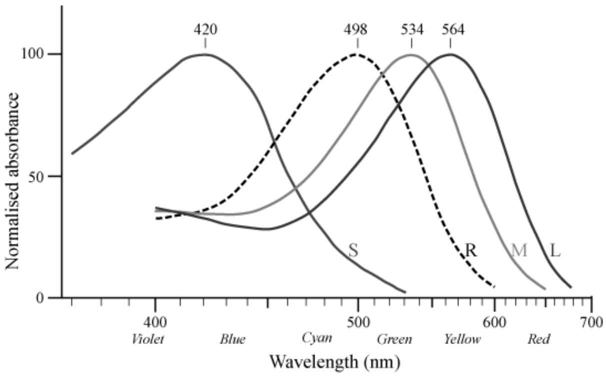

Fig. 3 - Sensitivity of three types of cones and rods (dashed line) to radiation of different wavelength

The pure colors shades differ depending on the intensity of the action: at long wavelength action the vary from the purple color up to the orange one, at MF band action - from the emerald color up to the yellow one, at short-wave action - from the blue color up to the violet one. The luminous flux, containing radiations from different length waves, causes different excitation of all three color perceiving units unequal on intensity, due to that a full color image is shaped in the visual centers of the cerebral cortex**.** The cerebral cortex synthesizes these excitations in uniform resulting color of the subject according to the laws of optical colors mixing, and the analysis and synthesis of the color perceiving happen permanently and simultaneously. The color perceiving is a function of the retina cones, a papilla-macular bundle of the optic nerve and cork vision centers of the cerebrum.

Relevance. At present a great attention is given to the problems, bound with learning psyxh-physical phenomenae, thus the object of research are as follows: human sensations; physical processes actiong on our organa sensoria and causing sensations; ratio, which link sensations to subjects of the external world, appropriate to them**.** The area of a science investigating conversions of the information by the organa sensoria is called psychophysics. Psychophysics has numerous practical and technical applications. The cybernetics, systems engineering, computer technology, automation, light engineering, engineering of cinema and television and many other areas of practical activity of a man rests on the outcomes of its researches**.** The mathematical description of sensations of a man sets before the researchers the task on development of the mathematical apparatus**.** The vision psychophysics classical task consists in learning the link between light radiation, i.e. visual patterns and characteristics of visual imoges (saturation, colour tone etc.) The main tool of the colorimetry, the science about colour measurements, is the colours matching method [10]**.**

SETTING OF THE RESEARCH PROBLEM

It is offered to utillize a special case of the method of matching, namely, the null - organ method, where the mathematical apparatus of predicates of a special sort is used, and the functional space will be utillized as entry spectrums of light radiations. According to this method [1, 5, 8], two small fields having common boundary, fig. 4, light radiations described by spectrums $b'(\lambda)$ and $b''(\lambda)$, respectively, are shown to an observer. The observer perceives these radiations as two adjoining colour spots.

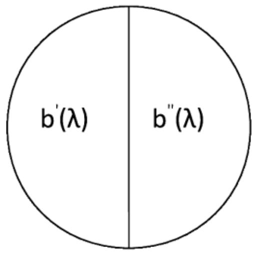

Fig. 4 - Light radiations described by spectrums $b'(\lambda)$ and $b''(\lambda)$ presented to an observer on two small fields having a common boundary

It is required from him to give an answer to the problem, whether the colours of the fields of matching coincide with each other or not. The answer formation is greatly facilitated bythe fact that the boundary between colour spots fades in case of the colours coincidence. Thus, the observer, as a matter of fact, makes a decision on coincidence or distinction of colours with the help of a very thin indicator - absence or presence of a visible boundary between fields of matching. Such fact testifies to a sharp response of the method of matching. If a pair of identical radiations is given on fields of matching $(b'(\lambda), b''(\lambda))$, the observer will register an equality of colours. But if a pair of radiations $(b(\lambda), 1,01\cdot b(\lambda))$ is presented, i.e. if we increase the energy level of radiation only by 1 % without change of a spectral distribution of light on the right-hand field, the observer with a normal vision will fix the colours difference clearly. It is found out, that using the method of matching it is possible to distinguish many millions of light radiations by colour. At first sight it may seem, that the colours are bound one-to-one to the light radiations generating them, and, consequently. the observer registering equality or inequality of colours thus finds out coincidence or difference of the appropriate light radiations.

However, there is a set of light radiations (completely different in the spectrum and in power, they are called metameric ones), which are purely not distinguishable in colour for eyes. It follows, that there are much less different colours, than different light radiations. The organ of vision reacts with the same colour to huge number of different light radiations. Thus, an eye, forming a colour, thus groups light radiations in some classes. Moreover, it is found out, that different observers classify light radiations not absolutely equally. Therefore light radiations, seen by one observer as monochrome ones, will look, as a rule, unequal in colour for other observer. It follows from these facts, that each man represents a special object for colorimetric inspection. Moreover, it appears, that the same observer can react differently in colorimetric experiences in different periods of the life. It means, that the parameters of the visual system of a man vary, evolve with time**.** Despite of these circumstances and that in colorimetric experiences it is necessary to deal with subjective sensations of the observer and with his subjectively formed solution about equality or inequality of colours, nevertheless these experiences are quite objective and can be classified as clearly objective physical experiments. The outcome of the colorimetric experiments does not depend at all on the obsrver's desire. Though the observer can arbitrary invent the answer or make an error when choosing an exact answer (for example at diverting attention during matching colours), but the researcher has all possibilities to detect such answers and to reject them, just as during processing outcomes of physical experiments it is possible to reveal and to eliminate gross errors of the experimenter. The observer in the colorimetric experiment operates quite machinelike: the repeated presentation reduces the same couple of light radiations results in the answer (if, certainly, to not expand carrying out of the experiment for many years, when the observer becomes the other). But in the special cases, namely, when the colours are on the boundary between equality and inequality, the element of chance in the answer is observed. But the same element of chance occurs in any physical experiment when it is necessary to work on a limit of possibilities of measuring instruments. In these cases the accuracy of the outcomes of physical experiments usually are increased at the expense of multiple repetitions of the same tests with the subsequent statistical processing of the outcomes of experiments. The same statistical processing of the subjects' answers is possible in colorimetric experimentse as well**.** The accuracy, achievable in the colorimetric experiments, is 2 - 3 signs, and at deep statistical processing four signs can be reached. Such an accuracy is at the level of the rather perfect physical experiment accuracy. It follows from the above From all said the output is those: in colorimetric experiments we have that, essentially amazing, case, when the subjective sensations of a man and his subjective actions, when matching the colours, are successfully researched with quite objective physical methods**.** In other words, the colorimetric experiments demonstrate a key possibility of objective learning of the subjective states of a man, give a specific precedent of such learning. This conclusion is a great responsibility, as it is possible to extract a number of far-reaching outputs from it**.** Really, if it so, then there is no insurmoumtable gapbetween the objective physical world and subjective world of a man. Thus, the concepts expressed by the words "objective" and "subjective" logically do not eliminate each other, and the second is absorbed by the first**.** It means also, that the subjective states can be studied quite objectively with physical methods only. In connection with such radical conclusions, the thesis about the possibility of successful objective learning of some subjective states of a man with the colorimetric experiments, on which these conclusions are grounded, should be subjected to a careful check.

The purpose of the work is to build a mathematical model of the organ of vision of a man on the basis of the matching method, which is described with the help of predicates, as well as executions of the predicates properties and numerical implementation of the tasks with the given method using a PC.

MATERIAL AND OUTCOMES OF RESEARCHES

Objectively registereg behaviour of a man is rseally studied in colorimetric experiements**.** An observer acts in them as a certain "black box" with two inputs and one outpu, fig. 5. The light radiations, characterized by the spectrums $b'(\lambda)$ and $b''(\lambda)$, arrive to the input of the "black box".

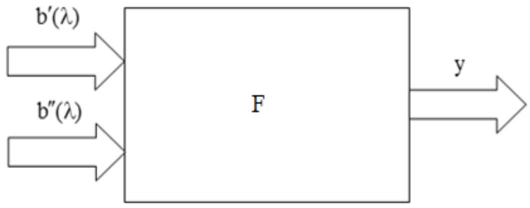

Fig. 5 - Man's vision organ as black box

From the mathematical point of view these spectrums represent some functions of the real argument λ, set at the interval $[\lambda_1, \lambda_2]$, with material values $b'(\lambda)$ and $b''(\lambda)$. The binary signal $y \in \{0,1\}$ is formed at the output of the "black box". Its value 1 will be interpreted as the answer "yes" of the observer, which means equality of colours on fields of matching, so the value 0 will mean inequality of colours. Thus, the observer implements some predicate by his behaviour**:**

$$y = F(b'(\lambda), b''(\lambda)), \qquad (1)$$

and the properties of this predicate are studied in the colorimetric experiments. Both the input signals $b'(\lambda)$ and $b''(\lambda)$, and output signal y can be registered by physical instruments and, consequently, give the quite objective information for establishment of the predicate F type. But still there is no place for subjective states of the observer in all this; not a single word is said about colours of visual sensations and about the operation of colours matching realized by consciousness of the observer.

But, using the subjective experience as a basis, we can state, that:

1) when the observer forms a signal $y = 1$, the colours of his sensations really are equal.

2) in this case the observer is really compares colours among themselves by some condition of his consciousness and comes to the conclusion about their equality.

Nevertheless, we can not be sure in the validity of these two statements by means of objective observations. How is it possible to deal with this opposition? It would lose force, if it was possible to us, outgoing only from is objective observable properties of predicate F, to prove somehow, that the signal converter presented in fig. 5, can be shown as the scheme in fig. 6.

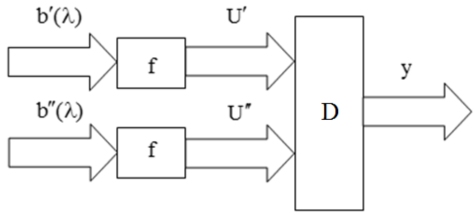

Fig. 6 - Block diagram of the man's vision organ

Here signals:

$$U' = (U_1', U_2', U_3') \text{ and } U'' = (U_1'', U_2'', U_3'') \qquad (2)$$

three-dimensional vectors with real components U_1', U_2', U_3' and U_1'', U_2'', U_3'', computed with the formulas:

$$U_1' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_1(\lambda)\, d\lambda,$$
$$U_2' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_2(\lambda)\, d\lambda, \qquad (3)$$
$$U_3' = \int_{\lambda_1}^{\lambda_2} b'(\lambda) K_3(\lambda)\, d\lambda,$$

and

$$U_1'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_1(\lambda)\, d\lambda,$$
$$U_2'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_2(\lambda)\, d\lambda, \qquad (4)$$
$$U_3'' = \int_{\lambda_1}^{\lambda_2} b''(\lambda) K_3(\lambda)\, d\lambda,$$

The formulas (3) and (4) mathematically describe the kind of the functions $U_1' = f(b'(\lambda))$ and $U_1'' = f(b''(\lambda))$. The character D marks predicate of equalities defined as follows:

$$D(U', U'') = \begin{cases} 1, if\ U' = U'' \\ 0, if\ U' \neq U'' \end{cases}, \qquad (5)$$

The presented predicate F is easily interpreted in the psychological terms. The signals U' and U'' can be understood as colours of the fields of matchings subjectively experienced by the observer. The function f is interpreted as a conversion of light radiation into a colour of visual sensation producible by the visual system of a man. Predicate D will be interpreted as the colours matching operation of the fields of matching realized by consciousness of the observer**.** If it would be possible to prove, that the predicate (1) can be represented as relations (2)-(5), it would give us the right to state, that:

1) the signals U' and U'' can be adopted as the mathematical description of colours on the fields of matching,

2) the function f can be adopted as the mathematical description of conversion of light radiation acting on the eye retina, in the colour of visual sensation originating in consciousness of the observer.

As a result the task of the logical substantiation with the objective methods of the mathematical model of colour vision (3)-(4), offered by Maxwell, would be completely solved. The described approach, however,also can be subjected criticism. The objection is that at this approach only the proof of the possibility to represent the signal convertion in the visual system as the block diagram shown in fig. 5 is taken into account. It is necessary to prove the necessity of such structure. According to this point of view it is necessary to prove, that the visual analyzer really has anatomic and physiological structures computing in the process of view the values of integrals (3)-(4), and that the colour of visual sensations actually is the triple of numerical codes materially introduced as some physical - chemical process**.** It is possible to reply to this objection as follows. No doubt, it would be very tempting to receive not only the functional, but also structural description of the visual analyzer. But derivation of mathematical dependences descibing only a way of the visual system functioning, it's also too much**.** Even in physics in most cases they restrict themselves to the functional (phenomenological) description of processes. Celestial mechanics, nuclear physics and many other important sections of physics go almost exclusively on this way. If we want to restrict ourselves to the functional part of the problem, then inevitabily it will have to content content only with possible mathematical models of the investigated processes. Thus, all possible identical formulas, distinguishing among themselves on structure, describing the same function should be considered equivalent. None of these formulas should be preferred at the functional approach, no matter how much they may differed from each other in their structure.

The choice of kinds of the investigating predicates is defined by practice and it is expedient to study converters most frequently met in the real life. One of such converters is the linear mapping

$$F: H \to R^n,$$

where: H - is Hilbert space of the input signals, R^n - is n-dimensional Euclidean space. The real systems, as a rule, as input signals have any functional dependencies and have properties of linearity. Thus, more often, the input signals themselves form a linear space, and the introduction on it of a scalar product makes it possible to describe linear functionals by easy way.

By virtue of the known Riesz theorem of the general view of the linear functional [7] each linear continuous functional $\alpha(x)$ on $L_2[\alpha, b]$ is set by the formula:

$$\alpha(x)=\int_{\alpha}^{b} g(t)x(t)dt,\ g(t)\in L_2\,, \qquad (6)$$

Hereinafter the following statement will be utilised.

Lemma 1. The linear continuous functionals $\alpha_i(x)\,(i=1,...,k)$ of the kind of (6) are linearly independent whenand only when the appropriate functions $g_i(x)$ are linearly independent.

The proof. If $\alpha_1 g_1(x)+...+\alpha_n g_n(x)$ - is a function equal to null almost everywhere on L_2, then by virtue of (6) $\alpha_1 f_1(x)+...+\alpha_n f_n(x)=0$**.** Vice versa, suppose, that $\alpha_1 f_1(x)+...+\alpha_n f_n(x)=0$**.** Then from (6) we obtained $\int_{\alpha}^{b}(\alpha_1 g_1(t)+...+\alpha_n g_n(t))\alpha(t)dt=0$ for all $x(t)\in L_2$**.** But it is known, that any function $y(x)\in L_2$, orthogonal to all functions from L_2, is equal to null almost everywhere**. The lemma is proved.**

Utillizing the obtained above auxiliary results let us study now special predicates $T(x,y)$. Let us introduce the predicate of equality:

$$D(x,y)=\begin{cases}0,\ if\ (x\neq y)\\ 1,\ if\ (x=y)\end{cases},\quad x,y\in L_2\,,\quad (7)$$

Let $A: L_2 \to H$ - be the continuous linear operator [2,4] mapping L_2 on a finite-dimensional vector space H above the field of real numbers. Main result of the present work is the axiomatic characterization of predicates $T(x,y)$ of the kind**:**

$$T(x,y) = D(A(x), A(y)), \qquad (8)$$

specified on the Cartesion square**.** First of all, let us note that if predicate $T(x,y)$ is presented as (8), it will also appear as:

$$T(x,y) = D(Px, Py), \qquad (9)$$

where P - is the operator of designing L_2 on some subspace H of the spaces L_2**.** Really, let S_1 - be the core of mapping $A: S_1 = \{x \in L_2, A(x) = 0\}$. As A - is the continuous operator, S_1 - is the closed subspace of the space L_2. According to the main theorem of homomorphic vector spaces the quotient space $L_2 | S_1$ is isomorphous to the space H**.** By virtue of S_1 closeness the orthogonal decomposition takes place:

$$L_2 = S_1 + S_2, \qquad (10)$$

where $L_2 | S_1 \cong S_2 \cong H$**.** In view of (9), for any unit $x \in L_2$ one significant decomposition takes place $x = x_1 + x_2$, where $x_1 \in S_1, x_2 \in S_2$, and $x_2 = P(x)$, where $P: L_2 \to S_2$ - is the operator of designing L_2 on S_2. The isomorphism between the spaces S_2 and H is set by the formula $P(x) \leftrightarrow A(x)\ (x \in L_2)$ and consequently:

$$T(x,y) = D(A(x), A(y)) = D(P(x), P(y)), \qquad (11)$$

Quod erat demonstrandum (Q.E.D.). From (11) it follows that for any $x \in L_2$ $T(x,(P(x)) = 1$, and if $T(x,y) = 1$ and $y \in S_2$, then $y = P(x)$.

Now proceed to the axiomatic characterization of predicates of the kind of (8). As the relation of the equality is a reflexive one, symmetric and transitive, the predicate $T(x,y)$ defines the relation with such properties:

1) $T(x,x) = 1$,

2) if $T(x,y) = 1$, then $T(y,x) = 1$,

3) if $T(x,y) = 1$ and $T(y,z) = 1$, then $T(x,z) = 1$.

The remaining properties of the predicate $T(x,y)$ should take into account linearity and continuity of the operator A, and also finite dimensionality of the space $H = A(L_2)$.

Theorem 1. Predicate $T(x,y)$, set on $L_2 \times L_2$, then and then will be presented as (8), when $T(x,y)$ obeys to the properties 1), 2), 3), and also the following three properties:

4) if $T(x_1,y_1) = 1$ and $T(x_2,y_2) = 1$, then $T(x_1 + x_2, y_1 + y_2) = 1$,

5) there is such a finite system of vectors: $l_1,...,l_n \in L_2$, that for each vector $x \in L_2$ there will be the unique n - dimensional vector $(\alpha_1,...,\alpha_n)$, for which

$$T(x_1\alpha_1 l_1 + ... + x_n \alpha_n l_n) = 1,$$

6) the functionals $\alpha_i(x)\ (i = 1,...,n) = 1$ are continuous on L_2.

The proof. Let us define, first of all, the necessity of conditions of the theorem. Let the predicate $T(x,y)$ be set by the formula (8). Then, evidently, the properties 1), 2), 3) are fulfilled.

If $T(x_1,y_1) = 1$**,** $T(x_2,y_2) = 1$**,** then by virtue of (8):

$A(x_1) = A(y_1)$**,** $A(x_2) = A(y_2)$**,** and then

$$A(x_1) + A(x_2) = A(y_1) + A(y_2)$$

or $A(x_1 + x_2) = A(y_1 + y_2)$**,** i.e.

$$T(x_1 + x_2, y_1 + y_2) = D(A(x_1 + x_2), A(y_1 + y_2)) = 1.$$

This proves the property 4). Let us present the predicate $T(x,y)$ as (11) $T(x,y) = D(Px, Py)$, where $P: L_2 \to H$ **-** is the operator of designing L_2 on some finite-dimensional subspace $H \subset L_2$**.**

Let $l_1,...,l_n$ - be the basis of the subspace H and $P(x) = (\alpha_1(x) l_1 + ... + \alpha_n(x) l_n)$**.**

Then $T(x, \alpha_1(x) l_1 + ... + \alpha_n(x) l_n) = 1$, from $T(x, j_1 l_1 + ... + j_n l_n) = 1$ it follows, that $j_1 = \alpha_1,.., j_n = \alpha_n$ and also $\alpha_1(x),..,\alpha_n(x)$ - are the continuous functionals on L_2, the operator of designing $L_2 \to H$ is continuous. Thus the properties 5), 6) of the predicate $T(x,y)$ are defined and the necessity of the theorem conditions is proved. Let us prove their sufficiency. Let $T(x,y)$ on $L_2 \times L_2$ satisfies the properties 1) $\div$ 6).

Then the vectors $l_1,...,l_n$ - are linearly independent. Really, if for $(j_1,..., j_k)\neq(0,...,0)$ $j_1 l_1+...+j_n l_n=0$, then by virtue of 1), $T(0,0)=1$, $T(0, j_1 l_1+...+\alpha_n l_n)=1$, and this equality contradicts the axiom 5). Let us denote by H the subspace of the space L_2, spanned by the vectors $l_1,...,l_n$ and let us assume, by virtue of 5), $A(x)=\alpha_1(x)l_1+...+\alpha_n(x)l_n$. We have:

$$T(x, A(x))=1, \qquad (12)$$

Then A - is the linear operator from L_2 on H, as from $T(x, A(x))=1$, $T(y, A(y))=1$ on the basis of 4) is obtained $T(x+y, A(x)+A(y))=1$, that by virtue of (12) gives $A(x+y)=A(x)+A(y)$. The continuity of the operator A follows from continuity of the functionals $\alpha_i(x)$ $(i=1,...,n)$. From equalities $T(x, y)=1$, $T(x, A(x))=1, T(y, A(y))=1$, by virtue of 1) ÷ 3), the formula follows:

$$T(x, y)=T(A(x), A(y)), \qquad (13)$$

Further:

$$T(A(x), A(y))=D(A(x), A(y)), \qquad (14)$$

Really, $T(\alpha_1 l_1+...+\alpha_n l_n, j_1 l_1+...+j_n l_n)=1$, where $(\alpha_1,...,\alpha_n)\neq(j_1,..., j_n)$, then comparison of this equality with the equality $T(\alpha_1 l_1+...+\alpha_n l_n, \alpha_1 l_1+...+\alpha_n l_n)=1$ contradicts the axiom 5). The equality (14) completes the proof of sufficiency of the theorem 1 conditions.

The remark. It is possible to write the equality (14) as:

$$T(x, y)=D(U(x),U(y)), \qquad (15)$$

where $U(x)=(\alpha_1(x),...,\alpha_n(x))$. On the Riesz theorem the functionals $\alpha_i(x)$ are set by the equalities:

$$\alpha_i(x)=\int_{\alpha}^{b} g_i(t)x(t)dt, \ (i=1,...,n), \qquad (16)$$

Thus, on **lemma 1** the linear independence of the functionals $\alpha_1(x),...,\alpha_n(x)$ are equivalent to the linear independence of functions $g_1(x),...,g_n(x)\in L_2$ (i.e.of the appropriate classes of functions). Therefore **theorem 1** admits the following equivalent statement**.**

Theorem 1. Predicate $T(x, y)$ on $L_2\times L_2$ then and only then can be presented as (15), (16) with linearly independent functionals $g_1(x),...,g_n(x)$, when it obeys to properties 1) ÷ 6).

The proof. If the properties 1) ÷ 6) are fulfilled, then, as it is shown, the equalities (15), (16) with linearly independent $g_1(x),...,g_n(x)\in L_2$ take place. Vise versa, if the equalities (15), (16) with linearly independent $g_1(x),...,g_n(x)\in L_2$ take place, the mapping $A(x)=(\alpha_1(x),...,\alpha_n(x))$ is the linear continuous operator from L_2 on n - dimendional vector space

$$H=\{(\alpha_1(x),...,\alpha_n(x))\},$$

and

$$T(x, y)=D(A(x), A(y)).$$

Then by virtue of theorem 1 the axioms 1) ÷ 6) are fulfilled. The theorem is proved.

In the specific case numerical mathematical methods [3, 6,] can be used for constructing a model of the vision on the basis of the obtained dependences.

CONCLUSION

1. The feasibility of the “method of matching " application to simulation of functions of human vision is realized.
2. The mathematical model for solution of the considered tasks is descibed with the help of the theory of predicates of a special form.
3. The properties of predicates are proved.
4. The theorem is proved, that the predicate: $T(x, y)=D(A(x), A(y))$, if all six conditions are feasible.

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ABOUT ONE APPROACH TO SOLVE THE PROBLEM OF MANAGEMENT OF THE DEVELOPMENT AND OPERATION OF CENTRALIZED WATER-SUPPLY SYSTEMS

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Abstract. One of the approaches to solve the problem of resource and energy saving in centralized water-supply systems by reducing the total excess pressure in water-supply nodes is examined in the present work. The implementation of this approach is provided due to re-engineering of water-supplies by their zoning and optimal flux-distribution in the water-supplies. Mathematical formulations of the problems to be examined and efficient algorithms of their solving are presented.

Key words: resource and energy saving, zoning, pressure regulators, pump station, adjustable drive.

INTRODUCTION

There are many problems in modern centralized water-supply systems (CWS) of Ukraine and main of them are out-of date technological equipment and a high degree deterioration of water-supply systems. This leads to significant loss of water due to undetected and unrepaired leaks, which are sometimes more than 50% of the volume of supplied water; to over-expenditure of the electricity, reagents spent on the preparation of drinking water. A radical solution of this problem is the replacement of the out-of-date technological equipment with more productive and less power-consuming one, the use of adjustable drives on the pump units, sanation and relining of water-supply systems. However, this requires huge material and financial costs.

It is known [1], that the volume of leakage is proportional to the value of excess pressure in the water-supply nodes. Therefore, any measures aimed to the reduction of excess pressure in water-supply nodes are effective means of resource and energy saving in CWS.

One of the approaches to solve the problem of resource and energy saving in centralized water-supply systems by reducing the total excess pressure in water-supply nodes is examined in the present work. The implementation of this approach is provided due to re-engineering of water-supplies by their zoning and optimal flux-distribution in water-supplies.

The principal feature of the proposed approach is the point that the problems of management of the development and operation of CWS are considered in their close functional interconnection and are solved at the different time intervals with different control parameters.

The problem of management of the development of CWS is solved at the time interval [0, T], where T = 1 year (365 days) and provides an optimal zoning of water-supply systems.

The problem of optimal management of the operation of CWS is solved at the time interval [0, T], where T = 1 day, in the form of a two-staged problem of nonlinear stochastic programming: the problem of effective planning of modes of transport and distribution of drinking water in CWS with prediction of 24 hours is solved on the first stage; on the second stage the problem of pressure stabilization in the dictating points of water-supply in the real time is solved. Let us examine sequentially each of these problems.

The actual operation modes of CWS are essentially non-stationary. In real conditions, the main disturbing factors are stochastic processes of water consumption, which depend on a huge number of non-controlled and non-controllable factors. The huge dimension of water-supplies (WS) and limitation of the operational data do not allow to evaluate adequately all the basic parameters of technological elements and the structure of WS. Moreover, the main parameters of the model of established flux-distribution, which are basic for solving the problems of engineering, management of the development and operation of CWS are estimated according to the experimental data of final length and therefore are themselves random varieties. This makes it necessary to use stochastic models of quasi-stationary modes of CWS at a given time lag [0, T], which take into account both the stochastic nature of the process of water consumption and the statistical properties of the model parameters [2, 3, 20]. This provides their much greater conformity and ability of their effective application to solve the problem of management of the development and operation of CWS.

THE PROBLEM OF ZONING OF CWS

The separation of WS on the zones is used while engineering of WS with a significant difference between the geodetic marks of the zones serviced by WS, different number of stores in buildings in city districts, therefore a significant difference of free heads required by different categories of consumers. The implementation of zoning of WS is provided either by the construction of additional pump stations (PS) which work on the selected zones, or by equipping with additional trunk pipelines or additional clean water reservoirs [1]. The appearance of new highly reliable and relatively inexpensive pressure regulators of high diameter allows to approach in a new way to solving of the problem of resource and energy saving in CWS. Zoning of WS using pressure regulators can significantly reduce the capital costs on the implementation of zoning of WS, significantly reduce the excess pressure in the nodes of WS and, therefore, the loss of water-leakage and reduce electricity consumption on PS.

At the substantive level, the problem of zoning of WS is the following: those nodes of water-supply in which excess pressure significantly exceeds the predetermined threshold value, it is necessary to group them into connected sections (zones) [4, 21], which should have at least two inputs (for providing of a given level of reliability and survivability of WS) and to provide the installation of pressure regulators at each of these inputs as well as to estimate the parameters of each regulator. If there are high-rise buildings (HRB) in the selected zone, it is necessary to provide each HRB with a pump station and to determine its parameters.

From the formal point of view, the solution of the problem of zoning of WS is taken to the decision of a number of interrelated problems:

- optimal load distribution between PS for the mode of maximum water consumption at the interval of management,
- selection of the zones of WS including HRB,
- determination of the parameters of pressure regulators installed at the inputs of the selected zones of WS,
- determination of the parameters of pumping stations for HRB in the selected zones.

While formulating the problem of zoning it is assumed that we know: the entire referenced data of the structure and parameters of WS (technological schemes, length and diameters of pipeline sections, type and characteristics of pump units (PU), adjustable and shut-off valves, etc.); operating data– minimum admissible values of heads (pressures) in all nodes of WS, predictable values of daily total water consumption in each node of WS, calculated at time zero with prediction of 365 days and their dispersion. We assume that the calculated values of predictions have normal distribution:

$$q_{i0}(\omega,l) \sim N\left(q_{i0}(l), \sigma^2_{q_{i0}}(l)\right), \qquad l = 1,2,...,365.$$

The formulation and solution of the problem of zoning performed using a stochastic model of quasi-stationary operation modes of WS [17] and the maximum water consumption, i.e. for:

$$q_{i0}(k): \quad q_{i0}(k) = \max_l q_{i0}(l), \quad l = 1,2,...,365.$$

At that the value of maximum water consumption $q_i^+(k)$ in i node determined according to the statement:

$$q_i^+(k): \; P(0 \le q_{i0}(k) \le q_i^+(k)) = 0{,}997. \quad (1)$$

To represent the structure of WS in the form of an orgraph G(V,*E*), where V – vertex set, E – arc set (*e*=Card(E), *v*=Card(V)), the real net is added by a zero top and fictitious chords which connect the zero top with all the inputs and outputs of WS [6, 7]. In order to construct the stochastic model of quasi-stationary operation modes of CWS let us produce the following net coding: choose a tree graph so that the fictitious sections of the net become chords. In this case, the actual sections will become partly chords, and partly – branches of the tree. Let us give number 1 to the branch of the tree with PS, the rest of the branches – from 2 to *v-1*, the chords of the actual sections – from *v* to $v+\eta_2-1$, the fictitious ones with given node costs from *v* to e, where η_2 - the quantity of chords of the actual sections. Under the above-mentioned coding, the stochastic model of quasi-stationary operation modes of WS takes the form [6, 7]:

$$\underset{\omega}{\mathrm{M}}\Big(sgn\, q_r(\omega) S_r\,(q_i(\omega)) q_r^2(\omega) + \sum_{i=1}^{v-1} b_{1ri}\, sgn\, q_i(\omega) S_i(q_i(\omega)) q_i^2(\omega)\Big) = 0, \quad (r = v,\dots,v+\eta_2-1), \quad (2)$$

$$\underset{\omega}{\mathrm{M}}\Big(h_r^{(c)}(\omega) - h_1^{(a)}(\omega) + \sum_{i=1}^{v-1} b_{1ri}(sgn\, q_i(\omega) S_i(q_i(\omega)) q_i^2(\omega) + h_i^{(g)})\Big) = 0, \quad r = (v+\eta_2,\dots,\, e), \quad (3)$$

$$M(q_i(\omega)) = \underset{\omega}{\mathrm{M}}\left(\sum_{r=v}^{v+\eta_2-1} b_{1ri} q_r(\omega) + \sum_{r=v+\eta_2}^{e} b_{1ri} q_r(\omega)\right), \quad (i = 1,\dots,v-1). \quad (4)$$

$$P\left(h_i^{(c)}(\omega) \ge h_i^+\right) \ge \alpha, \quad (\alpha \cong 1), (i = v+\eta_2,\dots,\, e), \quad (5)$$

$q_i(\omega)$, $S_i(q_i(\omega))$, $h_r^{(c)}(\omega)$, $h_1^{(a)}(\omega)$ - random varieties, characterizing: $q_i(\omega)$ - water flow on i- section of the pipeline, $S_i(q_i(\omega))$ - hydraulic resistance of the i- section of the pipeline, $(i = 1,\dots,v+\eta_2-1)$, $h_r^{(c)}(\omega)$ - free heads in the nodes of the net $r = (v+\eta_2,\dots,\, e)$, h_i^+ - minimum admissible heads in the nodes of the net, $h_1^{(a)}(\omega)$ -head of the pump station, $\underset{\omega}{\mathrm{M}}\{\cdot\}$ - mathematical expectation of a random variety $\{\cdot\}$.

To solve the system of equations (2) – (5) it is supplemented by the boundary conditions of the form:

$$q_r(\omega) \sim N(\bar{q}_r, \sigma^2_{q_r}), \quad r = (v+\eta_2,\dots,\, e). \quad (6)$$

In this case, the problem of zoning of WS CWS represented in the form:

$$\underset{\omega}{M} \sum_{i=1}^{v-1}\left(h_i^{(c)}(\omega,k) - h_i^+\right)^2 \to \min_{S\in\Omega}, \qquad (7)$$

where: the domain of allowable solutions Ω is a system of equations (2) – (5) of the stochastic model of quasi stationary operation modes of WS. To solve the equations (2) – (5) we will give the boundary conditions (6) in the form (1) $q_i^+(k)$, (i=1,…,*v*).

The problem (7) under the contingencies (2) – (5) and the conditions (6) belongs to the class of one-stage non-linear stochastic programming problems.

The formulation of the deterministic equivalent of the problem (2) – (7) is carried out by replacing the random varieties with their expectation values.

The solving of the deterministic equivalent of the problem (2) – (7) is carried out in two stages.

1) The optimal load distribution between pump stations, operating on the common WS is carried out by solving the following problem:

$$\underset{\omega}{M} \sum_{rV} \left(h_r^{(c)}(\omega) - h_r^{+} \right)^2 \to \min_{h_j^{(a)} \in \Omega}, \qquad (8)$$

where: $h_j^{(a)}$ - head values at the outputs of PS (j=1,…,n), n – the number of PS, operating together on WS; the area Ω is determined by the system of contingencies (2) – (5) and the condition (6).

If WS has significantly different number of stores, at this HRB is up to 5%, the values of the minimum admissible heads for the nodes with HRB are selected $h_{i*}^{+} = 10$ m ($i*$ - node number with HRB). At this, it is assumed that in case if there are these HRB in the selected zone, the pump stations will be installed.

2) ***The selection of zones.*** While zoning of WS we will allocate isolated zones of WS that have at least two inputs (for the providing of a given level of reliability and survivability of WS). Consider all the water-supply nodes in which the excess pressure exceeds a threshold value, which form the set P (card(P)=p). Nodes allocated to one zone must meet the following conditions:

1. $h_{izbi} = h_i^{(c)} - h_i^{+} \geq Porog$,

$$i \in P_k \quad (k = 1,..,p), \qquad (9)$$

where: h_{izbi} - excess pressure in i- node, $h_i^{(c)}$ - free head in i- node, h_i^{+} - minimum admissible pressure in i- node, *Porog* - excess pressure threshold.

2. All nodes of the allocated zone constitute one connected component.

Let the set P consist of k connected components (zones):

$$P = P_1 \cup P_2 \cup ... \cup P_k, \quad P_i \cap P_j = \varnothing \quad (\forall i \neq j). \qquad (10)$$

Let $M_{p \times p}$ – be an adjacency matrix composed for all the nodes from the set P.

For any node $i \in P_n$ (n=1,…,k) exists at least one node $j \in P_n$, for which $m_{ij} = 1$.

3) ***Determination of the pressure regulator's parameters for the zone P_κ.***

To determine the parameters of pressure regulators for each selected zone P_n, a subgraph $G_k(V_k, E_k)$ of WS is selected (where V_k: $v_k \in P_n$; $E_k = E_{1k} \cup E_{2k}$, E_{1k}, E_{2k} - the set of real arcs corresponding to inputs into the zone P_n and the set of fictitious arcs corresponding to outputs from the zone P_n), and we solve the problem of optimal load distribution between pressure regulators $E_{1\kappa}$, installed at the inputs into the zone P_n, i.e. the problem (8) with contingencies (2) - (6) for the subgraph $G_k(V_k, E_k)$ with replacement of head values at the outputs of pump stations $h_1^{(a)}$ with heads at the outputs of the regulators h_{Ri}, $i \in E_{1k}$. As a result we obtain minimum necessary values of heads to be stabilized at the outputs of the regulators and maximum admissible values of consumption trough them. Based on this information, in accordance with [9] optimal type of regulators and minimum admissible head at its input $h_{\min Ri}$ are determined. The ends of the arcs $E_{1\kappa}$ with defined pressure regulators are assigned with new numbers of nodes of WS, for which as a minimum admissible heads values h_l^{+} ($l \in E_{1k}$) the values of minimum admissible head at the input of the regulator are taken $h_i = h_{\min Ri}$. As the mathematical expectation flow in these nodes the maximum flow value at the output of the regulator is taken $q_i = q_{\max Ri}$, and as a measure of dispersion flow – the sum of dispersion flows in the nodes of the zone P_κ,:

$$\sigma_{q_i}^2 = \sum_{r \in P_k} \sigma_{q_r}^2 .$$

All remaining nodes of the selected zone are removed from the graph of WS.

4) Definition of the pumping station's parameters. As a result of solving the problem of load distribution between the pressure regulators we obtain the values of the actual free heads in all nodes of the set P_n zone, including nodes with HRB h_{i*}^{c}. The calculation of the required head of the pumping station for the nodes with HRB is determined in accordance with the statement:

$$\text{Hst}=h^{+}_{i*}-h^{c}_{i*}, \tag{11}$$

h^{+}_{i*}, h^{c}_{i*} - minimum admissible pressure in i- node of HRB and actual pressure in i- node with HRB of the zone P_{κ}.

THE EFFECTIVE PLANNING PROBLEM OF THE CWS OPERATION MODES

The problem of effective planning of operation modes of CWS should be solved at time zero with the prediction of 24 hours. The initial data for the solution of the problem of planning are: all referenced data about the structure and parameters of WS (technological schemes, length and diameters of pipeline sections, type and characteristics of PU, adjustable and shut-off valves, etc.) with the replacement of the selected zones with their equivalent characteristics; operating information – minimum admissible values of heads in all nodes of WS, predicted values of hour volumes of water consumption in each node of WS, calculated at time zero with the prediction of 24 hours and their dispersion [11-13].

The problem of effective planning of the operation modes of CWS is considered as two interrelated problems: the problem of optimal load distribution between PS operating on the common WS CWS and the problem of optimization of the operation modes of each PS of CWS.

THE PROBLEM OF OPTIMAL LOAD DISTRIBUTION BETWEEN PS

The problem of optimal load distribution between PS is solved at the interval of planning [0, T], (T=24). Taking into account the specificity of water consumption the interval of planning is divided into 4 subintervals: $[t_1,t_2]$ – period of minimum water consumption, $[t_2,t_3]$ – transition from minimum to maximum water consumption, $[t_3,t_4]$ – period of maximum water consumption, $[t_4,t_5]$ – transition from maximum to minimum water consumption.

For each subinterval $[t_i,t_j]$ the estimations of expectation of the predicted volume of water consumption by all categories of consumers of WS and their dispersions:

$$q_{i0}(l), \sigma^2_{q_{i0}}(l), l=1,...,24, \ i=1,\dots,4$$

are calculated:

$$\bar{q}_{10}=\frac{1}{t_2-t_1}\sum_{l=t_1}^{t_2} q_i(l),$$

$$\sigma^2_{q10}=\frac{1}{t_2-t_1}\sum_{l=t_1}^{t_2}\sigma^2_{q_i}(l), \tag{12}$$

$$\bar{q}_{20}=\frac{1}{t_3-t_2}\sum_{l=t_2}^{t_3} q_i(l),$$

$$\sigma^2_{q20}=\frac{1}{t_3-t_2}\sum_{l=t_2}^{t_3}\sigma^2_{q_i}(l), \tag{13}$$

$$\bar{q}_{30}=\frac{1}{t_4-t_3}\sum_{l=t_3}^{t_4} q_i(l),$$

$$\sigma^2_{q30}=\frac{1}{t_4-t_3}\sum_{l=t_3}^{t_4}\sigma^2_{q_i}(l), \tag{14}$$

$$\bar{q}_{40}=\frac{1}{t_5-t_4}\sum_{l=t_4}^{t_5} q_i(l),$$

$$\sigma^2_{q40}=\frac{1}{t_5-t_4}\sum_{l=t_4}^{t_5}\sigma^2_{q_i}(l). \tag{15}$$

Without breaking the generality, we consider the mathematical formulation and algorithm of solving the problem of load distribution between PS for the interval $[t_i,t_j]$:

$$\underset{\omega}{M}\sum_{rmV^*}\left(h_r^{(c)}(\omega)-h_r^{+}\right)^2 \to \min_{h_j^{(a)}\in\Omega}, \tag{16}$$

$$\underset{\omega}{M}\sum_{r\in E_{i2}} q_r(l,\omega)=\bar{q}_{i0}, \text{ (i=1,…,4)}, \tag{17}$$

where: V^* - the set of nodes of the graph of WS G(V,E), with the exception of the nodes of all selected zones P_n (n=1,…,k) and adding the nodes corresponding to the inputs of each of the pressure regulators in all zones of P_n; $q_r(l,\omega)$ - predicted value of consumption by rconsumer at the interval [t_i, t_j]; $h_j^{(a)}$ - head values at the outputs of PS (j=1,…,n1), n1 – number of PS operating together on WS; the area Ω is defined by the system of contingencies (2) – (5) and condition (6) with an additional balance condition (17). The problem is solved using Nelder-Mead's modified algorithm [14]. As a result of solving of the problem of load distribution between PS we obtain the estimations of expectations of optimal values of consumption $\bar{q}_{iNS}$ and heads $\bar{h}_{iNS}$ at the outputs of each PS and their dispersions. These values are the starting points for solving the problem of optimization of operation modes of PS.

THE OPTIMIZATION PROBLEM OF PS OPERATION MODES

On the PS for pumping water several single-type or multi-type pump units (PU) working simultaneously are used. The number of simultaneously operating on a common pipeline PU can be different, depending on the desired operation mode of PS. The direction of the operation mode of PS is carried out by switching PU on/off; changing the degree of opening of the adjustable valve (AV) at the output of PU; changing of the speed of shaft rotation of PU, if PU is equipped with an adjustable drive.

In the work [21] we propose the strategy of effective planning of operation modes of PS at the given time slice [t_i,t_j] based on a stochastic model of quasi-stationary operation modes of PS.

In the present work this strategy is applied for planning of operation modes of PS consisting of m working simultaneously PU in which:

1) the operation of all m PU is controlled with the help of AV;

2) one PU is equipped with adjustable drive, the rest are controlled with AV;

3) all m PU are equipped with an adjustable drive.

To represent the structure of PS in the form of an orgraph $G_{NS}(V,E)$, where V – a vertex set, E – an arc set (e=Card(E), v=Card(V), a real PS is added by a zero top and fictitious chords connecting the zero top with the input and output of PS. For the mathematical formulation of the problem, the following coding of PS is made: each branch of the tree graph contains one PU, one AV (adjustable valve) and passive areas connecting PU and AV with the input and output of PS. The vertices of the graph of PS are the connection points of two or more elements [22, 23].

The arc set E of the net graph PS can be presented as:

$$E=L\cup M\cup K\cup R,$$

where: L – arc set of the net graph corresponding to the areas with PU; M – arc set of the net graph corresponding to the passive areas, K – set of the fictitious areas of the net, R – arc set of the net graph corresponding to the adjustable valves (AV). For this labeling the stochastic model of quasi-stationary operation modes of PS is the following:

(The adjustment of PU is carried out by means of AV.)

$$\underset{\omega}{\mathrm{M}}\Bigg(h_r(q_r(\omega)) + \sum_{i\in L} b_{1ri} h_{NAi}\ (q_i(\omega)) + \\ + \sum_{i\in R} b_{1ri} h_{RZi}\ (q_i(\omega)) + \sum_{i\in M} b_{1ri} h_i\ (q_i(\omega)) \Bigg) = 0, \\ (r = v,\ldots, v+\eta_2 - 1)\,. \tag{18}$$

$$\underset{\omega}{\mathrm{M}}\Big(\bar{h}_{NS} - h_{vh} + h_{NAr}(q_r(\omega)) + h_{NAr}^{(g)} + h_{RZr}(q_r(\omega)) + \\ + h_{RZr}^{(g)} + \sum_{i\in M} b_{1ri}\,(h_i(q_i(\omega)) + h_i^{(g)}) \Big) = 0, \\ (r = 1,\ldots,\ \mathrm{n})\,. \tag{19}$$

$$\underset{\omega}{\mathrm{M}}\left(\sum_{r=v}^{e} b_{1ri} q_r(\omega) \right) = \bar{q}_{NS}\,, \\ \left(i = 1,\ldots, v-1 \right), \tag{20}$$

$$q_i\left(\omega\right) > 0, \quad i \in M. \tag{21}$$

$$h_{NAi}(q_i(\omega)) = a_{0i}(\omega) + a_{1i}(\omega) q_i(\omega) + \\ + a_{2i}(\omega) q_i^2(\omega), \quad i \in L, \tag{22}$$

$$h_{RZi}(\mathrm{q}_i(\omega)) \ = \ \frac{\mathrm{q}_i(\omega)\mathrm{C}_i(\omega)}{\mathrm{E}_i^2}, \quad \mathrm{i} \in \mathrm{R}, \tag{23}$$

$$h_i(q_i(\omega)) = sgn\, q_i(\omega) S_i(\omega) q_i^2(\omega),$$
$$\bar{S}_i = 0,001736 \frac{l_i}{d_i^{5,3}}, \quad i \in M, \qquad (24)$$

$$\eta(q_i(\omega)) = d_0(\omega) + d_1(\omega) q_i(\omega) +$$
$$+ d_2(\omega) q_i^2(\omega), \quad i \in L, \qquad (25)$$

In this equations: the random varieties characterize $q_i(\omega)$ – water flow on i- section of the pipeline, $S_i(\omega)$ – hydraulic resistance of i- section of the pipeline ($i \in M$), $\bar{h}_{NS}$ –head at the output of PS, $h_{NAi}(q_i(\omega))$ – head of i- PU, $h_{RZi}(q_i(\omega))$ – drop of head on i-AV; $\eta(q_i(\omega))$ – efficiency of i- PU, $a_{0i}(\omega), a_{1i}(\omega), a_{2i}(\omega), d_{0i}(\omega), d_{1i}(\omega), d_{2i}(\omega)$ – parameters of PU ($i \in L$), $C_i(\omega)$ – parameters of AV ($i \in R$), h_{vh} – head at the input in PS, E_i – degree of opening of AV (E ϵ (0,1]), l_i, d_i, $h_i^{(g)}$ – length, diameter and geodetic mark of i- section of the pipeline ($i \in M$), b_{1ri} – cyclomatic matrix element, $\bar{q}_{NS}$ - evaluation of the expectation of the flow at the output of PS, $\underset{\omega}{M}\{\cdot\}$ - mathematical expectation of the random variety {.}.

If PU is equipped with an adjustable drive it is necessary to use in the system of contingencies (18) – (25) models (26) and (27) instead of the models (22) and (25).

The mathematical model of PU with an adjustable drive is:

$$h_{NAi}(q_i(\omega)) = a_{0i}(\omega) \left(\frac{n_1}{n_0} \right)^2 +$$
$$+ a_{1i}(\omega) q_i(\omega) \frac{n_1}{n_0} + a_{2i}(\omega) q_i^2(\omega), \quad i \in L, \qquad (26)$$

$$\eta(q_i(\omega)) = 1 -$$
$$- \frac{1 - d_0(\omega) - d_1(\omega) q_i(\omega) - d_2(\omega) q_i^2(\omega)}{\left(n_0 / n_1 \right)^{0,36}}, \quad i \in L, \qquad (27)$$

where: n_0, n_1 – nominal and operating speed (r/min) of the rotor drive motor.

The mathematical formulation of the problem of optimization of the operation modes of i- (i=1,…,n1) of SW CWS at the interval of adjustment $[0,T]$ can be represented as a non-linear stochastic programming problem:

$$\underset{\omega}{M} \sum_{t=1}^{4} \sum_{j=1}^{m} \frac{9,81 \cdot h_{NAtj}(q_j(\omega)) \cdot q_{tj}(\omega)}{\eta_{NAtj}(q_j(\omega))} \to \min_{S, E_{ij} \in \Omega}, \qquad (28)$$

with statistical conditions (18) – (20) and additional conditions (21) – (27). Where the area Ω for i- of PS is determined by the system of PS (18) – (27), m – the number of PU on the selected PS, S – the structure of PS, i.e. the line-up of PU operating on PS, t – the interval of planning (1,2,3,4):

t=1 corresponds to the interval $[t_1,t_2]$,
t=2 corresponds to the interval $[t_2,t_3]$,
t=3 corresponds to the interval $[t_3,t_4]$,
t=4 corresponds to the interval $[t_4,t_5]$.

The building of a deterministic equivalent of the problem (18) – (28) is carried out by replacement of the random varieties with their expectations. As a result of solving of this problem we obtain S_{NSj}^*, n_{1jz}^* - optimal structure of i- PS, optimal values of the expectations of the drive speed z- PU, position of each AV.

THE PROBLEM OF THE PRESSURE STABILIZATION IN THE WS DICTATING POINTS

The statement of the problem of pressure stabilization in the dictating points of WS is the following:

$$\underset{\omega}{M}\, c \sum_{t=1}^{24\cdot 60} \sum_{i=1}^{v^*} \left(h_i^{(c)}(t, \hat{\omega}) - h_i^+ \right)^2 \to \min_{\Delta n_{1jz} \in \Omega}, \qquad (29)$$

where: v^* - quantity of the dictating points in WS, c – normalizing factor; Δn_{1jz} - modification of the value of rotation z- PU on j- PS; area of the contingencies Ω is determined by the equation (30).

Problem (29) – is one-stage problem of stochastic programming.

Contingencies for the problem of pressure stabilization in DP:

Suppose $y_{jit}(\omega)=h_{ji}^{(c)}(t,\hat{\omega})-h_{ji}^{+}$ - the value of pressure increase in j - DP, caused by the change of pressure $h_{itNS}^{P}(\omega)$ on i - PS at the moment of time t. Then the values $y_{jit}(\omega)$ and $h_{itNS}^{P}(\omega)$ can be connected with each other by the dependence:

$$h_{ji}^{(c)}(t,\hat{\omega})-h_{ji}^{+}=V_{ijt}h_{iNS}(\omega)+N_{it}(\omega), \quad (30)$$

where: V_{ijt} - operator of the linear discrete transfer function, connecting the change of pressure in j - DP with the change of pressure on i - PS at the moment of time t; $N_{it}(\omega)$ - the value of noise, determined by the stochastic nature of water consumption in the load zone i - PS and the actual operation mode of CWS at the moment of time t.

RESULTS AND DISCUSSION

Without breaking the generality let us examine as an example the segment of WS (fig. 1) with two PS- PS1 and PS2, each of which consists of m=3 connected in parallel PU such as AD4000-95-2. The schemes of PS1 and PS2 are shown in the fig. 2, the parameters of PU are given in the Table 1. The parameters of WS (l, d, h^g – length, diameter and geodesic mark of the section of WS, h^+, q_{i0} – minimum admissible pressure and predicted flow in the nodes of WS) are given in the Table 2.

The results of solving of the problem of optimal load distribution between PS1 and PS2 for each interval of planning are in the Table 3 ($\bar{h}_{NSi}$ – mathematical expectation of head of the pump station (i=1,2), $\bar{q}_{NSi}$, $\bar{Q}=\bar{q}_{NS1}+\bar{q}_{NS2}$ – mathematical expectation of water flow on the pump stations and in the water-supply).

The problem of zoning we will solve for the mode of maximum water consumption: $\bar{Q}$=7,06 m^3/s, $\bar{h}_{NS1}$=74,97 m, $\bar{q}_{NS1}$=3,56 m^3/s, $\bar{h}_{NS2}$=78,61 m, $\bar{q}_{NS2}$=3,5 m^3/s.

In the present WS (fig. 1) for a threshold value of the excess pressure value Porog=23 zone P* has been selected (selected in the Table 4 and in the fig. 1). $h^{(c)}$, $h^{(c)*}$ - free heads in the nodes of water-supply initially and after zoning (m); h_{izb}, h_{izb*} - excess pressure in the nodes of water-supply initially and after zoning (m).

In the node №11 the pumping station is provided.

As a result of solving of the problem of load distribution between the pressure regulators installed at the inputs in the zone, we determined the value of pressure regulators “behind”, at the inputs in the zone h_{R1}=41,47, h_{R2}=52,59 м, and based on these data we identified the type of the pressure regulator: Honeywell D15P.

The pressure of the pumping station in the node 11 Hst =26,944 m.

The analysis of the obtained results in the Table 5 and in the fig. 3.

After zoning, installation of pressure regulators at the inputs in the selected zone and the pumping station, the sum of squares of the excess pressures in the nodes of WS has decreased on 74,88%.

Table 6 – Table 9 are presented results of solving the problem of effective planning of the operation modes of PS1 and PS2. Where F - the function of power inputs:

$$F=\sum_{k=1}^{4}\sum_{i=1}^{3}\frac{9{,}81\cdot\bar{h}_{NAi}\cdot\bar{q}_i}{\bar{\eta}_{NAi}}.$$

These results may be effectively used in many real situations in centralized water-supply systems.

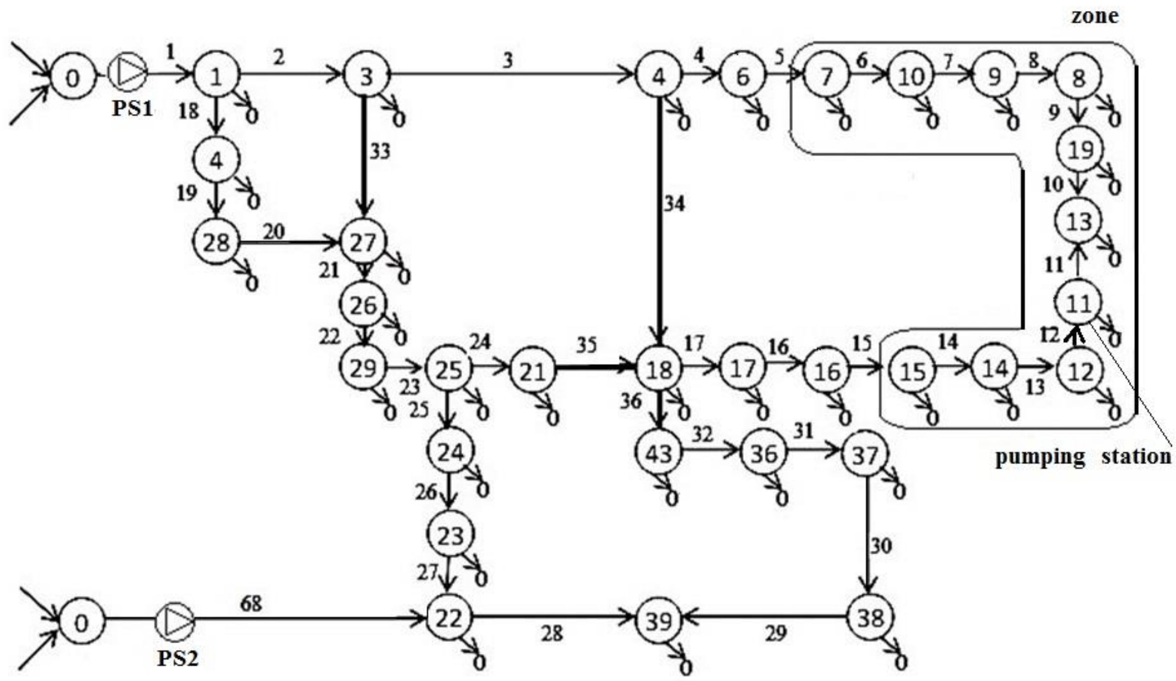

Fig. 1. The graph of water-supply with two pump stations PS1 and PS2

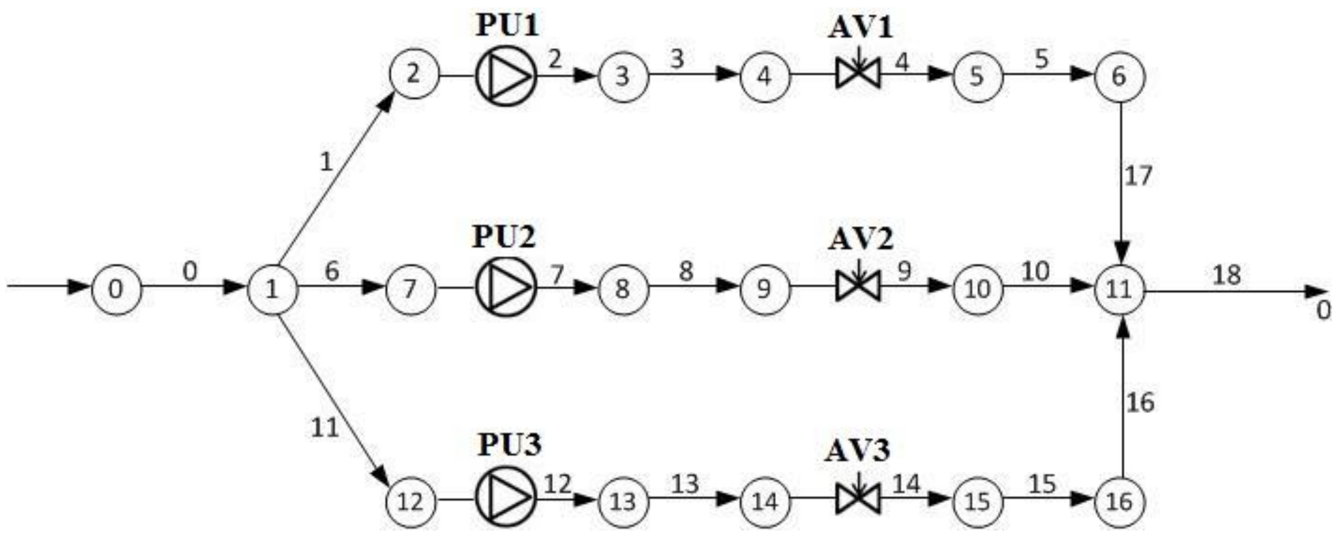

Fig. 2. The scheme of pump stations PS1 and PS2

Table 1. The estimated coefficient of approximation of the characteristics of PU

PU	a_0	a_1	a_2	d_0	d_1	d_2
PU1	108	-2,03524	-8,94861	0	147,0046	-61,030796
PU2	100	-2,03524	-8,94861	0	147,0046	-61,030796
PU3	95	-2,03524	-8,94861	0	147,0046	-61,030796

Table 2. The parameters of water-supply

Node couple	l, m	d, m	$h^{(g)}$, m	№ of the node	h^{+}, m	q_{i0}, m^3/s			
						$[t_1,t_2]$	$[t_2,t_3]$	$[t_3,t_4]$	$[t_4,t_5]$
0-1	0	0,8	0	-	-	-	-	-	-
1-3	478	0,8	0,79	1	52,6	0,1143	0,2857	0,4	0,3077
3-4	1036	0,8	1,19	3	51,309	0,0286	0,0714	0,1	0,0769
4-6	600	0,6	-8,99	4	49,464	0,0286	0,0714	0,1	0,0769
6-7	686	0,6	-12	6	48,048	0,1143	0,2857	0,4	0,3077
7-10	1693	0,8	-6	7	51,368	0,0114	0,0286	0,04	0,0308
10-9	566	0,25	-3	10	46,887	0,0069	0,0171	0,024	0,0185
9-8	302	0,3	-1	9	45,432	0,0023	0,0057	0,008	0,0062
8-19	500	0,3	-1	8	46,401	0,0057	0,0143	0,02	0,0154
13-19	442	0,3	-1	19	48,281	0,0033	0,0084	0,0117	0,0090
11-13	313	0,3	-2	13	49,835	0,0046	0,0114	0,016	0,0123
12-11	454	0,3	-2	11	85 (50)	0,0034	0,0086	0,012	0,0092
14-12	445	0,3	-1	12	50,919	0,0023	0,0057	0,008	0,0062
15-14	471	0,3	-1,99	14	52,915	0,0011	0,0029	0,004	0,0031
16-15	895	0,3	-4	15	54,946	0,0003	0,0007	0,001	0,0008
17-16	290	0,3	-10	16	56,379	0,0046	0,0114	0,016	0,0123
18-17	209	0,3	-10	17	52,8	0,0011	0,0029	0,004	0,0031
1-40	181	0,6	0,28	18	49,986	0,0011	0,0029	0,004	0,0031
40-28	212	0,6	-4,23	40	52,124	0,2286	0,5714	0,8	0,6154
28-27	432	0,6	-0,74	28	56,198	0,1429	0,3571	0,5	0,3846
27-26	375	0,6	1,15	27	56,783	0,0857	0,2143	0,3	0,2308
26-29	250	0,5	2,63	26	55,526	0,0914	0,2286	0,32	0,2462
29-25	266	0,4	2	29	52,74	0,1143	0,2857	0,4	0,3077
25-21	300	0,4	-0,1	24	49,5	0,1143	0,2857	0,4	0,3077
25-24	217	0,3	0,1	23	51,568	0,1429	0,3571	0,5	0,3846
24-23	215	0,3	0	22	54,288	0,2857	0,7143	1	0,7692
23-22	305	0,3	0,1	39	48,063	0,1429	0,3571	0,5	0,3846
22-39	671	0,25	-0,1	38	55,277	0,1371	0,3429	0,48	0,3692
38-39	497	0,3	0,1	37	53,842	0,0286	0,0714	0,1	0,0769
37-38	525	0,3	0	36	51,418	0,0286	0,0714	0,1	0,0769
36-37	274	0,3	0,1	43	50,848	0,0823	0,2057	0,288	0,2215
43-36	219	0,3	0	25	49,825	0,0286	0,0714	0,1	0,0769
3-27	400	0,6	-5,48	21	49,876	0,0286	0,0714	0,1	0,0769
4-18	700	0,6	-0,69	amount	-	2,02	5,04	7,06	5,43
21-18	350	0,4	0,3	-	-	-	-	-	-
18-43	214	0,4	-0,3	-	-	-	-	-	-

Table 3. The planning modes of PS1 and PS2

Interval of planning	$[t_1,t_2]$ min	$[t_2,t_3]$	$[t_3,t_4]$ max	$[t_4,t_5]$
PS1	$\bar{h}_{NS1}$=57,94 m $\bar{q}_{NS1}$=1,02 m^3/s	$\bar{h}_{NS1}$=65,65 m $\bar{q}_{NS1}$=2,54 m^3/s	$\bar{h}_{NS1}$=74,97 m $\bar{q}_{NS1}$=3,56 m^3/s	$\bar{h}_{NS1}$=67,205 m $\bar{q}_{NS1}$=2,74 m^3/s
PS2	$\bar{h}_{NS2}$=57,168 m $\bar{q}_{NS2}$=1 m^3/s	$\bar{h}_{NS2}$=67,1 m $\bar{q}_{NS2}$=2,5 m^3/s	$\bar{h}_{NS2}$=78,61 m $\bar{q}_{NS2}$=3,5 m^3/s	$\bar{h}_{NS2}$=68,92 m $\bar{q}_{NS2}$=2,69 m^3/s
The total supply of WS	$\bar{Q}$=2,02 m^3/s	$\bar{Q}$=5,04 m^3/s	$\bar{Q}$=7,06 m^3/s	$\bar{Q}$=5,43 m^3/s

Table 4. The results of solving of the problem of zoning for the modes of minimum and maximum water flow

№ of the node	$h^{(c)}$	h_{izb}	$h^{(c)*}$ (in the zone)	h_{izb*}	$h^{(c)}$	h_{izb}	$h^{(c)*}$ (in the zone)	h_{izb*}
	$[t_3,t_4]$ max				$[t_1,t_2]$ min			
1	74,962	22,362	-	22,362	57,941	5,341	-	5,341
3	66,283	14,974	-	14,974	56,491	5,182	-	0,66
4	59,904	10,44	-	10,44	54,822	5,358	-	1,237
6	64,915	16,867	h_{R1}=41,47	16,867	63,437	15,389	-	11,278
7	76,674	25,306	53,226	1,858	75,41	24,042	41,595	2,2
10	82,611	35,724	59,162	12,275	81,402	34,515	53,568	12,673
9	82,921	37,489	59,472	14,04	84,11	38,678	59,56	16,836
8	83,53	37,129	60,081	13,68	85,065	38,664	62,268	16,822
19	84,407	36,126	60,959	12,678	86,05	37,769	63,223	15,926
13	83,406	33,571	59,957	10,122	85,049	35,214	64,207	13,371
11	81,505	31,505	58,056	0	83,059	33,059	63,206	3,032
12	79,915	28,996	56,466	5,547	81,102	30,183	61,216	8,34
14	79,557	26,642	56,108	3,193	80,169	27,254	59,259	5,411
15	78,395	23,449	54,946	0	78,265	23,319	58,326	1,476
16	76,042	19,663	h_{R2}=52,59	19,663	74,435	18,056	-	14,224
17	67,031	14,231	-	14,231	64,536	11,736	-	7,93
18	57,841	7,855	-	7,855	54,619	4,633	-	0,846
40	64,783	12,659	-	12,659	56,842	4,718	-	0,099
28	66,684	10,486	-	10,486	60,858	4,66	-	0,193
27	67,136	10,353	-	10,353	61,563	4,78	-	0,388
26	63,384	7,858	-	7,858	60,166	4,64	-	0,488
29	60,075	7,335	-	7,335	57,464	4,724	-	0,754
24	63,222	13,722	-	13,722	55,861	6,361	-	5,561
23	72,858	21,29	-	21,29	56,731	5,163	--=	6,323
22	78,607	24,319	-	24,319	57,168	2,88	-	4,716
39	68,557	20,494	-	20,494	56,392	8,329	-	9,664
38	55,634	0,357	-	0,357	55,277	0	-	0
37	53,842	0	-	0	55,089	1,247	-	0,167
36	53,628	2,21	-	2,21	55,153	3,735	-	1,912
43	53,913	3,065	-	3,065	55,18	4,332	-	0,974
25	60,463	10,638	-	10,638	55,691	5,866	-	3,696
21	59,061	9,185	-	9,185	55,635	5,759	-	2,697

Table 4. (continuation) The results of solving of the problem of zoning for transient conditions

№ of the node	$h^{(c)}$	h_{izb}	$h^{(c)*}$	h_{izb*}	$h^{(c)}$	h_{izb}	$h^{(c)*}$	h_{izb*}
	$[t_2,t_3]\uparrow$				$[t_4,t_5]\downarrow$			
1	65,643	13,043	-	13,043	67,205	14,605	-	14,605
3	60,827	9,518	-	9,518	61,734	10,425	-	10,425
4	56,993	7,529	-	7,529	57,467	8,003	-	8,003
6	63,949	15,901	-	15,901	64,099	16,051	-	16,051
7	75,816	24,448	53,468	2,1	75,947	24,579	53,44	2,072
10	81,78	34,893	59,432	12,545	81,906	35,019	59,399	12,512
9	83,296	37,864	60,948	15,516	83,212	37,78	60,706	15,274
8	84,076	37,675	61,728	15,327	83,963	37,562	61,456	15,055
19	85,004	36,723	62,655	14,374	84,881	36,6	62,374	14,093
13	84,002	34,167	61,653	11,818	83,879	34,044	61,372	11,537
11	82,054	32,054	59,705	1,515	81,938	31,938	59,431	1,25
12	80,272	29,353	57,923	7,004	80,188	29,269	57,681	6,762
14	79,614	26,699	57,265	4,35	79,58	26,665	57,073	4,158
15	78,064	23,118	55,716	0,77	78,095	23,149	55,588	0,642
16	74,941	18,562	-	18,562	75,1	18,721	-	18,721
17	65,467	12,667	-	12,667	65,703	12,903	-	12,903
18	55,898	5,912	-	5,912	56,197	6,211	-	6,211
40	60,314	8,19	-	8,19	61,061	8,937	-	8,937
28	63,356	7,158	-	7,158	63,909	7,711	-	7,711
27	63,94	7,157	-	7,157	64,468	7,685	-	7,685
26	61,458	5,932	-	5,932	61,767	6,241	-	6,241
29	58,461	5,721	-	5,721	58,711	5,971	-	5,971
24	59,121	9,621	-	9,621	59,755	10,255	-	10,255
23	64,128	12,56	-	12,56	65,508	13,94	-	13,94
22	67,079	12,791	-	12,791	68,917	14,629	-	14,629
39	61,996	13,933	-	13,933	63,016	14,953	-	14,953
38	55,277	0	-	0	55,277	0	-	0
37	54,31	0,468	-	0,468	54,18	0,338	-	0,338
36	54,236	2,818	-	2,818	54,087	2,669	-	2,669
43	54,387	3,539	-	3,539	54,269	3,421	-	3,421
25	57,72	7,895	-	7,895	58,142	8,317	-	8,317
21	57,015	7,139	-	7,139	57,328	7,452	-	7,452

Table 5. The results of zoning (f – sum of squares of the excess pressures in all nodes of WS)

f, м^2	$[t_1,t_2]$	$[t_2,t_3]$	$[t_3,t_4]$	$[t_4,t_5]$	Amount	f, %
f initially	11881	12410	14352	12644	51286	100%
f after zoning	1911	3144	4969	3370	13394	26,12%

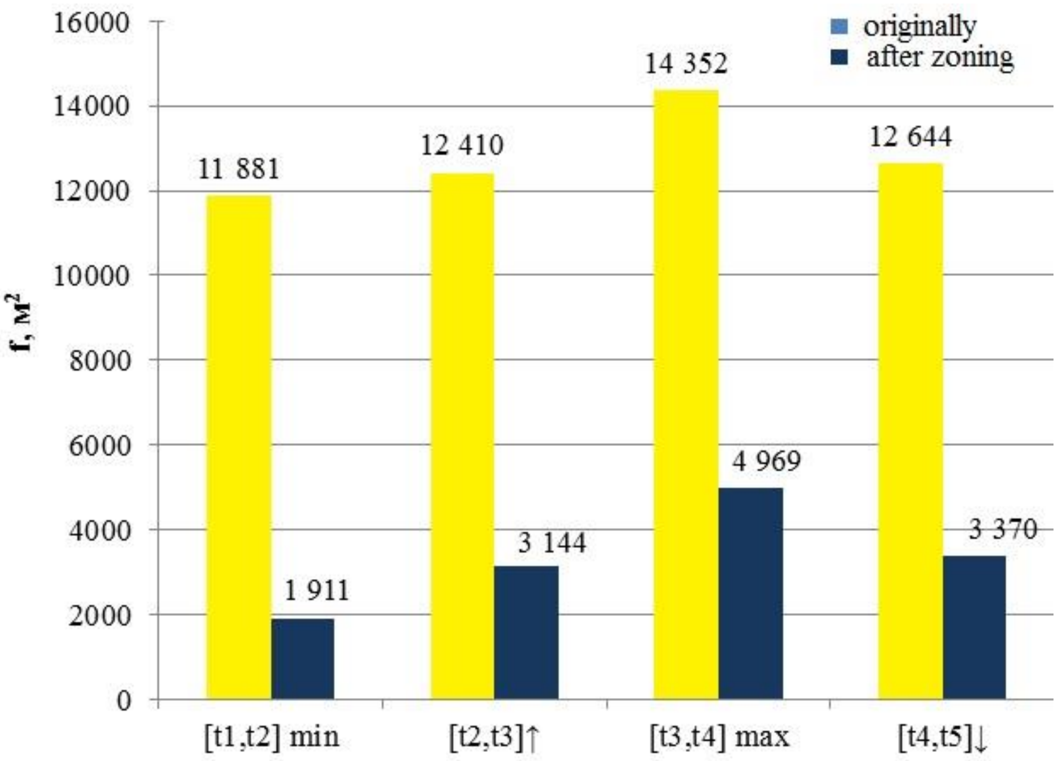

Fig. 3. The results of the zoning of WS CWS

Table 6. The results of solving of the problem of effective planning of the operation of PS 1 (the adjustment is carried out with the valves). (q_i – water flow on i- PU, E_i – the degree of opening of the adjustable valve (i=1, 2, 3))

PS1				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]$ ↑	$[t_3,t_4]$max	$[t_4,t_5]$ ↓
PU1	----------------	--------------	q_1=1,155 m^3/s E_1=0,45	q_1=0,731 m^3/s E_1=0,25
PU2	---------------	q_2=1,213 m^3/s E_2=0,5	q_2=1,238 m^3/s E_2=0,75	q_2=0,93 m^3/s E_2=0,35
PU3	q_3=1,02 m^3/s E_3=0,351	q_3=1,327 m^3/s E_3=0,841	q_3=1,167 m^3/s E_3=0,969	q_3=1,079 m^3/s E_3=0,481
F, kw	967,8	2215,97	3392,8	2952,9
PS 2				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]$ ↑	$[t_3,t_4]$max	$[t_4,t_5]$ ↓
PU1	-------------	-------------	q_1=1,254 m^3/s E_1=0,6	q_1=1,368 m^3/s E_1=0,5
PU 2	-------------	q_2=1,229 m^3/s E_2=0,5	q_2=1,19 m^3/s E_2=0,9	-------------
PU 3	q_3=1 m^3/s n_1=590	q_3=1,271 m^3/s E_3=0,672	q_3=1,055 m^3/s E_3=0,99	q_3=1,322 m^3/s E_3=0,892
F, kw	958,7	2245,7	3376,76	2500,57

Table 7. The results of solving of the problem of effective planning of operation of the pump station (one PU3 is with an adjustable drive). (n_1 – rotation of PU3 (r/min))

PS1				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]\uparrow$	$[t_3,t_4]$max	$[t_4,t_5]\downarrow$
PU1	----------------	--------------	q_1=1,232 m^3/s E_1=0,5	q_1=1,324 m^3/s E_1=0,45
PU 2	---------------	q_2=1,213 m^3/s E_2=0,5	q_2=1,26 m^3/s E_2=0,8	--------------
PU 3	q_3=1,02 m^3/s n_1=596	q_3=1,327 m^3/s n_1=660	q_3=1,068 m^3/s n_1=666	q_3=1,416 m^3/s n_1=675
F, kw	750,2	2177,9	3370,98	2508,4
PS2				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]\uparrow$	$[t_3,t_4]$max	$[t_4,t_5]\downarrow$
PU 1	-------------	-------------	q_1=1,206 m^3/s E_1=0,55	q_1=1,368 m^3/s E_1=0,5
PU 2	-------------	q_2=1,229 m^3/s E_2=0,5	q_2=1,216 m^3/s E_2=1	-------------
PU 3	q_3=1 m^3/s n_1=590	q_3=1,271 m^3/s n_1=660	q_3=1,079 m^3/s n_1=680	q_3=1,322 m^3/s n_1=671
F, kw	958,7	2178,6	3365,6	2468,5

Table 8. The results of solving of the problem of effective planning of operation of the pump station (all PU are equipped with an adjustable drive)

PS1				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]\uparrow$	$[t_3,t_4]$max	$[t_4,t_5]\downarrow$
PU1	----------------	--------------	n_1=682	-------------
PU 2	---------------	n_1=621	n_1=633	n_1=639
PU 3	n_1=596	n_1=649	n_1=661	n_1=668
F, kw	750,24	2072,95	3253,49	2330,74
PS 2				
Time slice	$[t_1,t_2]$min	$[t_2,t_3]\uparrow$	$[t_3,t_4]$max	$[t_4,t_5]\downarrow$
PU 1	-------------	-------------	n_1=692	--------------
PU 2	-------------	n_1=624	n_1=643	n_1=641
PU 3	n_1=590	n_1=652	n_1=671	n_1=671
F, kw	730,7	2069,4	3332,9	2316,7

Table 9. The function of power inputs per 24 hours under the different ways of adjustment of operation of the pump station

PS	Adjustment with AV F, kw	One PU3 with an adjustable drive F, kw	All PU with an adjustable drive F, kw
PS1	9529,5	8807,48 (decreased on 7,6%)	8407,42 (decreased on 11,8%)
PS2	9081,73	8971,4 (decreased on 1,2%)	8449,7 (decreased on 7%)

CONCLUSIONS

The examined approach to solve the problem of resource and energy saving in CWS owing to optimal reengineering of WS by means of their zoning and optimal adjustment of the water flow in WS is an effective means of resource and energy saving in CWS.

The obtained results confirm that the efficiency of operation of CWS can be increased:

1) due to zoning of WS CWS and installation of the pressure regulators at the inputs in isolated zones,

2) due to installation of the pumping stations for HRB in the selected zone,

3) due to using of two-staged system of effective operation including the problem of effective planning of the operation modes of CWS and the problem of pressure stabilization in the dictating points of WS.

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GEOINFORMATIONAL ANALYTIC CONTROL SYSTEM OF THE COLLECTION OF MUNICIPAL SOLID WASTE

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A b s t r a c t : One of the effective approaches to solve the problem of optimal control of the collection of municipal solid waste in cities and towns on the basis of geoinformational analytic system is studied in the present work. Its structure is shown in the form of three interconnected subsystems: geoinformational system, analytic system, subsystem of monitoring of the refuse collection vehicles. The main attention is focused on the analytic subsystem. The mathematical formulation of the problem of effective planning of the traffic routes of the refuse collection vehicles and efficient algorithm to solve it are set here. The control of implementation of the planned routes is carried out by the subsystem of monitoring of the refuse collection vehicles using GPS-navigation.

K e y w o r d s : municipal solid waste, optimization of routes, subsystem of monitoring, energy saving, refuse collection vehicle.

INTRODUCTION

The problem of control of the collection and recycling of municipal solid waste (MSW) is the most priority one, ranking in the system of urban economy the second place for the costs and investments after the sector of water supply and sewerage.

MSW (municipal) are waste generated in the residential sector, in trade enterprises, office buildings, office blocks, institutions, offices, kindergartens and schools, cultural and sports facilities, railway and bus stations, airports, river ports.

Nowadays on the territory of Ukraine every city dweller annually emits between 100 and 400 kg of MSW, posing a serious sanitary and epidemiological threat. In recent years, the volume of MSW has dramatically increased. The collection of MSW is usually carried out with using of standard containers installed on the container platforms. The location of the container platforms in different parts of the city and the number of containers installed on them depends on the density of population in the attended area and the intensity of filling containers with MSW at the platform [1,2,5].

The removal of MSW from the filled containers is carried by means of specially equipped refuse collection vehicles (RCV), which implement the loading of MSW in the body (bin) of RCV and its pressing. After filling the body, RCV is used as a special truck to deliver collected MSW to the place of recycling.

On the verbal level, the main point of the problem of control of the collection of MSW is in the following:

- in time synchronization of processes of filling of the containers with MSW on each container platform and removal processes of MSW,
- in minimization of material and energy costs for the collection of MSW.

The process of filling of the containers with MSW is generally a non-stationary random process, depending on a number of chronological (weekdays, holidays, seasons), meteorological (temperature, precipitations) and organizational factors (area cleaning). The sense of synchronization is in the point that the time of delay between the end of the process of filling of the containers with MSW and the moment of their removal must not exceed a few (1-5) hours. Increasing of the time of delay is unacceptable, as it can contribute to serious pollution of the cities due to the overflow of containers, to increasing the risks of environmental disasters and to increasing of additional labor and time costs for the collection and loading of MSW spilled out of the containers. The removal of MSW from not fully filled containers is also undesirable because this leads to underloading of RCV and therefore to additional fuel expense by RCV.

The ensuring of minimization of material and energy costs for the collection and recycling of MSW is carried out by:

- effective planning of the optimal amount and minimum extent of daily traffic routes of each RCV with its maximum load,
- effective control by means of GPS-navigation over the strict implementation by each RCV of each planned route for it.

The structure and functionality of geoinformational analytic control system of the collection of MSW is examined in the present work (GIACS MSW).

THE STRUCTURE OF THE SYSTEM

GIACS MSW includes three interrelated subsystems – geoinformational, analytic and subsystem of monitoring of RCV.

Geoinformational subsystem. Spatial distribution of the city or town and the need in complete and accurate information about the spatial location of each container platform and each container, the structure and parameters of city building-up, approach roads, public transport routes led to the fact that the multi-level electronic cards have become the invariant core of all information databases. Each electronic card includes a topographic base and a plurality of associated layers. Each layer has a specific subset of spatially distributed objects of city districts or attended areas. Each object of the attended area, including container platforms, is a graphic image, which is associated with text databases containing all the necessary static information about this object: address, number of floors, the owner, etc. for block of flats, buildings and structures; number, type, and volume of each container and geodesic mark for each container platform. The dynamic information in the form of time series about actual intensity of filling of the containers located on this platform is additionally introduced for each container platform. Besides, the dynamic information is also entered for the state of the roads sections (open / closed for travel) and approach roads to each container platform. Each layer can be combined with the topographic or to any other layer, and also with their arbitrary combination.

Such structural organization of the databases is an essential information basis on which the decision of all analytical problems is based.

Analytic subsystem implements resource-saving and environmentally friendly technologies of the collection and recycling of MSW. The main element of the analytic subsystem is the subsystem of effective planning of routes of RCV.

THE EFFECTIVE PLANNING PROBLEM OF RCV TRAFFIC ROUTES

Mathematical model of human-vehicle park of RCV. Municipal utility company (MUC), occupied with the removal of solid waste has a park of RCV $V = \{1,2,...,p,...,m\}$, drivers of these vehicles and workers. The most suitable model of human-vehicle park of RCV is a statistic model which allows to take into account not only technical characteristics of each RCV, but also the features of its maintenance by drivers and workers. To construct such a model they daily include statistic data about the actual work performed by each RCV and its staff as well as all types of costs for this work in GIACS MSW database. While constructing the model of human-vehicle park of RCV it is assumed that all statistic data are implementations of the relevant random varieties $X(\omega)$ with normal distribution $X(\omega) \sim N(\bar{x}, \sigma^2)$, which are completely characterized by two parameters: mathematical expectation $\bar{x}$ and dispersion σ^2. $\omega \in \Omega$: (Ω, B, P) - probable space, where Ω - the space of elementary events, B - σ - algebra of events from Ω, P - probability mass on B. As a result of statistic data processing we obtain the model of human-vehicle park of RCV in the form of a set of family of estimations of the mathematical expectation of the following technical characteristics for each p RCV:

$S^P(\omega)$ - lifting capacity of RCV, i.e. maximum quantity of the containers, the contents of which can be loaded in RCV,

$B^P(\omega)$ - fuel cost (winter - B_z^p, summer - B_l^p), consumed by RCV per 1 km of way,

$m^P(\omega)$ - engine oil cost (winter - $m_z^p(\omega)$, summer - $m_l^p(\omega)$), consumed by RCV per 1 km of way,

$b^P(\omega)$ - fuel cost (winter - $b_z^p(\omega)$, summer - $b_l^p(\omega)$), consumed by special equipment of RCV to perform the loading and unloading of MSW,

$t_p^p(\omega)$ - time spent on unloading (loading in the body) of RCV of one container,

$t_r^p(\omega)$ - time spent on unloading of RCV,

$V^P(\omega)$ - velocity of RCV on the route,

k^p - maximum quantity of the routes, which RCV can perform during a day.

Since the evaluations of the mathematical expectations obtained from the samples of finite length, they are themselves random varieties with normal distribution, the mathematical expectation of which coincides with the estimates obtained and the dispersion equal to σ^2 / n, where n, the volume of the sample.

MATHEMATICAL MODEL OF THE TRANSPORT NETWORK

The basic model of the transport network in the problem of effective planning of the traffic routes of RCV for the removal of MSW in the city is undirected weighted graph $H = (V,U)$ with top set V from n elements, numbered by figures $1, 2, ..., n$ and ribs set U.

Each top $i \in V$, $|V| = n$ is associated with:

- address, allowing to position the container platform located in this top on the city map,
- random parameter $d_i(\omega)$, corresponding to the weight of MSW in all filled containers set out in this top;

To the top set V of the graph $H = (V,U)$ we add two additional tops: $\{n+1\}$, corresponding to the garage, in which there are all RCV and $\{n+2\}$ - corresponding to the place of recycling (unloading) of MSW. The place of recycling of MSW can be: polygon (refuse dump), garbage recycling plant, rubbish selecting or rubbish transloading station.

Let's introduce the set:

$$N = V \bigcup \{n+1\} \bigcup \{n+2\}, \text{ i.e. } |N| = n+2.$$

Tops i and j form in the graph $H = (N,U)$ ribs $\{i, j\}$. The graph $H = (N,U)$ doesn't contain loops i.e. ribs $\{i, i\}$.

Each rib $\{i, j\}$, connecting top i and top j, assign the length $d_{ij} \in R_0^+$ of the way from i to j, i.e. the distance which RCV must overcome while moving from the top i to the top j.

In general case:

$$d_{ij} \neq d_{ji},\ i, j = \overline{1, n+2} \text{ and } d_{ij} = d_{ij}(\omega),$$

i.e. the distance which RCV must overcome while moving from the top i to the top j is considered as a random [8, 10, 16].

MATHEMATICAL MODEL OF THE PROCESS OF FILLING THE CONTAINER WITH MSW

As it was previously mentioned the intensity of emission of MSW by inhabitants is non-stationary random process depending on a great number of chronological, meteorological and organizational factors. As the quantity of the containers installed on each container platform and their volume are known a priori instead of the intensity of emission of MSW the index connected linearly with it, but more effective is used in GIACS MSW – the time of full filling of all of the containers with MSW on the container platform. To predict the time of filling of all of the containers with MSW on each container platform in GIACS MSW an integrated multifactor model ARIMA introduced in [3] for forecasting the processes of consumption of targets products in the engineering networks is used. Time prediction of full filling of all of the containers with MSW is calculated in the form of conditional mathematical expectation for each container platform and is approximated for the nearest integer value: day, two days, …, L days. As a result of solving the problem of prediction to each top V of the graph $H=(V,U)$ the index $l_i=\{1,2,3,4...,L\}$, corresponding to the predicted time (in days) of full filling of all of the containers installed on this top is put. The interval of effective planning of traffic routes of RCV accept equal to the length of the maximum interval $l_i=L$, i.e. the interval of planning $T=[1,L]$.

MATHEMATICAL FORMULATION OF THE PROBLEM OF THE EFFECTIVE PLANNING OF TRAFFIC ROUTES OF RCV

Determine the value of the mathematical expectation $\bar{d}$ of the quantity of containers which must be removed daily on the interval of planning:

$$T=[1,L]:\ \bar{\mathrm{d}} \le \underset{\omega}{M}\sum_{i=1}^{n} d_i(\omega)/l_i\,, \qquad (1)$$

In real conditions the value $\bar{d}$ is considerably bigger than one of the lift capacity $\bar{S}^P$ of any p - RCV, i.e. $\bar{d} >> \bar{S}^P$. In practice, it means that for the daily removal of all filled containers some number of RCV, each of which can carry no more than k^p runs per a day, is necessary.

Divide the set N into a series of random subsets $N_p^k(l,\omega)$ so, that:

$$N=\bigcup N_p^k(l,\tilde{\omega}):\quad \bigcap N_p^k(l,\tilde{\omega})=n+2,$$
$$k=1,2,...,k^p,\quad p=\{1,2,...,m\}, \qquad (2)$$
$$l=\{1,2,...,L\}.$$

For each fixed $\tilde{\omega}$ each of the subsets $N_p^k(l,\tilde{\omega})$ is a set of nodes of the network graph $H=(N,U)$, included in k - route, carried by p - RCV in the first day of the planned period $T=[1,L]$.

Under the above labeling, the mathematical formulation of the problem of effective planning of the traffic routes of RCV can be presented as:

$$\underset{\omega}{M}\sum_{l=1}^{L}\sum_{k=1}^{k^p}\sum_{p=1}^{m}\sum_{i\in N_p^k(l)}\tau_i(N_p^k(l,\omega))d_{ij}(\omega)\to\ , \qquad (3)$$
$$\to \min_{<k^p,m,\tau(\bullet)>\in\Omega}$$

$$\Omega: P\Big(\sum_{i\in N_p^k(l)} d_i(\omega)\le S^p(\omega)\Big)\ge\alpha,\ k=1,2,...,k^p,$$
$$p=\{1,2,...,m\},\ l=\{1,2,...,L\},\ j=\tau_i(i). \qquad (4)$$

where: $\tau_i(N_p^k(l,\omega))$ - cyclic rearrangement, defining the order of passing $N_p^k(l,\omega)$ of the tops k - route, carried by p - RCV on the l - day of the planned period $T=[1,L]$, α - a constant, which determines the probability of implementation of the condition of overload of p - RCV.

The problem (3), (4) belongs to the class of problems of discrete stochastic programming M type with interline probabilistic constrains. To solve the problem (3), (4) the deterministic equivalent is constructed by means of replacement of all random varieties included into the objective function by their mathematical expectations and recalculation of the probability inequalities (4) into deterministic inequalities on the basis of known statistical properties of random varieties $d_i(\omega)$ and $S^P(\omega)$.

The deterministic equivalent of the problem (3), (4) refers to the class NP-complete problems, exact algorithms solutions of which do not exist at the present time [17]. Therefore, to solve the deterministic equivalent of the problem (3), (4) the close algorithm is used.

Algorithm of solving the problem of effective planning of the traffic routes of RCV includes the following steps:

1. Calculate the value m of the necessary quantity of RCV for daily removal $\bar{\mathrm{d}}$ of the containers under the condition:

$$\sum_{k=1}^{k^p}\sum_{p=1}^{m}\bar{S}_k^p \ge \bar{d}\,. \qquad (5)$$

2. Divide the set N into a number of subsets so, that:

$$N = \bigcup N_p^k(l): \ \bigcap N_p^k(l) = n+2,\ k = 1,2,...,k^p,$$
$$p = \{1,2,...,m\},\ l = \{1,2,...,L\}.$$

$$\sum_{i \in N_p^k(l)} \bar{d}_i \le \bar{S}^p . \qquad (6)$$

Each of the subsets $N_p^k(l)$ is a set of nodes, belonging to k-route, carried by p- transport facility (TF) on l-day of the planned period $T = [1, L]$.

The separation of the set N into subsets $N_p^k(l,\tilde{\omega})$ is performed by the hierarchical method of 3D-clustering [6].

3. The length $d_{ij} \in R_0^+$ of the shortest way between all the tops $i, j \in N_p^k(l,\tilde{\omega})$ taking into account all traffic rules while implementing of the conditions of possibility of passing this way by a particular TF is determined according to the map of the city.

4. The construction of each detailed route of minimum length passing through the set of tops $N_p^k(l,\tilde{\omega})$, is performed by the Little's method [10]. Each received route is presented in the form of circular permutation $\tau_i(N_p^k(l,\tilde{\omega}))$, at that the minimum length of each route is:

$$L_p^k(l) = \sum_{i \in N_p^k} \tau_i^*(N_p^k(l,\tilde{\omega})\, \bar{d}_{ij}, \quad j = \tau^*(i). \qquad (7)$$

Thus the solution of the problem (3), (4) is:

- minimum required value of the number of runs carried by each RCV on each l-day of the planned period,
- minimum value p_l^* - the number of RCV necessary for removal of MSW on each l-day of the planned period,
- $\tau_i^*(N_p^k(l,\tilde{\omega}))$ - k-optimal route of minimum length carried by p-RCV on l-day of the planned period.

5. The calculation of the additional parameters for each detailed route:

- average fuel cost for the implementation of all k_l^{p*} routes by p- RCV on l-day of the planned period:

$$\bar{B}^p(l) = \sum_{k=1}^{k^{p*}} \bar{B}^p \sum_{i \in N_p^k(l,\tilde{\omega})} \tau_i^*(N_p^k(l,\tilde{\omega}))\, \bar{d}_{ij} + \bar{b}^p , \qquad (8)$$

- average time spent by p-RCV for the implementation of k_l^{p*} route on l-day of the planned period:
-

$$\bar{T}_p^k(l) = L_p^k(l) / \bar{V}^p + \bar{t}_r^p + \bar{t}_p^p \sum_{i \in N_p^k(l,\tilde{\omega})} \bar{d}_i . \qquad (9)$$

SUBSYSTEM OF MONITORING OF RCV

The subsystem of monitoring of RCV is an integrated in GIACS MSW system of collecting, recording and supplying of the information about the location of RCV on the territory of the city.

The main aim of the subsystem of monitoring of RCV is providing of the supervisory service and the direction of MUC with actual and effective information about the location of RCV under their control, the realization of efficient methods of collecting, processing and distributing of the information, procedures and technical means of data exchange. The main functional tasks of the subsystem of monitoring of RCV are:

- control over the location and condition of RCV,
- coordination of the activity of the departments which support RCV of the company,
- preventing of no-purpose use of RCV of the company,
- ensuring of the maintenance of the technical conditions of the exploitation of RCV.

Subsystem basis is the geoinformational system that allows to solve the problems of collection, storage and data processing of various types and origin.

Coordinates and location of RCV on a city map are determined by means of GPS technology using satellite system NAVSTAR. Nowadays it is the only completely deployed and operable satellite navigation system. Data about the location of RCV are fed to the board radio terminal which collects and processes simultaneously the information about the status of various systems and units of the vehicle. Data transfer from RCV to the server is provided through the network of mobile communication GSM 900 standard using the GPRS protocol of data transmission.

The subsystem of monitoring of RCV is an integrated set of functional modules, information resources and standardized technological procedures of information processing, telecommunication and computing environment.

The subsystem is designed to allow its further modernization with minimization of time and financial costs without changing of system platform and data communications protocols.

The subsystem has a wide range of possibilities of graphic display of vector models of the place and wide set of report forms.

The subsystem of monitoring of RCV gives an opportunity to follow any changes of RCV status and its characteristics. The database structure allows to carry out the expansion of information structure about RCV.

The subsystem basis is a centralized data depository with distributed processing.

The subsystem is constructed according to the modular approach that allows, in case of necessity, to replace certain subsystem modules keeping its operability in general (Fig. 1).

Terminal ND GPS terminal 031 produced by Ltd «Navidev» (http://navitron.mobi) is used in the subsystem.

In the basic configuration this subsystem allows to evaluate the following parameters of each RCV under control: current location, passed route, route deviation from the agreed one, speed, time of movement, time and places of parking, exit and entrance into the area (Fig. 2).

With appropriate sensors it is possible to control the following additional parameters: fuel quantity in the tanks, travel fuel consumption, fuel filling and fuel draining, engine speed, engine running time, the state of “panic button”, loading level, positions of mechanisms (for special equipment), opening of the door etc.

The fuel sensor ДУ-01М produced by «Orgtechavtomatica» Ltd is used in the system.

The subsystem also allows to control various devices: lock the door, turn off the engine etc. on condition of appropriate hardware component.

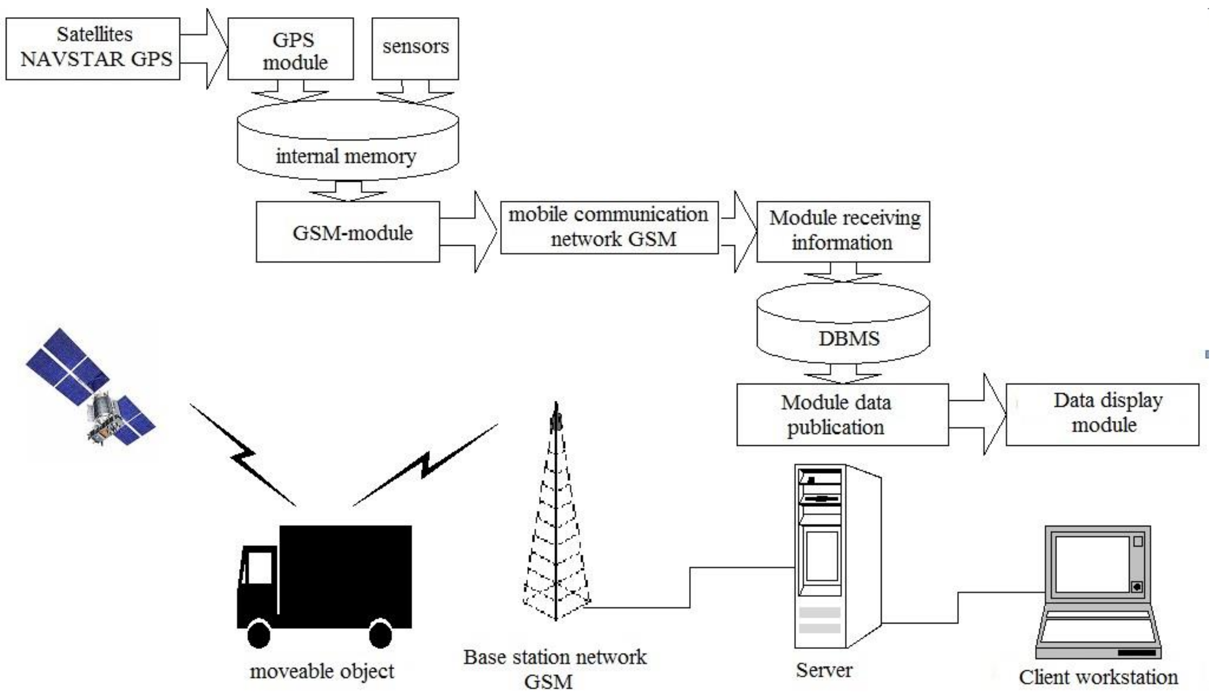


Fig. 1. Subsystem of monitoring of RCV

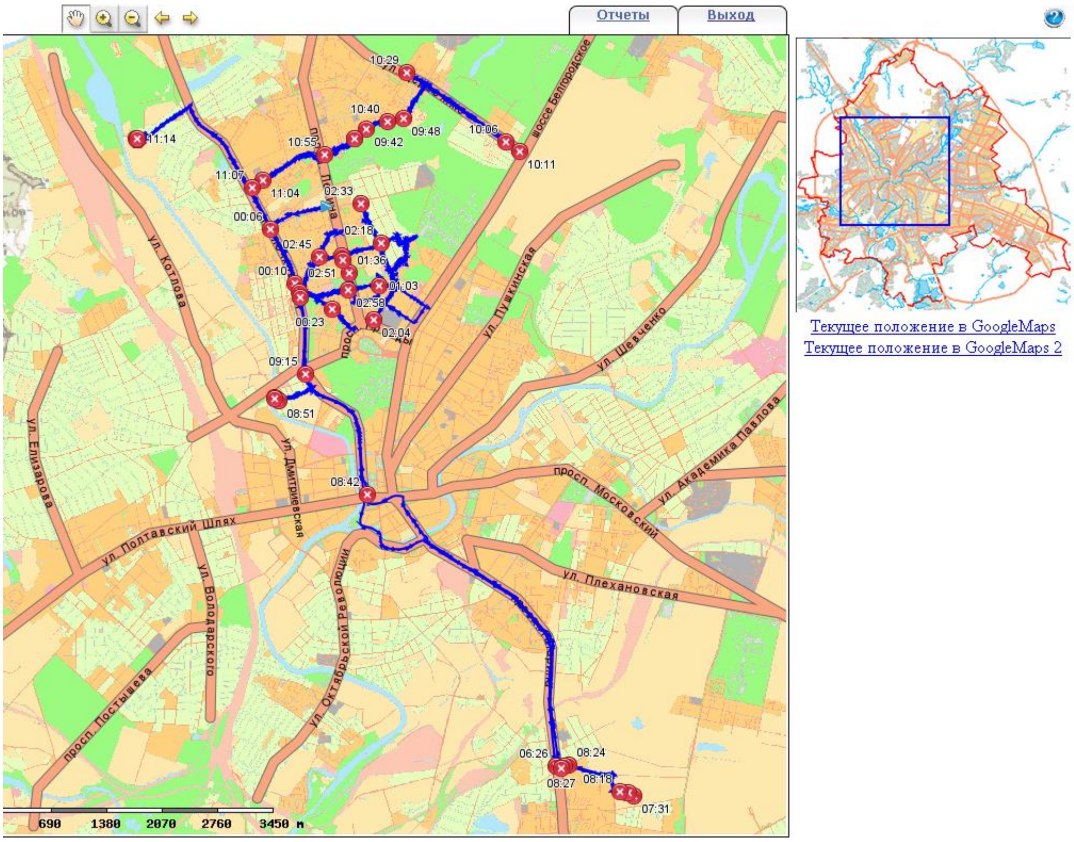


Fig. 2. System of RCV monitoring

The effects of the implementation of the system: POL saving and other resources saving connected with the exploitation of RCV, optimization of the operating functions of the controller, reducing of unproductive and dummy run, monitoring and prevention of misuse of RCV, adoption of the management decisions on the basis of reliable statistic reporting data, increasing of the discipline level of drivers and controllers, reducing of accidents.

RESULTS AND DISCUSSION

Let's present the results of solving the problem of effective planning of traffic routes of RCV for one of the cities of Ukraine with population about 40 thousand people and the quantity of the containers 431. As the input data are given the addresses of the container platforms (103), the quantity of the containers on them (1-13), the intensity of their filling (1, 2, 3, 7, 15, 30 days), as well as the set of constrains on the traffic routes of RCV.

As a result of solving the problem of effective planning of the routes of removal of all of the containers it is turned out that it was enough two vehicles: RCV KAMAZ with parameters S^1=40 containers, B_z^1=0,318 l/km, B_l^1=0,334 l/km, b_z^1=14,0 l, b_l^p=13,3l and RCV GAZ with parameters S^2=16 containers, B_z^2=0,309 l/km, B_l^1=0,294 l/km , b_z^2=6,1 l, b_l^2=5,8 l. Both RCV must perform 4 runs a day.

In the tables 1 - 3 the parameters of developed detailed routes are shown under V^P=37 km/h, t_p^p=3 min, t_r^p=10 min. The fragment of optimal routes are shown in the fig. 3

Table 1. Optimal routes of RCV1 (KAMAZ) for the first day (l=1)

Flight 1

№	Address	Number of containers	Intensity of filling (days)
1	Garage		
2	Lomonosov str. 26	3	1
3	Lomonosov str. 3	10	2
4	Oktiabrskaya str. 21	1	1
5	Jubileyniy distr. 27Б, 9, 16	17	1
6	School lane	4	1
7	Komsomolskaya str. 42	2	1
8	Ovrazniy lane	1	1
9	River lane	2	1
10	Polygon		
	amount of containers	40	

Flight 2

№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	Кavкаz str.	1	2
3	Michurin str. beach	2	2
4	Michurin str. 46(1)	3	2
5	Michurin str. 1	1	2
6	v. Golubovka PS	2	45
7	Michurin str. 123, 144, 159	3	2
8	Mechnikov str.15, Michurin str. 181	2	2
9	Selskiy lane 44	1	2
10	Ucraine str. 55	1	1
11	Lenin str. 49,52,47,60	12	1
12	Communist str. 6	7	1
13	Lenin str. 42	4	1
14	"Obriy", world furniture	2	15
15	Usikovskaya str.	1	2
16	Polygon		
	amount of containers	38	

Flight 3

№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	Lenin str. 19	8	30
3	1 May str. 19	1	15
4	1 May str. 37	1	30
5	Communist str. 30	1	2
6	Pushkin str.	2	40
7	Sports lane	1	1
8	Lenin str. 114	1	30
9	Lenin str. cemetery	4	15
10	Lenin str. 103 Ukrautogaz	2	30
11	Jubileynaya str.	3	1
12	1 May str. 57,55	7	1
13	1 May str. 59 vetclinic	2	40
14	1 May str. 59. School 34	4	15
15	Technical lane. 3	3	1
16	Ukrainian str. 48, .2a	2	1
17	October sq. 12, 20	10	1
18	1 May str. 34, Department of Culture	2	15
19	Lenin sq. 1, music school	1	30
20	Komsomolskaya str. 50	1	1
21	Khmelnitsky str. 6	1	2
22	Polygon		
	amount of containers	29	

Flight 4

№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	North str.2	1	1
3	Hospital str. 10(1) 28(2); Bugorny lane	5	1
4	Sidorenko str. 56	1	1
5	Jubileynaya str. 87	2	1
6	Demkina str. 5	2	2
7	Davydov Luchitskiy str. 1a, 1д	6	1
8	60 years of USSR str.119(1), 50(1)	2	3
9	60 years of USSR str. 55(1), 75(1)	2	3
10	60 years of USSR str. 19(2)	2	3
11	Gardening str. 100	2	1
12	Gardening str. 7	2	2
13	Gardening str. 7	2	1
14	Kharkiv str. 9	4	1
15	Chapaev str .2	1	2
16	Chapaev str. 30	2	2
17	Komsomolskaya str. 34	4	3
18	Polygon		
19	Garage		
	amount of containers	40	

Table 2. Optimal routes of RCV12 (GAZ) for the first day (*l*=1)

Flight **1**

№	Address	Number of containers	Intensity of filling (days)
1	Garage		
2	Lenin sq. 39 АТВ-мarket	3	2
3	Komsomolskaya str. 5	13	1
4	Polygon		
	amount of containers	**16**	

Flight 2

№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	Kolhoznaya str. 44	1	2
3	Vorovskiy str д.9	2	2
4	Lermontov str. 41	3	2
5	Rovenskaya str. 110	2	2
6	Shevchenko str. 19	1	2
7	Karl Marx str.12	1	2
8	Flower str.	1	2
9	Forest Glade str.	1	2
10	Dobrolyubov str.	1	2
11	Artem lane. 1	1	2
12	entry Komsomolskiy 10	1	2
13	Akhtyrsky lane 6	1	2
14	Polygon		
	amount of containers	16	

Flight **3**

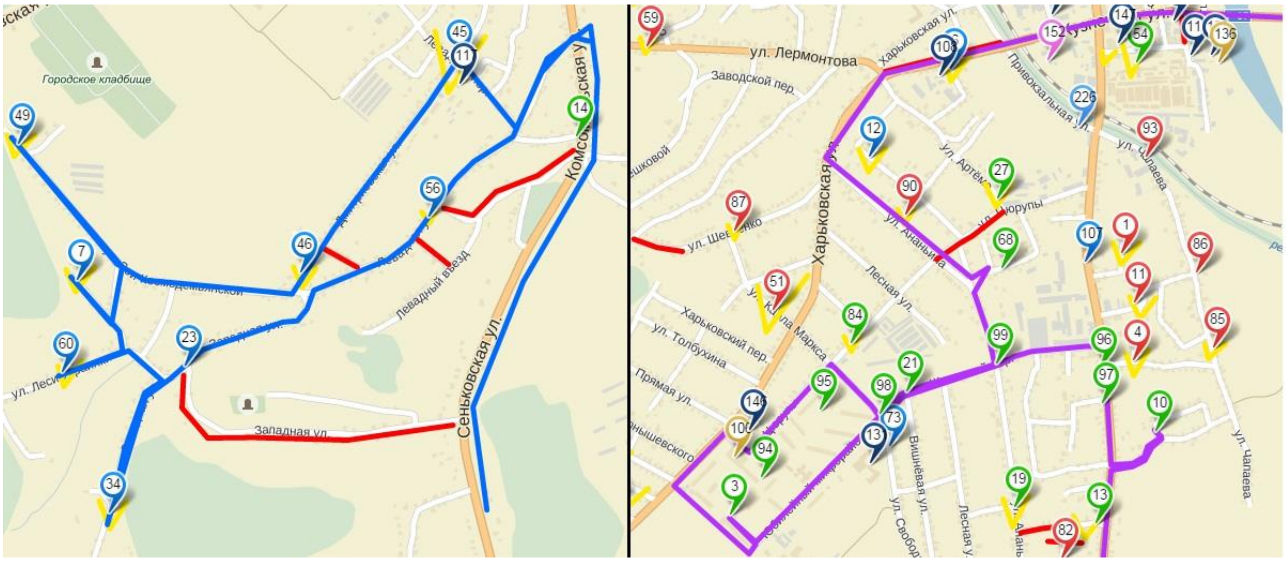
№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	Podgorniy lane 1	2	1
3	1 May str. 39	4	1
4	1 May str. 4	4	1
5	Artem str. 38	2	1
6	Curupi str.. 20a	3	1
7	Sholkoviy lane	1	1
8	Polygon		
	amount of containers	**16**	

Flight 4

№	Address	Number of containers	Intensity of filling (days)
1	Polygon		
2	Krivoy lane 1	1	3
3	Dmitrievskaya str. 1	2	3
4	Dmitrievskaya str. 68	1	3
5	Kosmodemyanskoy str. 71, 57, 31	3	3
6	Lesya Ukrainka lane 10	1	3
7	Lesya Ukrainka str. 30	1	3
8	West str. 65	1	3
9	West str. 2, 16	3	3
10	Levadny str. 66	3	3
11	Polygon		
12	Garage		
	amount of containers	16	

Table 3. Characteristics of routes

Car	Flight number	Route length (km)	Fuel consumption (l) summer	Fuel consumption (l) in winter	The number of containers	Time, h
KAMAZ	flight 1	14,4	17,88	18,81	40	2h 28м
	flight 2	25	21,25	22,35	40	2h 42м
	flight 3	26	21,57	22,68	36	2h 31м
	flight 4	28	22,20	23,35	40	2h 46м
Sum	-	93,40	82,90	87,20	156	10h 27м
GAZ	flight 1	11,2	9,09	9,56	16	1h 12м
	flight 2	26,5	13,59	14,29	16	1h 32м
	flight 3	19,75	11,61	12,20	16	1h 23м
	flight 4	30	14,62	15,37	16	1h 36м
Sum	-	87,45	48,91	51,42	64	5h 43м
In total	-	180,85	131,81	138,62	220	-



a) b)

Fig. 3. Fragments of optimal routes of GAZ flight 4 - a), KAMAZ flight 1 - b)

CONCLUSIONS

The examination of the effectiveness of work of GIACS MSW was held for MUC, attended one of the cities of Ukraine with the population of about 40 thousand people. Using GIACS MSW for a given number of containers and predicted intensity of their filling allowed: to justify the duration of the interval of planning and minimum quantity of RCV; to develop effectively detailed traffic routes of RCV providing collection and removal of all of the filled containers with MSW for each day of the planned period, which are: implemented, i.e. they can be carried by suitable RCV; minimum according to the length among all possible routes passed through the specified addresses of the container platforms; optimal in the number of loaded containers in each RCV on each route, i.e. the lifting capacity of each RCV is used to the maximum; optimal to the fuel costs for RCV.

Developed detailed traffic routes of RCV exceeded in all indexes traffic routes of RCV providing collection and removal of all of the containers with MSW actually used by MUC. Practical implementation of the developed plan of realization of the detailed routes for the removal of MSW allowed to reduce considerably the necessary park of RCV (from three to two), to reduce significantly the total length of the routes carried by each RCV and to reduce the actual fuel costs for the planned period of one month by 35%.

General conclusion. Implementation of GIACS MSW is an effective means to improve the environmental safety, energy and resource saving of MUC.

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STRUCTURAL METHOD OF DESCRIBING THE TEXTURE IMAGES

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Abstract. This article describes the histograms and polarograms obtained from the different types of textures using BRVAL filters. Comparative analysis of the polarograms and histograms showed that BRVAL filters description for textures in a wide range of distances and the light does not depend by the researched factors, and the level of detail segments on polarograms are inversely proportional to the increasing the number of absolute zero tending picture elements.

Key words: texture, polarogram, histogram, filter, pattern recognition, the two-dimensional convolution.

INTRODUCTION AND OBJECTIVE

Pattern recognition problems [15, 19] are very diverse as to their belonging to a particular domain [2, 11, 14] and the approach used to analyze them, for simplifying a partial or complete solution [13, 19], so and in features used mathematical methods [12, 17, 20], software and hardware tools.

An important role in successfully solving any problems related to the pattern recognition [2, 3, 11] are knowledge about features used (analyzed) the initial information, the preparation conditions and the presentation formats.

The complex structure of space observations cannot effectively solve the data analysis problem directly by the spectral features. Spectral objects portraits of the earth's surface are transient, because they depend on many factors, such as topography, soil type, climate, geographical position of the area. To increase the reliability of the accepted decisions we need to use a priori information about the shooting geometry, on one side, and context information from images - on another.

Observation context information is expressed in the spatial organization of objects elements and their boundaries. Comprehension of the task context, that is, the limitations that imposed on the mutual relationship between the components of the image and increases the efficiency of decision rules. The simplest form of contextual information for a pixel on the image is the neighborhood of the pixel. In [14] proved the assertion that the object decision rule when the fragment is taken, as a whole is effectively then pixel decision rule.

Another form of contextual information is the concept of texture. An **image texture** is a set of metrics calculated in image processing designed to quantify the perceived texture of an image. Image Texture gives us information about the spatial arrangement of color or intensities in an image or selected image region.

There are two ways to analyze an image texture in computer graphics: Structured Approach and Statistical Approach [4].

A structured approach analyzes an image texture as a set of primitive texels in some regular or repeated pattern.

A statistical approach analyzes an image texture as a quantitative measure of the arrangement intensities in a region. In general, this approach is easier to compute and is more widely used, since natural textures are made of irregular patterns sub elements [18].

The preference textural features are potential opportunities to aggregate context information of this type with certain invariance properties for a specific task of pattern recognition [1, 6 - 10].

One of the tasks in texture analysis is precise texture definition. To do this, need to determine the dependence of the textures description on the following factors:

1) distance (the distance to the analyzed object),

2) direction (the angle of observation of the object),

3) lighting (relative to the time of shooting information, namely, day, evening, etc.),

4) nature (species) texture.

There are three main types of texture:

a) texture with regular repetition of elements (masonry, mosaic),

b) texture with a random distribution of elements (gravel, hay),

c) image texture of polygonal objects (aerial photos of forests, fields).

In this article we focused on the structural description of the images and the experimental study of the texture characteristics, depending on these factors.

DESCRIPTION OF TEXTURE IN THE IMAGE

As the original textures pictures, in order to compare with the results which described in article [18], were selected (bricks, mosaic, hay, slate, etc.) and added new ones. All of these pictures (Fig. 1) were made with different distances (10m, 20m), with different angles of fixing the image (45°, 90°, 135°), with different lighting (morning, evening, afternoon, sunny, cloudy). The size of the texture is the same - 640×354 pixels.

For the experiment were selected large values filters (BRVAL - The big range of values) [3], which mask size is $N\times N$ pixels. The set of characteristic masks forms a kind of alphabet. The choice of the alphabet was determined by the following considerations: separate point of the image cannot held significant information, the information contains in the distribution of light location fragment relative to its central element. Filter is a square matrix with size $(3\times3,\ 5\times5,\ \ldots n\times n)$, consisting of zeros and ones elements, each letter of the alphabet can be obtained through the use of logic operation "or" to the other two letters (filters) of the same alphabet. The choice of the values "0" and "1" due to necessity of limiting the number of filters and process calculations optimization. The choice of filters and sequences of their use caused by possible changes for the scanned image at a predetermined pitch.

An example of possible mask filters (the alphabet) in the case of 3*3 matrix shown in Fig. 2. There are the basic concepts that will be used in the future. The letter - a simple indivisible character (some non-derivative element), that presents as an element of selected alphabet. The only limitation is a finite number of symbols of the alphabet.

The set of all words over the alphabet A is called the closure and is denoted by A^* [16].

$$A_i^* = ((((a_{i,j}^0 \cup a_{i,j+1}^{k_{z1}}) \cup a_{i,j+2}^{k_{z2}}) \cup a_{i,j+3}^{k_{z3}}) \cup \ldots \cup a_{i,j+n-1}^{k_{zn-1}}) \cup a_{i,j+n}^{k_{zn}} = \bigcup_{k_{zn=0}}^{zn=\infty} a_i^n$$

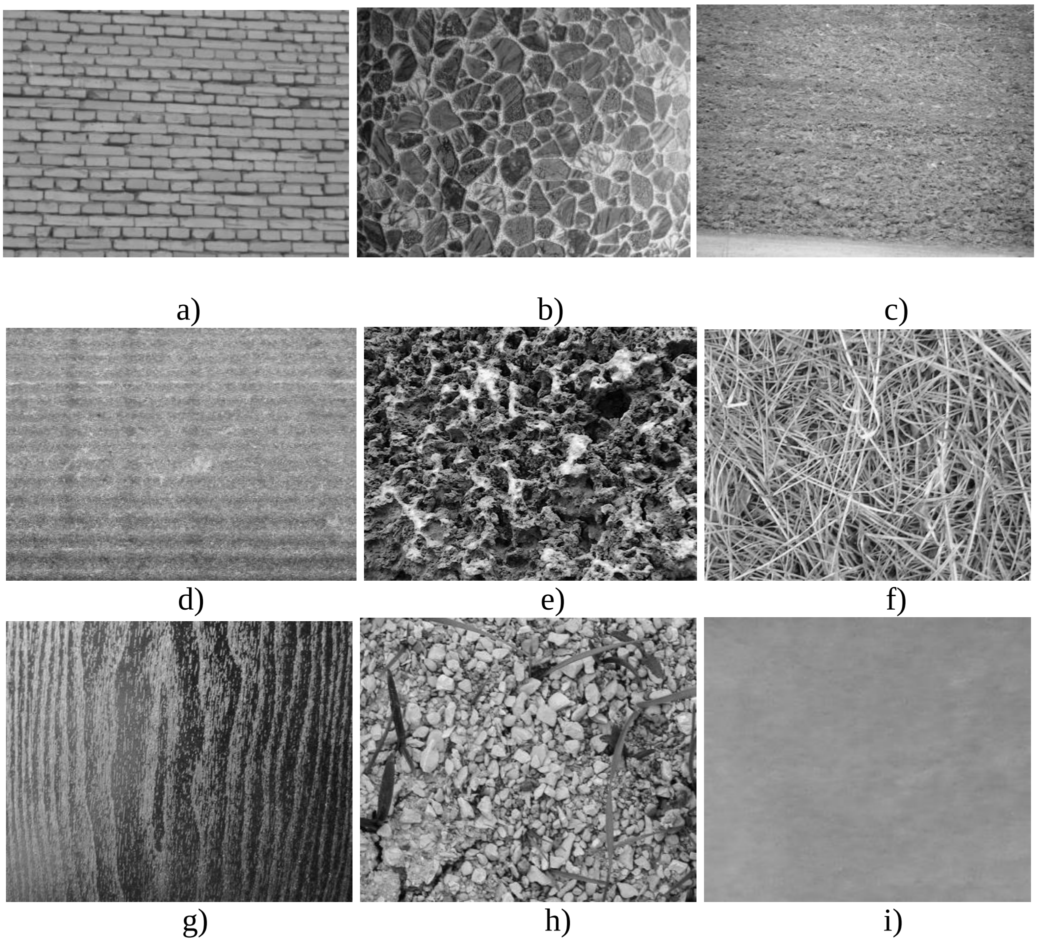


Fig. 1. Photos of different textures: a) a brick wall, b) mosaic,a) arable land, g) slate, d) coquina, e) hay, g) slice of wood, h) gravel, i) walls – plaster

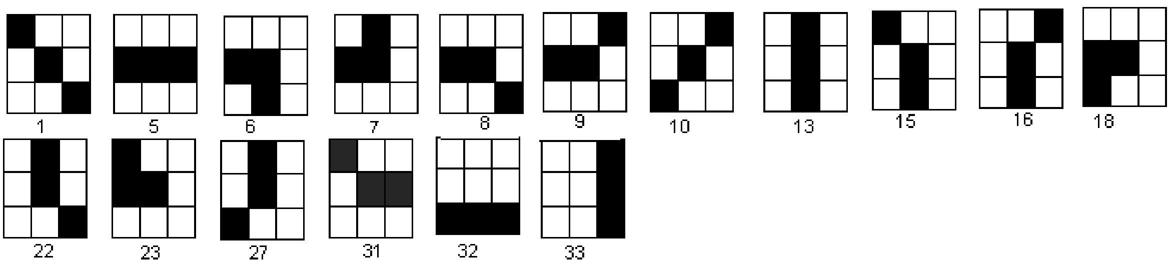


Fig. 2. Example of possible mask filters (the alphabet)

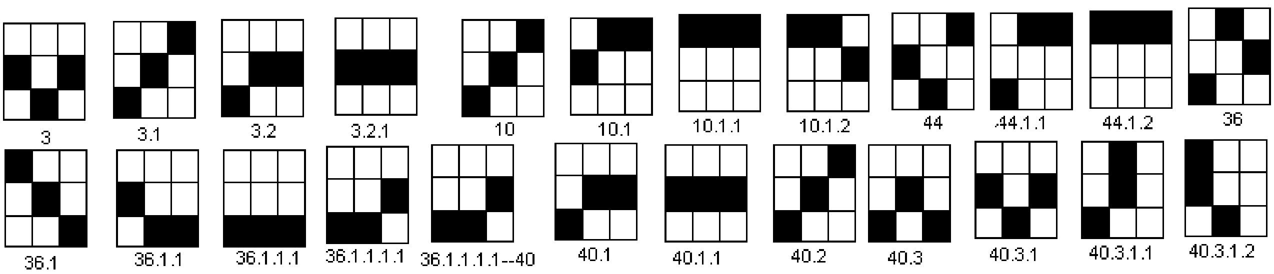


Fig. 3. Example of possible sequences of transformations

As in ordinary languages, the basic operations over words (strings) is a concatenation. Formally, it can be defined as a binary operation Θ on the set of elements of words as follows:

$$\Theta : (a^{k_{z1}}, a^{k_{zi}}) \xrightarrow{p_i} a^{k_{z1}} a^{k_{zi}} ,$$

where: p_i the probability of transition k_{z1} into k_{zi}.

The result of the merger of alphabet letters (words) in the following procedure: the letter $a^{k_{z2}}$ imposed on off-center letter $a^{k_{z1}}$ with some step. Fig. 3 shows a graphic illustration of the obtained results.

It should be noted that the procedure described above assumes the linear independence of the rows that for the image is not true (Fig. 4 illustrates this).

$$A^*_{ij} = \bigcup_j \bigcup_{k_{zn}=0}^{k_{zn}=\infty} a_i^n \quad j$$

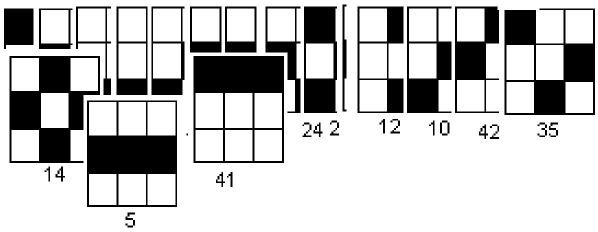


Fig. 4 - Example of a possible transformation of the alphabet letters

The processed image is discretized monochrome images that digitally describes the matrix

$$B = \lfloor b_{ij} \rfloor , \; i \in I , \; j \in J .$$

The value of the image b_{ij} (i - row number, j - column number) is quantized at the K level of the brightness value represented by the nodes of the sensor or display device and additive noise superimposed on it. Image processing can be represented as a transformation of one set of strings consisting of symbols - the brightness values in the other set.

The initial matrix B being transformed and as a result of which we obtain a new matrix $G = \lfloor g_{ij} \rfloor$ with sizes $(I-1)\times(J-1)$, each element of which is the result of a convolution of a local fragment with sizes $L \times K$ with a set of masks - alphabet BRVAL.

DIGITAL TWO-DIMENSIONAL CONVOLUTION OPERATION

Mathematically, the two-dimensional convolution can be described as follows:

$$g^{a_z}_{i,j} = \sum_{m=-M}^{M} \sum_{n=-N}^{N} b_{i-m,j-n} \omega^{a_z}_{m,n} \qquad (1)$$

where: $B = [b_{i,j}]$ - the original image, wherein the brightness values is representted in the sensor nodes or the forming device (display) images and additive noise imposed on it, $G = [g^a_{i,j}]$ - code representtation of the image dimension $(I-1)\times(J-1)$, the dimension of weights matrix $W = [\omega^{a_z}_{m,n}]$, is $(m,n) \in (2*M+1)(2*N+1)$, a - the set of filters masks, z- filter number (letter).

The process of classifying and identifying objects in the image, based on the above described approach which is consist of finding a statistically significant chains of alphabet letters (words).

In the simplest case, identification can be obtained by analyzing the histogram of features derived from the filters BRVAL.

Histogram features fragment image with size $I \times J$ - $H_{IJ} = \{h_{ij}(a)\}$- we will be called the empirical probability distribution of the filter response:

$$h_{ij}(a) = P\{g_{ij} = a \mid g_{ij} \in W_{mn}\}; \quad \sum_{a=0}^{A} h_{ij} = 1.$$

TEXTURE COMPARISON ON THE IMAGE

Often textures are present in the images of natural scenes, containing both natural and man-made objects. Bricks, tiles, pebbles and many other objects form the texture image content [18].

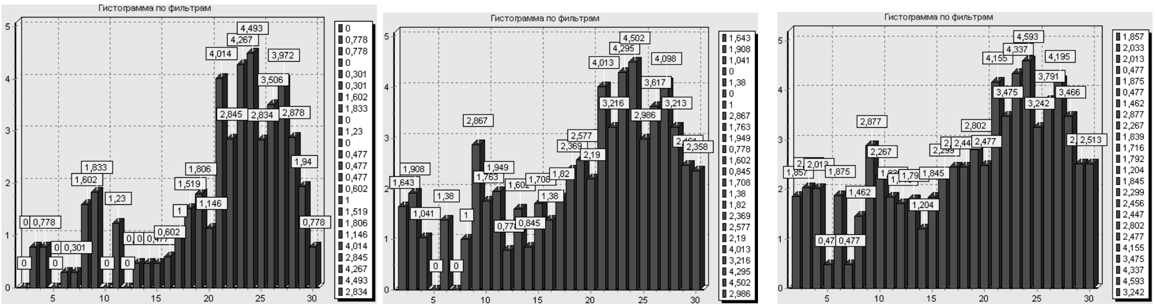
Consider a brick wall photos, taken from different angles in the evening and at a distance of 10 meters. The results of research we will be displayed in the form of histograms and polarograms. Under the polarograms we will understand the histogram constructed in the polar coordinate system on which the data are divided into groups and form a local coordinate system for each group.

After the experiment we obtained the histograms (Fig. 6) and polarograms (Fig. 7) which showing the distribution of the values features for different textures. From this examples we can see changes on polarograms, depending on the direction of image registration, but these differences are not essential to the assigned in this article task of identification texture. In addition, we observe amplitude change, but not the whole histogram generally (Fig. 8 - comparison textural features on the polarograms with the help of the brick wall photos which were made from the left and right side in the evening and on the different distance).

Similar results were obtained for different textures, such as pebbles, mosaics, hay, slate that are presented below (Fig. 9-17). They show the constancy of the global distribution of the filter values response in researched textures.

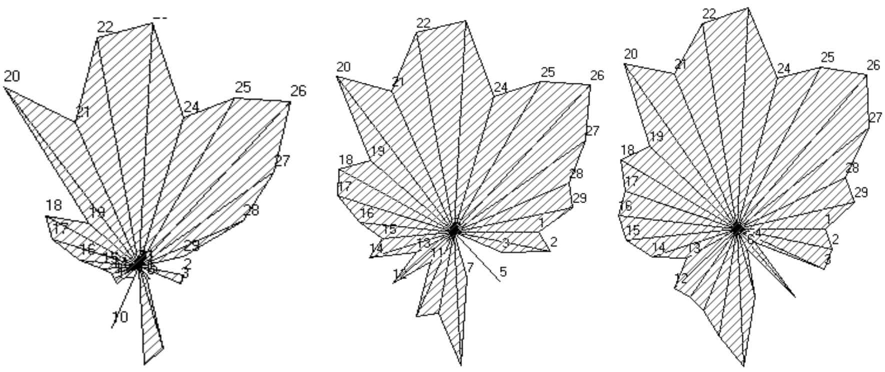
a) b) c)

Fig. 5. Photo of a brick wall in the evening at distance of 10 meters from different directions: a) from the left side, b) from the right side, c) straight



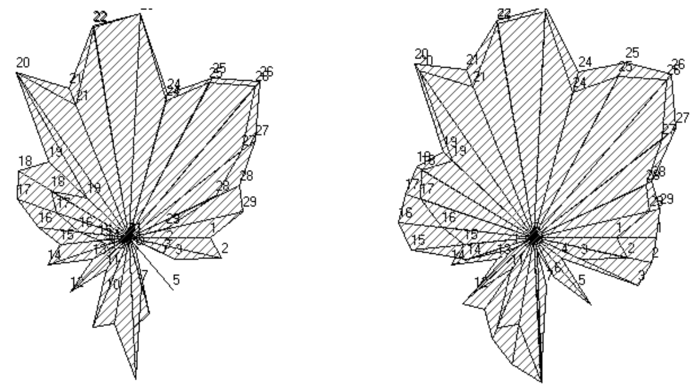
a) b) c)

Fig. 6. Histograms of a brick wall in the evening at distance of 10 meters from different directions: a) from the left side, b) from the right side, c) straight



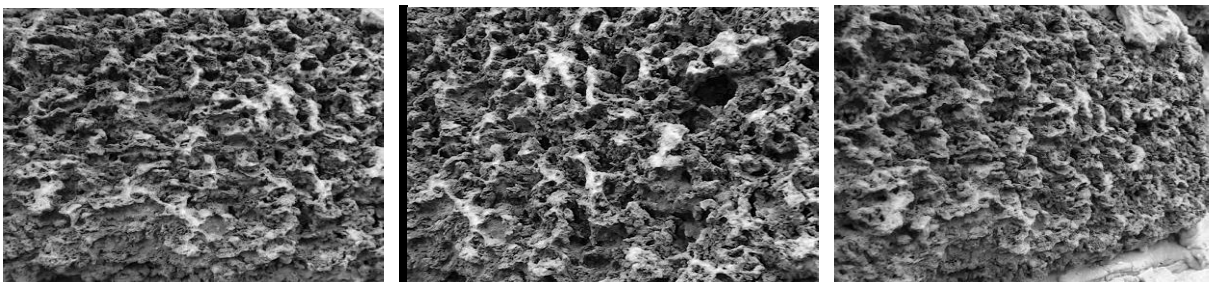
a) b) c)

Fig. 7. Polarograms of a brick wall in the evening at distance of 10 meters from different directions: a) from the left side, b) from the right side, c) straight



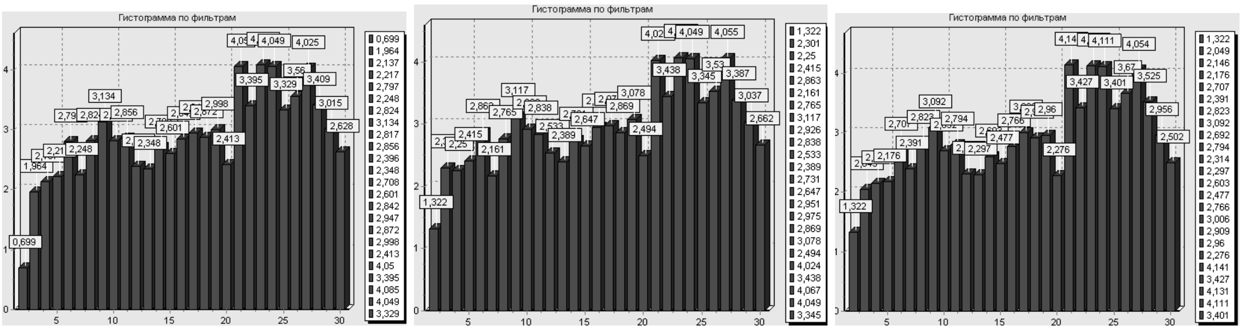
a) b)

Fig. 8. Comparison textural features on the polarograms wall photos from the left and right side on the different distance: a) 10 meters, b) 20 meters



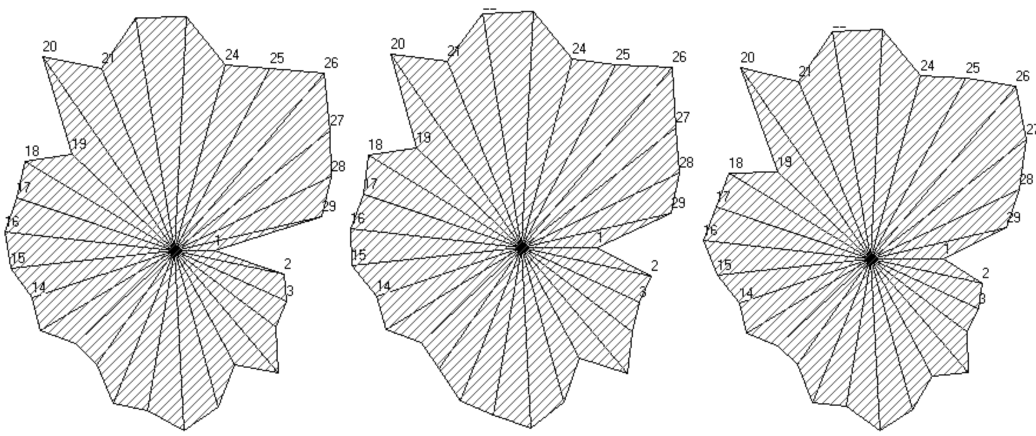
a) b) c)

Fig. 9. Photo of the shell rock: a) straight, b) from the right side, c) from the left side



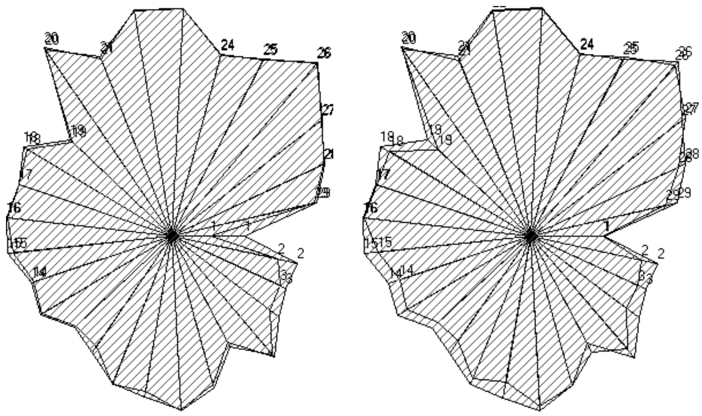
a) b) c)

Fig. 10. Histograms of the shell rock: a) straight, b) from the right side, c) from the left side



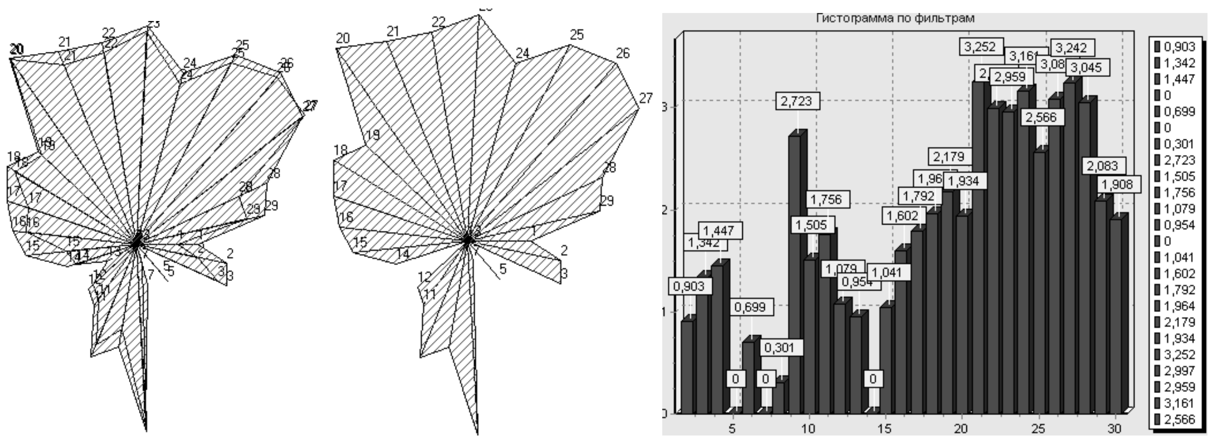
a) b) c)

Fig. 11. Polarograms of the shell rock: a) straight, b) from the right side, c) from the left side



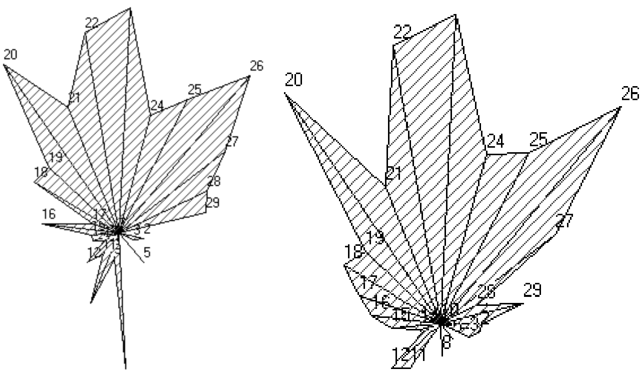
a) b)

Fig. 12. Comparison the textural features on the polarograms with the help of the shell rock photos, which were made from the different sides: a) straight and left, b) left and right



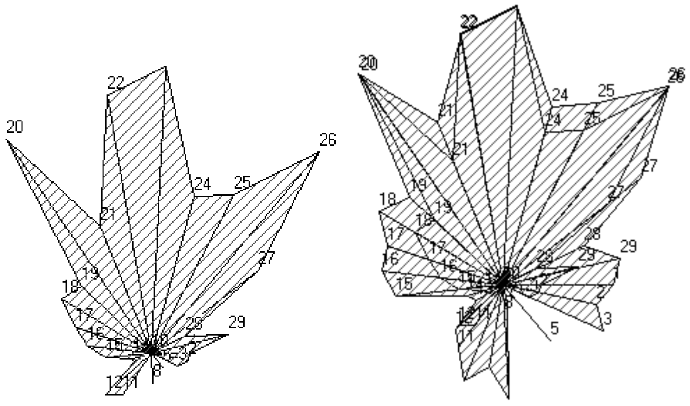
a) b) c)

Fig. 13. Comparison the textural features on the polarograms with the help of the brick wall photos: a) the straight wall and the wall at a distance of 20 meters and at an angle of 45 °, b) wall at a distance of 20 meters and at an angle of 45 °, a) histogram of the brick wall texture



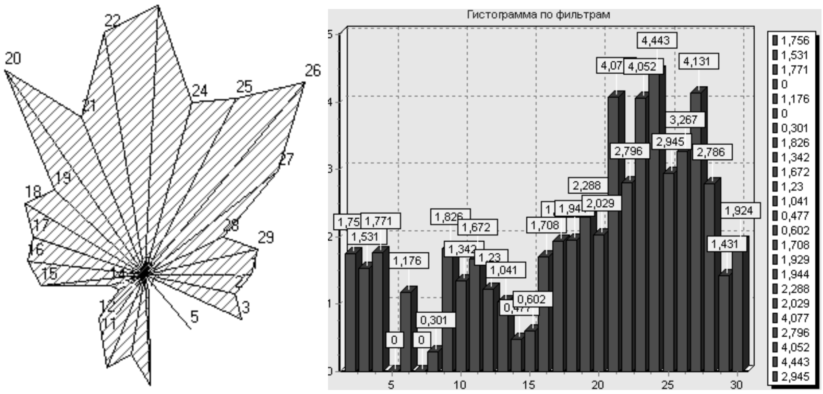
a) b)

Fig. 14. Polarograms of the wall and smooth concrete textures at a distance of 10 meters and at an angle of 45 °: a) wall, b) smooth concrete



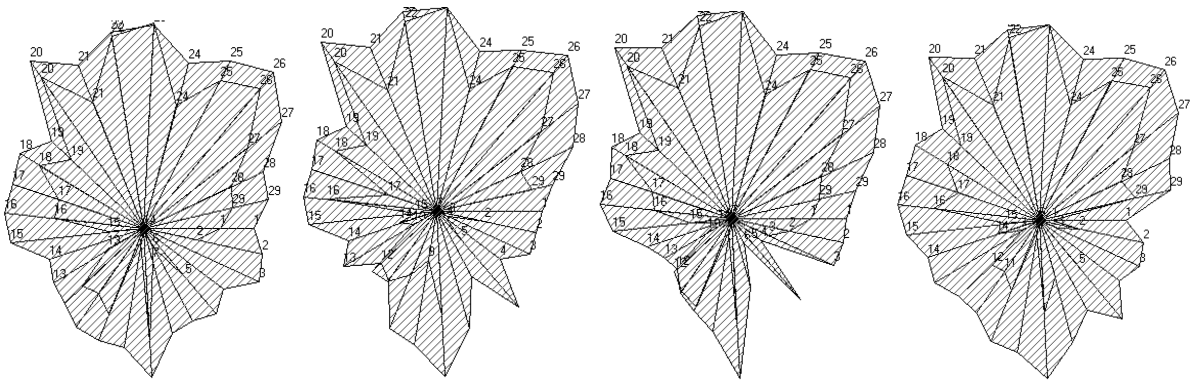
a) б)

Fig. 15. Comparison of the textural features on polarograms wall and smooth concrete textures: a) wall at a distance of 10 meters and at an angle of 45°; the concrete at a distance of 10 meters and at the angle of 90 °, b) smooth concrete and wall at a distance of 10 meters and at the angle of 90°



a) б)

Fig. 16. Comparison of the textural features with the help of the wall and smooth concrete textures at a distance of 10 meters and at an angle of 135 °: a) on the polarogram, b) on the histogram



a) b) c) d)

Fig. 17. Comparison the textural features of the brick wall texture on the polarograms with: a) the slate, b) shell rock, c) mosaics, d) plowed land

The analysis of the experiments shows that for the BRVAL filters describe texture in a wide range of distances and lighting independent of researched factors. Thus, we can say that under a texture image we will understand the set of points (distribution of a random process) image, between which there is a statistically significant association.

Of course, at distances when information structural elements of the image are merged, distribution statistics will be violated. It concerns also the field angle, but such experiments are not the purpose of this work.

The most important in this experiment is the mutual comparison of the research textures. Various textures, as seen from the results of experiments, are essentially differ from each other, which is the indication of their recognizability.

As can be seen from the experimental results observed the following trend: the more fine detail, the less visible changes on the polarographic curves.

CONCLUSION

There were investigated different types of textures and analyzed a number of texture images using polarograms and histograms. The algorithm description of texture images using textural elements presented in the form of the alphabet distribution filters that allow us to describe the texture images in a wide range of invariant factors: distance, viewing angle and lighting conditions. It was given the definition of texture based on the proposed algorithm.

The results obtained in this paper provide a basis for further research in identifying texture of the images, including analysis of the position, direction and speed of objects in time.

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FUZZY MODELS PRESENTATION AND REALIZATION BY MEANS OF RELATIONAL SYSTEMS

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Abstract. The feature of the research is the proposed fuzzy models representation method by means of relational systems, which unlike the known approaches, can solve integrated data mining problems in databases and fuzzy systems.

Key words: fuzzy set, fuzzy logic, fuzzy relational databases, intelligent systems, data mining.

INTRODUCTION

Computer technologies of intelligent algorithms are experiencing their heyday. This is due to the flow of new ideas coming from computer science, which was formed at the intersection of artificial intelligence, statistics and database theory [5]. The traditional mathematical statistics has long claimed to be the main tool for data analysis, but now because of the heterogeneity of the source data, disruption of traditional statistical assumptions, substantial prior uncertainty, etc. it was not efficient enough to process information in many applied problems.

However, the methods of mathematical statistics were useful mainly for testing pre-formulated hypotheses (verification-driven data mining). Methods of multivariate data mining designed to find patterns in multivariate data (or sequence of one-dimensional data). These include: visualization of data sets, cluster analysis, factor analysis, multidimensional scaling, Log-Linear analysis, canonical correlation, stepwise linear and nonlinear regression, correspondence analysis, time series analysis, classification trees and etc. The areas of application of these methods are the various areas of research: technological data, medicine, biomedicine, metallurgy, chemistry, molecular genetics, and others.

Modern technology intellectual Data Mining (discovery-driven data mining) is the process of extracting previously unknown, nontrivial, practically useful and interpretable knowledge from the "raw" unstructured data in large arrays or databases. In general, the problem being solved by Data Mining consist from finding patterns in data of different nature. The composition of the most common problems are such as forecasting, the designing of mathematical models, classification, clustering, rule generation, aggregation, reduction of dimension, visualization of data [4].

One of the areas of Data Mining is an exploratory data analysis, which is used to find systematic relationships between variables in situations where there are no (or are inadequate) a priori ideas about the nature of these connections [2].

Usually, the exploratory analysis takes into account and compared a large number of variables, and for the search of patterns variety of methods is used.

Many scientists research the problem of designing fuzzy databases, particularly on the basis of the relational model. Various versions of fuzzy relational database models have been designed. The results of the surveys, can be found in works of next authors: Buckles B.P., Petty F.E., Shenoi S., Melton A., Vila M.A., Lipski W. Jr., Prade H.

Systems built by combining the databases and fuzzy logic can significantly extend the functionality and range of tasks of data mining. The theory of fuzzy databases is not yet complete from a mathematical point of view, and there are still many issues that require resolution.

DEVELOPMENT OF A RELATIONAL MODEL REPRESENTATION AND STORAGE OF FUZZY DATA

Intelligent technologies are becoming an integral part of modern decision support systems, and the relevant question becomes - what requirements must meet the database to the data and structures in which they are stored to be as focused on methods of processing these data. For example, let's consider the task of designing an integrated intelligence system which combines a relational database technology and fuzzy logic. Systems built by combining the databases and fuzzy logic, can significantly extend the functionality and range of tasks of data mining [6].

To solve the problems of storing fuzzy data we will define a specialized type of relation. The scheme of such relation must meet two conditions: compatibility with the requirements of the classical relational data model and effective storage and representation of linguistic variable model [7]. Now let’s look into it’s graphical representation (Fig.1).

Any line on a coordinate plane can be represented as a binary relation, where Dom R is represented by the values of the x-axis, and Im R - the y-axis.

There are three indicators in the fuzzification problem that should be considered when forming relation.

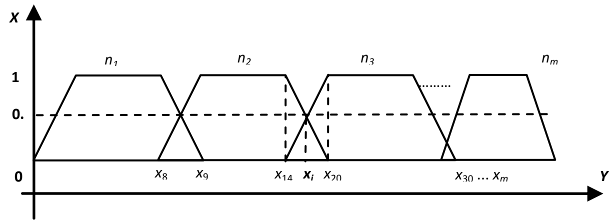

Fig.1. Graph of the values of linguistic variable

Define the fuzzy variable as a set (N, X, Y), where N - is the name of a variable, X - the area of research, Y - a fuzzy set on X. Using this definition, we define the three domains corresponding to elements of the variable.

Assume that:

$N = \{n_1, \dots, n_m\}$,

$Y = \{0, 0.1, \dots, 1\}$,

$X = \{x_0, \dots, x_k\}$.

The X and Y correspond to the selected scale sampling of the coordinate axes and represent the region belonging to the parameter N. For this particular case we define the respective domains in order to present the values of fuzzy variable [3]:

$D_1 = \{n_1, n_2, n_3, \dots, n_m\}$,

$D_2 = \{x_0, \dots, x_8, x_9, \dots, x_{14}, \dots, x_i, \dots, x_{20}, \dots, x_{30}, \dots, x_m\}$,

$D_3 = \{0, 0.1, \dots, 1\}$.

We define a set of domain names and mapping to formulate a set of attributes: for a set of names $A = \{A_1, A_2, A_3\}$, - mapping $\rho : (A_1 \to D_1; A_2 \to D_2; A_3 \to D_3)$ determines the set attributes $A = \{A_1, A_2, A_3\}$ (for example Fig.2), which corresponds to the relation scheme:

$$S(A_1, A_2, A_3).$$

Thus, in general, we can talk about a universal relation, which includes the full set of tuple resulted by domains of Cartesian product $D_1 \times D_2 \times D_3$ [1].

A_1	A_2	A_3
n_2	x_{14}	0
n_3	x_{14}	1
n_2	x_i	0.9
n_3	x_i	0.9
n_2	x_{20}	1
n_3	x_{20}	0
...	...	…

Fig.2. Fragment of the values of linguistic variable

On this basis, we can conclude that the key of such relation will be a set of all attributes:

$$K = \{A_1, A_2, A_3\}.$$

Obviously, the informative of tuples is defined by the values in a chart of fazzification. Based on the conditions of the problem, must take into account another parameter - the set of values of sample from the database, for which diagram was made. That is, it is necessary to establish relation between the corresponding domains of fazzification relation - R^f and integrated into the database.

DEVELOPING A MODEL OF INTEGRATION FUZZY RELATION AND RELATIONAL DATABASE

Let's look at the problem in general. Let U $(R_1,\ldots, R_n)$ – database that stores basic data about the studied domain, $R^f(A_1, A_2, A_3)$ – fuzzy relation. The problem will make sense if there is an attribute in U database, fuzzification is based on.

In order to provide simultaneous work of two databases U and $R;$ we need to formalize the process of integration, based on the gradual normalization technique. The structure of U relation is based on basic functional dependencies

$$F = \{M_i \to N_i\}$$

where: $M_i, N_i \in U$.

Now select one of the dependencies, which includes an attribute with fuzzy parameters, and define it as a $W \to V$, where W and V, in global case, may be sets. Relation R^f contains one dependence:

$$F' = \{A_1, A_2, A_3 \to A_1, A_2, A_3\}.$$

Now we can obtain an equivalent set:

$$F' = \{A_1, A_2, A_3 \to A_1; A_1, A_2, A_3 \to A_2; \\ A_1, A_2, A_3 \to A_3\}$$

using output axioms.

Let's assume that the parameter corresponds to the fuzzy attribute A_2, then to determine the type of communication we need to get a set $F = F \cup F'$ and study two cases affecting the rules of normalization.

1. $A_2 \in W$ - search for parent relations: if functional relations and $\xi \to \zeta$, and $\omega \to \zeta$ take place - then the dependence of $\omega \to \zeta$ is incomplete.

2. $A_2 \in V$ - search for transitively dependent elements: if the functional relations $\xi \to \omega$ and $\omega \to \zeta$ take place - then an element ζ is transitive dependent. The existence of such relations will allow to make correct decomposition and to establish connection between U and R^f databases. If $A_2 = W$ or $A_2 = V$, then the process of decomposition leads to the second or third normal form. If equation is not satisfied, then it's impossible to organize support of connected data uniqueness, because the association between relations will be "many-to-many".

Usually, in practice, the condition of equality is not meets, and for the normalization is necessary to allocate basis F and repeat the process of decomposition. Considering the fact that the structure of the database should not be changed, fazzification relation R^f and U must be linked without restructuring the data schema. Using a diagram of the model "entity-relationship", we represent R^f and U as the main entities.

Thus, in general to analyze data that's stored in relational databases, it is enough to build a fuzzy relation and establish a connection with the attribute (attributes) whose values must be properly analyzed.

To destroy one-to-many relationship «N:M» let's introduce more cohesive entity that will solve the problem of supporting the integrity of data by defining new types of relationships. The entity relationship will contain one attribute - binding for R^f and U, for objective reasons, it will be the key. Fig. 3 shows a diagram of integration explored database and the fazzification relationship supporting the uniqueness of relation.

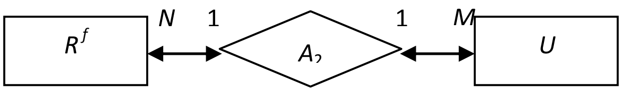

Fig. 3. Graph “entity-relationship"

Based on the description of a conceptual scheme that the correct connections for R^f and U needs to design an intermediate relation. This approach ensures data consistency for all types of fazzification settings.

We show that for this problem is quite correct results when the join relations with the association of type «N:M». Possible values of attribute $A_1 \in U$ can be repeated as many times as given value crosses the boundaries of the Y-axis on fazzification graph. That is, each A_1 attribute value corresponds to the line of unique data. If A_1 is not the key, and the values are repeated, then, by definition of sets, in a row must be at least one different value. In terms of the problem being solved it is needed to analyze all these lines. A_1 attribute's value relation R^f may also be repeated, at all sorts of combinations.

Thus, in a general way, to analyze the data accumulated in a relational database, is enough to design a fazzificationed relation and establish relation with the attribute (attributes) by values of which is needed to be conduct an appropriate analysis/

CONCLUSIONS

Based on analysis of relational data model’s features we described the main problems of building integrated information systems that combine the database as a source of primary data mining tools that enable to extract knowledge from data. This article proposes an efficient method for designing relational data model schema to represent the membership functions of linguistic variables. The proposed approach describes data analysis using fuzzy queries.

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Table of contents

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