ELABORATION THE MATHEMATICAL MODELS TO OBTAIN THE OPTIMUM PERFORMACE OF THE AGRICULTURAL TRACTOR ENGINE

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Summary. The phenomena that take place during the functioning of diesel engines having a classic type fuel injection are complex ones, therefore the corresponding mathematical models can be elaborated mostly based upon the experimental results and data (known as system identification). In order to improve the performances for vehicles (cars and tractors) equipped with this kind of diesel engine, during the last years, researches have been conducted to obtain some specific mathematical algorithms. These researches are based upon the advantages offered by the analytical solution from the viewpoint of experimenting with compression ignition engine.

With the help of classic implementations of higher mathematics together with adequate software, the algorithm is elaborated in the paper, which defines the optimum working curve for the 33,1 kW diesel engines for the Romanian tractors U 445. The elaboration of the optimum working curve for a diesel engine constitutes the most important step in the conception of an on-board information system which helps reduce engine fuel consumption.

Key words: diesel engine; optimum working curve; on-board information system

INTRODUCTION

In order to determine the optimum-working curve for the D115 diesel engine, one can consider it to be the locus of the optimum points. These are obtained from the condition of equality between the \( T = f(n) \) curve slopes for constant power \( (P_c = \text{const.}) \) and those for constant specific fuel consumption \( (c_e = \text{const.}) \), representing the equal slope theorem:

\[
\left( \frac{\partial T}{\partial n} \right)_{P_c=\text{const.}} = \left( \frac{\partial T}{\partial n} \right)_{c_e=\text{const.}}
\]  

The specific fuel consumption is considered to be an integral rational function of torque and engine speed:
\[ c = \sum_{i=0}^{h} \sum_{j=0}^{h} a_{ij} \cdot T^i \cdot n^j \]  

(2)

MATHEMATICAL MODELS FOR THE DETERMINATION OF THE OPTIMUM WORKING CURVE

A number of \( h \) tests were made in order to determine the values for specific fuel consumption \( c_e \) (g/kWh) dependant on different values of engine speed \( n_e \) (rot/min) and the effective engine torque \( T_e \) (Nm). \( T_{ek}, c_{ek} \) and \( n_{ek} \) represent the values of the parameters for the \( k \) measurement.

In order to conceive a proper mathematical algorithm, it is necessary to establish a correspondence among the variations among the specific fuel consumption \( c_{ek} \), engine speed \( n_{ek} \) and torque \( T_{ek} \).

The coefficients \( a_{ij} \), \( i = 0; p, j = 0; s \), are determined so that the values of the \( c(T,n) \) function would be the nearest to the experimental values. Therefore, the function is considered:

\[ f = \sum_{k=1}^{h} \left[ c(T_k,n_k) - c_e \right] \]  

(3)

By replacing the function from (2) with \( c_e(T_{ek},n_{ek}) \), it results:

\[ \sum_{k=1}^{h} \sum_{i=0}^{p} \sum_{j=0}^{s} \left( a_{ij} \cdot T_k^i \cdot n_k^j \cdot T_{ek}^i \cdot n_{ek}^j \right) = \sum_{k=1}^{h} c_e \cdot T_k^i \cdot n_k^j; \quad l = a, p, \quad t = 0, s \]  

(4)

Using Excel program to study the dependencies between the effective engine torque \( T_e \) and the effective engine speed \( n_e \) (5) and between the effective engine fuel consumption \( c_e \) and the effective engine speed \( n_e \) (6), it is possible to elaborate the mathematical algorithm for the specific fuel consumption \( c_{ek} \) as a second degree function dependant on \( T_{ek} \) and \( n_{ek} \) engine speed:

\[ y = -2E-0.5x^2 + 0.0682x + 90.827; \quad R^2 = 0.94; \]  

(5)

\[ y = 1E-0.5x^2 - 0.0265x + 265.27; \quad R^2 = 0.95 \]  

(6)

The system of equations with \( a_{ij} \) unknowns is:

\[ c = a_{00} + a_{10} \cdot T + a_{01} \cdot n + a_{11} \cdot T \cdot n + a_{20} \cdot T^2 + a_{02} \cdot n^2 \]  

(7)

Thus (3) becomes:

\[ f = \sum_{k=1}^{h} \left[ a_{00} + a_{10} \cdot T_k + a_{01} \cdot n_k + a_{11} \cdot T_k \cdot n_k + a_{20} \cdot T_k^2 + a_{02} \cdot n_k^2 - c_e \right] \]  

(8)
ELABORATION THE MATHEMATICAL MODELS...

The stationary condition is:

$$\frac{\partial f}{\partial a_j} = 0; \quad l = 0, \rho; \quad t = 0, s$$

(9)

Using (8) and (9), the system results:

$$\left( \sum_{i=1}^{k} a_{i0} + \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij} \right) \cdot a_{i0} + \left( \sum_{i=1}^{k} n_i \right) \cdot a_{i0} + \left( \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij} \right) \cdot a_{i0} + \left( \sum_{i=1}^{k} n_i \right) \cdot a_{i0} + \left( \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij} \right) \cdot a_{i0} + \left( \sum_{i=1}^{k} n_i \right) \cdot a_{i0} = \sum_{i=1}^{k} c_i$$

(10)

For a 45 HP engine (D115), the determination of the dependence $c_{el}(T_{el}, n_{el})$ as it results from (2) has to be done by solving (10) in order to obtain $a_j$ coefficients.

Therefore, the values are considered as input data for this mathematical model, that have been determined by summing up the resulted data from the full load engine speed characteristic.

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<th>$\Sigma(n_c)$</th>
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The system (10) has been solved using proper computer programs obtaining:


(11)

The values of the coefficients from (11) are used in the relation (7) and thus it results in the final form of the dependencies $c_{el}(T_{el}, n_{el})$: 
\[ c = 436.5382 + 0.0000179n^2 + 0.0059171128T^2 + 10.12663n - 1.978583T + 0.0004016Tn \]  

The obtained dependence \( c(T, n) \) represents the ellipse equation. Based on (1) and (2), the partial derivative of fuel consumption dependant on engine speed becomes:

\[
\left( \frac{\partial c}{\partial n} \right) = \sum_{i=0}^{p} \sum_{j=0}^{q} a_{ij} \left( t^{i-1} \cdot \frac{\partial T}{\partial n} \cdot n^i + T^i \cdot t^{j-1} \right)
\]  

(13)

Because the equal slope theorem is applied for the case of constant fuel consumption, it results:

\[
\left( \frac{\partial c}{\partial n} \right) = 0
\]

(14)

The dependence among power, torque and engine speed is:

\[
P = k \cdot T \cdot n
\]

(15)

where:

\( k \) – connection constant.

By differentiating (13) with respect to the engine speed \( n \) at constant power it results:

\[
\frac{\partial P}{\partial n} = c \left( \frac{\partial T}{\partial n} + T \right) = 0 \quad \Rightarrow \quad \frac{\partial T}{\partial n} = -\frac{T}{n}
\]

(16)

In order to determine the optimum working curve, \( \frac{\partial T}{\partial n} \) from (16) is introduced in (13), for the given second degree function resulting:

\[
\frac{\partial C}{\partial n} = a_{10} \cdot \frac{\partial T}{\partial n} + a_{01} + a_{20} \cdot 2 \cdot T \cdot \frac{\partial T}{\partial n} + 2 \cdot a_{02} \cdot n + a_{11} \cdot \frac{\partial T}{\partial n} \cdot n + a_{11} \cdot T = 0
\]

(17)

Using (13) and regrouping the terms results in the optimum of the engine’s efficient work throughout the optimum working curve, considered to be the following function:

\[
2a_{20} \cdot T^2 + a_{10} \cdot T - 2a_{02} \cdot n^2 - a_{01} \cdot n = 0
\]

(18)

The coefficients obtained from (11) are introduced in (18), thus resulting in the curve that illustrates the optimum working of the D115 engine (torque – engine speed
dependence for a minimum specific consumption for every point of the engine speed characteristic).

\[ 0.0118343 \cdot T^2 + 1.9785853 \cdot T - 0.000035902856 \cdot n^2 + 0.1012663 \cdot n = 0 \quad (19) \]

Using computer programs, one can obtain the graphic illustration of the engine’s optimum working curve.

As it results from above, the optimum working curve is defined as a function of torque for which the fuel consumption is the minimum for each level of constant power.

![Graph](image)

**Fig.1. Optimum working curve for D 115 diesel engine:**

1 – isoconsumption curves; 3 – constant power curves; 3 – optimum working curve

Based on this definition, it is necessary to study the optimum working curve with respect to the different levels of constant power. It is well known that the maximum power (studied on the relative engine speed characteristic) is obtained for the rated operating conditions and is indicated by:

\[ P_{\text{max}} = P_n = \frac{T_n \cdot n_n}{9554} \text{ kW} \quad (20) \]

where:

- \( T_n \) – the rated torque, Nm;
- \( n_n \) – the rated engine speed, rot/min.

Knowing the characteristics of the D115 diesel engine (\( P_n = 33.1 \text{ kW} \) and \( n_n = 2400 \text{ rot/min} \)), the graphic illustration for different levels of constant power can be
made (Fig. 1). Again, using the computer programs, one can study the constant power for a specific level, also obtaining the graphic illustration.

CONCLUSIONS

Beginning with the system identification and using superior mathematical elements as well as specific programs, the mathematical algorithms for each diesel engine can be obtained in order to determine the specific fuel consumption dependant on an effective engine torque and engine speed as well as to elaborate the optimum working curve.

The optimum working curve of the diesel engine represents the function of engine torque dependant on speed engine for which the specific engine fuel consumption is minin for each power level.

The determined mathematical algorithms are useful for the on-board diagnostics and real-time engine management.

The mathematical model of the optimum working curve of the engine represents a fundamental element in the designing of an on-board informational system (in case of vehicles equipped with the type of transmissions that provides gear shifting in full load situation or with hydrostatic transmission).

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