MODEL OF TURBO-CHARGING WITH PULSE SUPPLY OF TURBINE

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Summary. This paper will present issues related to the modelling of a turbo-charging system of traction engines. The method of analytical description of wave phenomena in the outlet system of an engine has been presented, based on the basic equations of thermodynamics and gasodynamics, the solution of which enables determination of the pressure and temperature curves in the manifold with taking conditions of charge flow through the engine valves into account.

Keys words: diesel engine, modeling, turbo-charging.

INTRODUCTION

Mathematical modelling, despite numerous limitations resulting from difficulties in analytical description of a series of phenomena and insufficient experimental knowledge forcing for application of many simplifications in models, allows for provision of a series of information as to the curve of operational cycle of a combustion engine. Particular difficulties take place in case of modelling of a motor vehicle engine with turbo-charging. This results from the gas connection between the engine with pulsating flow and the turbocharger with continuous flow. It is required to apply the mathematical description not only of the engine but also of the turbocharger and the turbine. In such case, the simplest way is to formulate a mathematical model through determination of function dependence by approximating the characteristics of the engine and of the turbocharger with polynomials of greater degrees, with making use of the multiple regression [Ćwik, Szczeciński 1993, Danilecki 2006, Данилов, Руденко 1979, Wisłocki 1986]. The coefficients of polynomials may be determined on the basis of digitisation of the characteristics being at disposal or by making use of the approximated dependences obtained through identification methods during engine testing. Functions obtained in such a way may be directly used for calculations of the cycle parameters. They provide a satisfactory image of the engine operation quality with specified adjustment in steady states.

In order to obtain the required accuracy of calculations in the engines with pulse turbine, the influence of the stream pulsation on the power and on the efficiency of the turbine is to be taken into account. The increase of the turbine power in the pulsating stream of exhaust gases may be taken into account by means of pulsation coefficient determined on the basis of identification tests [Wislocki 1986, Wanscheidt et. al. 1977].
ASSUMPTIONS FOR TURBO-CHARGING MODEL

On the basis of the assumptions presented above, for simulation and assessment of operation of an engine with supercharging, an empirical-mathematical model of the engine-turbo-charging unit has been elaborated, based on average parameters of the engine cycle [Danilecki 2007 a]. The calculations carried out, aiming at verification of the model show that the prepared model of a turbo-charging system based on average cycle parameters allows for calculation of conditions of the engine co-operation with a turbo-charging unit with sufficient accuracy. However, suitability of such model is limited only to the engine of specified constructional and adjustment parameters, for which values of identification coefficients have been determined. Taking the above mentioned limitations into account, assumptions for a turbo-charging model have been worked out where a series of essential phenomena have been taken into account, and that significantly influence the conditions of the engine co-operation with a turbocharger. The basis of the turbo-charging analytical model is the assumption of quasi-stationary flow of charge in the inlet system, in the cylinder and in the outlet system of an engine, and it comes down to the determination of the pressure and temperature curves of exhaust gases in the outlet manifold on the basis of a calculated energy balance. At the same time the existing works and achievements are used, by referring to the qualitative and quantitative descriptions of phenomena that accompany turbo-charging, however supplemented with proposals of analytical approach to certain issues. The analysis covers heat exchange, cylinder scavenging and backflows through the valves, with taking the timing angles and geometrical parameters of the outlet system of the engine as well as the pulsating character of the turbine supply.

DESCRIPTION OF TURBO-CHARGING MODEL EQUATIONS

The basis for calculations of the turbo-charging system that allows for determination of the pressure and temperature curves in the outlet manifold is the analysis of phenomena and proposal of their analytical description proposed in [Симсон 1964]. It is based on the system of differential equations of the volumetric balance presented in the paper [Глаголев 1950]. Momentary pressure changes $dp$ and temperature changes $dT$ of the charge in the outlet manifold with the volume of $V$, can be presented by means of system of equations in the form of:

\[
\frac{dp}{V} = \frac{k_s}{V_s} \left( \dot{\hat{V}} - \dot{\hat{V}}_{in} + \dot{\hat{V}}_{out} \right),
\]

\[
\frac{dT}{V} = T_s \left( \frac{V \cdot dp}{p} + \dot{\hat{V}} - \dot{\hat{V}}_{in} \right).
\]

In the equations (1) and (2), the particular expressions specify the pressure and temperature changes in the manifold related to: $\dot{\hat{V}}_{in}$ – change of the volume of charge flowing from the cylinder to the manifold, $\dot{\hat{V}}_{out}$ – change of the volume of charge flowing from the manifold to the turbine at the pressure $p$ and the temperature $T$, $\dot{\hat{V}}_{in}$ – related to the change of the volume as the result of heat exchange with the outlet system walls, $p$ and $T$ – the pressure and the temperature of exhaust gases in the outlet manifold, identical within the whole volume but changing in time.

Change of the exhaust gases volume in the outlet manifold, as the result of heat exchange can be determined on the basis of the energy balance and the equation of state after transforming it into the form of:

\[
\dot{\hat{V}}_{out} = \frac{R \cdot \dot{\hat{V}}_{in}}{p \cdot m \cdot c_p}.
\]
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In the dependence (3) \( \frac{\partial Q}{\partial x} \) includes heat lost in the outlet pass of the head \( \frac{\partial Q_1}{\partial x} \), and the inlet box of the turbine \( \frac{\partial Q_2}{\partial x} \).

The heat \( \frac{\partial Q_1}{\partial x}, \frac{\partial Q_2}{\partial x} \) given up by the exhaust gases with the temperature of \( T_i \) to the walls with the temperature of \( T_{st} \) can be calculated from the known Newton formula in the general form of:

\[
\frac{\partial Q}{\partial x} = \alpha_i \cdot F_i \cdot \left( T_i - T_{st} \right) \frac{d\alpha}{6 \cdot n}.
\]  (4)

The heat transfer coefficient for a known velocity of flow of charge \( c_i \) in relation to the pass wall with the thermal conductivity \( \lambda_i \) and the diameter \( d \) can be determined from the dependence [Wiśniewski 1988]:

\[
\alpha_i = 0.023 \cdot \frac{\lambda_i}{d} \cdot Re^{0.8} \cdot Pr^{0.3}.
\]  (5)

One may assume the Prandtl number \( Pr \) equal to 0.7. The Reynolds number \( Re \) is determined by the expression:

\[
Re = \rho_i \cdot c_i \cdot d / \eta_i.
\]  (6)

For calculations of the coefficient of dynamic viscosity of the charge in the outlet system, an empirical dependence can be applied [Budzik, Jaskólski 2004]:

\[
\eta = 3.3 \cdot 10^{-8} \cdot T_i^{0.7}.
\]  (7)

The wall conductivity can be calculated from the dependence [Budzik, Jaskólski 2004]:

\[
\lambda_{st} = 7.36 \cdot 10^{-5} \cdot T_i.
\]  (8)

Numerous authors [Cimrman 1964, Zawadzki J. 1982, Zinner K. 1985] say that the temperature of the charge in the outlet pass from the cylinder can be taken as equal to the temperature in the cylinder as the temperature drop during exhaust of gases as the result of partial decompression is compensated by the temperature increase at the kinetic energy exchange into heat. At the same time, the temperature of gas in the outlet pass can change within a significant temperature range – from the temperature close to the temperature in the cylinder at the beginning of exhaust gases outflow up to the temperature of the scavenging air at its final period. Heat exchange in the turbine inlet box takes place at practically constant temperature of inflowing exhaust gases. This results from mixing of gases flowing out from all the cylinders, which then supply the turbine. At the same time, the quantity of heat lost in the inlet box is usually lower than the quantity of heat lost in the outlet passes of particular cylinders. This allows for assuming that the heat exchange in the inlet box of the turbine will be coming down to the decrease of the exhaust gases temperature in the manifold supplying the turbine, by the constant value of \( AT \). The temperature drop in the manifold with the specific heat \( c_{p0} \) at the temperature of the turbine box walls \( T_w \) and the area \( F_w \) can be calculated on the basis of the Newton formula from the dependence:

\[
\Delta T = \frac{\alpha_w \cdot F_w \cdot \left( T_w - T_{st} \right)}{m_k \cdot c_{p0}}.
\]  (9)

The heat transfer coefficient in the turbine box \( \alpha_w \) can be calculated from the dependence (5). The mass of exhaust gases \( m_k \) taking up the volume of the manifold at the average pressure \( p_t \)
and the temperature $T_t$ can be calculated from the equation of state. Finally, taking into account the influence of the heat exchange in the inlet box of the turbine on the pressure changes in the outlet manifold will be coming down to the increase of volume taken up by the charge leaving the manifold and flowing into the turbine in relation $T_t + \Delta T_t/T_t$.

The heat exchange in the outlet pass of the head, similarly to the inlet box of the turbine, can be simplified to the determining the charge temperature drop in the outlet passes by the constant value of $\Delta T_2$ that – at the known temperature of the pass walls in the head $T_{gw}$ – can be calculated from the dependence:

$$\Delta T_2 = \frac{\alpha_g \cdot F_g \cdot (T_f - T_g)}{m_s \cdot c_{p(t)}}.$$ (10)

The heat transfer coefficient in the outlet pass of the head $\alpha_g$ can be calculated from the dependence (5). Including in the calculations the influence of heat exchange in the outlet passes of the head on the pressure variations in the manifold will be coming down to the reduction of the volume taken up by the charge flowing out from the cylinder to the outlet manifold in relation $T_t + \Delta T/T_t + \Delta T + \Delta T_t$.

The quantity of heat exchanged by the gas in the outlet pass of the head can be also determined approximately as a part of the heat delivered to the circuit together with fuel. According to [Глазов, 1950, Симсон, 1964, Zinner, 1985], the heat lost by gas constitutes approximately 2-4% of heat delivered together with fuel. In the paper [Zawadzki, 1982], the author shows a possibility to include the heat exchange in the outlet pass and in the inlet box of the turbine by means of so called cooling numbers, the values of which are 0.96 – 0.99.

The temperature $T_t$ can be determined on the basis of the energy balance in the cylinder and in the manifold during exhaust of gases:

$$dQ_{exf} = dQ_e + dQ_g + dQ_s + dQ_k.$$ (11)

The heat conveyed from the cylinder within the elementary interval of exhaust period is defined by the expression:

$$dQ_{exf} = c_p \cdot T \cdot dm_{exf},$$ (12)

where: $T$ – charge temperature in the cylinder.

The heat conveyed in the outlet manifold is defined by the expression:

$$dQ_e = c_{p(t)} \cdot T_t \cdot m_s.$$ (13)

The heat transferred through the walls of the outlet pass of the head $dQ_g$ and the inlet box of the turbine $dQ_s$, is defined by the expression (4). The quantity of heat corresponding to the kinetic energy as referred to an average speed of charge in the outlet manifold $c_{kch}$:

$$dQ_k = \frac{c_{k}}{2} \cdot m_s.$$ (14)

The total quantity of heat conveyed in the outlet manifold:

$$Q_e = \int c_{p(t)} \cdot T \cdot dm_{exf} - \alpha_g \cdot F_g \cdot (T_f - T_g) \cdot d\alpha =$$

$$- \alpha_s \cdot F_s \cdot (T_s - T_f) \cdot d\alpha - \frac{c_{k}}{2} \cdot m_s.$$ (15)
Mixing of gases flowing out from particular cylinders causes that the temperature of exhaust gases in the outlet manifold is practically constant. The results of calculations presented in [Simson 1964, Simson et al. 1979] show that for the taken assumptions, momentary changes of temperature in the manifold determined on the basis of the dependence (2) do not exceed 2-3% of the average value even in case, where the charge mass in the manifold exceeds the mass flowing out from the cylinder just twice. Therefore, the influence of momentary changes of the temperature on the pressure pulsations in the manifold can be neglected assuming that the heat exchange in the manifold takes place at constant average temperature.

Taking into account the taken simplifying assumptions related to the heat exchange in the outlet pass of the head and the inlet box of the turbine, the differential equation determining the pressure pulsations in the manifold can be presented in the form of:

\[
\frac{d\pi}{d\alpha} = \frac{\kappa_t \cdot p_t}{V_h} \left( \partial_{\alpha d} V \cdot \frac{T_r + \Delta T}{T_r + \Delta T + \Delta T_0} - \partial_{\alpha d} V \cdot \frac{T_r + \Delta T_0}{T_r} \right),
\]

(16)

The analytical model assumes use of the experimental characteristics of the turbine that determines dependence of the flow parameter \( F_p = \frac{G_t \cdot \sqrt{T_r^*}}{p^*} \) in the function of the expansion ratio \( \pi \) and the criterial rotational speed of the rotor \( n_{w, k} = n_w \sqrt{T_r^*} \):

\[
F_p = \frac{G_t \cdot \sqrt{T_r^*}}{p^*} = f\left( \pi, \frac{n_{w, k}}{T_r^*} \right),
\]

(17)

where: \( T_r^*, p^* \) – temperature and pressure of exhaust gases accumulation before turbine.

In such case, momentary changes of the charge volume flowing out from the manifold to the turbine \( \partial_{\alpha d} V \) for the elementary range of the rotation angle of a crankshaft \( d\alpha \) can be presented in the form of dependence:

\[
\partial_{\alpha d} V = V_t \cdot \frac{d\alpha}{6 \cdot n}.
\]

(18)

The volume of the \( V_t \) charge flowing through the turbine referred to \( p^*, T_r^* \), can be determined on the basis of the turbine characteristics for known speed of the turbocharger rotor \( n_w \) from the dependence:

\[
V_t = \frac{G_t}{p^*} = F_p \cdot R_t \cdot \sqrt{T_r^*}.
\]

(19)

If we neglect the heat exchange then the change of the volume of charge flowing out from the engine will also result from throttling of the charge flow through valves, leading to its decompression. At the same time, the flow conditions depend on the differential pressure in the spaces, through which the charge flows. It is therefore necessary to take the periods of supercritical and subcritical flows into account in the calculations.

Change of the pressure resulting from the change of the volume of charge flowing out from the cylinder to the manifold \( \partial_{\alpha d} V \) is defined by the dependence:

\[
\partial_{\alpha d} V = c_1 \cdot F_i \cdot \frac{d\alpha}{6 \cdot n}.
\]

(20)

The velocity of the charge \( c_1 \) flow in the place of the outlet pass of the head, where the stream fully fills the intersection of the pass \( F_i \) can be calculated from the equation of continuity. For the minimum intersection \( F_w \) of the stream flowing through the outlet valve and the intersection \( F_i \), filled with the charge:
For the period of supercritical flow, the equation of continuity (21) can be transformed into the form of:

\[
\frac{c_n \cdot F_n \cdot p_n}{T_n} = c_i \cdot \frac{F_i \cdot p_i}{T_i}.
\]  

(21)

By substituting (22) for (20) and assuming that the kinetic energy during flow is fully converted into heat, the following entry is possible:

\[
\partial_{\alpha} V = \mu f(\alpha)_{\text{ref}} \cdot \left[ R \cdot T \cdot \kappa_i \cdot \left( \frac{2}{\kappa_i + 1} \right)^{\frac{k_i+1}{\kappa_i-1}} \cdot \frac{p}{p_i} \cdot \frac{T_i + \Delta T_i}{T_i + \Delta T_i + \Delta T_f} - F_p \cdot R \cdot \sqrt{T_i^*} \right].
\]  

(22)

Pressure pulsations in the manifold can be therefore presented in the form of expression:

\[
\frac{dP}{d\alpha} = \frac{\kappa_i \cdot p_i}{6 \cdot n \cdot V_e} \left[ \mu f(\alpha)_{\text{ref}} \cdot \left[ R \cdot T \cdot \kappa_i \cdot \left( \frac{2}{\kappa_i + 1} \right)^{\frac{k_i+1}{\kappa_i-1}} \cdot \frac{p}{p_i} \cdot \frac{T_i + \Delta T_i}{T_i + \Delta T_i + \Delta T_f} - F_p \cdot R \cdot \sqrt{T_i^*} \right] \right].
\]  

(23)

Pressure pulsations in the manifold during such period will be defined by the dependence:

\[
\frac{dP_t}{d\alpha} = \frac{\kappa_i \cdot p_i}{6 \cdot n \cdot V_e} \left[ \mu f(\alpha)_{\text{ref}} \cdot \left[ 2 \cdot R \cdot T \cdot \frac{\kappa_i}{\kappa_i - 1} \cdot \left( 1 - \left( \frac{p_i}{p} \right)^{\frac{k_i-1}{k_i}} \right) \cdot \frac{p}{p_i} \cdot \frac{T_i + \Delta T}{T_i + \Delta T + \Delta T_f} - F_p \cdot R \cdot \sqrt{T_i^*} \right] \right].
\]  

(24)

Results of calculations

For calculation of the pressure curve in the outlet manifold from the dependences (24) and (26) it is necessary to know the variations of pressure \( p \) and of the temperature \( T \) in the engine cylinder during outflow of exhaust gases. Calculations of these parameters for the quasi-stationary flow of gases can be carried out on the basis of the charge exchange model based on known equations of energy balance, mass balance for an open system and equations of state [Piątek 1986, Sobieszczański 2000, Tasaka and Matsuoka 1980, Zinner 1985] with taking into account the earlier-presented assumptions.

Equation of the energy balance in the cylinder for the period of charge exchange, taking into account the heat exchanged through the cylinder walls \( dQ_w \), the energy delivered together with a fresh charge \( dl_{\text{fresh}} \), the energy of charge carried away from the cylinder \( dl_{\text{wyl}} \) and the internal energy of charge in the cylinder \( dU \) is defined by the expression:
\[ dQ_w + dQ_{cyl} + dQ_{g} = dU + p \cdot dV. \] (27)

In the model of the charge exchange process with the total quantity of heat exchanged by the cylinder walls \( dQ_w \), the heat absorbed by the cylinder liner \( dQ_{cyl} \) with a variable area \( F_{cyl} \), the piston and the head \( dQ_{g} \) with an area \( F_{g} \), calculated taking into account the piston shape factor depending on its construction [Maćkowski 1990], are to be considered separately. The average temperature of the head and the piston surfaces \( T_{g} \) and the cylinder liner \( T_{cyl} \) were determined by means of empirical dependences [Cháčian and Siniavski 2001].

The heat transfer coefficient during the period of charge exchange was calculated according to the modified Woschni formula [Woschni 1967].

If we neglect the kinetic energy of the medium flowing through the cylinder, the energy balance equation will have the form of:

\[
\alpha_1 (T - T_{g}) F_{g} / dn + \alpha_2 (T - T_{cyl}) F_{cyl} / dn + c_p \cdot T \cdot dm_{cyl} + c_v \cdot m \cdot dT + c_v \cdot T \cdot dm + p \cdot dV.
\] (28)

The equation of the mass balance in the cylinder \( dm \) taking into account the mass flowing through the inlet valve \( dm_{in} \) and the outlet valve \( dm_{out} \) for the charge exchange period:

\[ dm = dm_{in} - dm_{out}. \] (29)

Equation of state for the working medium in the cylinder:

\[ p \cdot V = m \cdot R \cdot T. \] (30)

This model was supplemented by own proposal of analytical description of chemical composition changes of the charge flowing through the engine valves during the charge exchange caused by mixing of air with exhaust gases during backflows. Assumptions for that model were presented in the paper [Danilecki 2007 b].

Solving of the system of equations describing changes of thermodynamic parameters and the composition of charge in the cylinder as well as in the outlet pass allows for the determination of the pressure and temperature variation curves for exhaust gases supplying the turbine.

Fig. 1. Example of pressure curve calculations: in cylinder – \( p \) (curve 1), in outlet manifold – \( p_{t} \) (curve 2)
By solving the system of equations (24) and (26) and the energy balance equations as well as the mass balance equation for an open system (engine – manifold – turbine) (28), (29), (30) with use of numerical integration one can determine the pressure curve in the cylinder and the pressure of exhaust gases in the outlet manifold of the engine that supply the turbine. Due to the interdependence of these figures it is necessary to apply iterative calculation methods.

Example results of calculations of the pressure in the cylinder and in the outlet pass of the engine were presented in Fig. 1.

SUMMARY

In the simplified analytical models of turbo-charging based on average cycle parameters, taking into account the influence of pulsation of the exhaust gases pressure in the outlet pass on the turbine parameters, for the purpose of assessment one can use the pulsation coefficients available in the literature, worked out on the basis of the condition of probability of flow through the outlet system of similar structure. In the presented methodology of calculations that is based on the gas dynamic analysis of phenomena at the flow of exhaust gases in the outlet system, the possibility of obtaining characteristics of the pressure changes in the outlet manifold $p_f(a)$ and in the cylinder $p_f(a)$ during the whole period of exhaust gases outflow allow for specifying calculations of the turbine power at pulsating flow of exhaust gases and the cylinder scavenging conditions. Quantitative determination of these factors that – particularly during the scavenging period – essentially influence the exhaust gases temperature in the outlet system is of decisive influence on the precision of calculations in the engines with turbo-charging.

REFERENCES


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